Adaptive Windowing of Insufficient CP for Joint Minimization of ISI and ACI Beyond 5G

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Abstract—Using minimum, even insufficient guards are proposed to achieve the spectral efficiency and latency requirements of cellular communication systems beyond 5G. This leads to interference in both time and frequency domains. In this paper, a partial-non-orthogonal multiple accessing scenario in which the desired user is experiencing both intersymbol interference (ISI) due to insufficient cyclic prefix (CP) and adjacent channel interference (ACI) caused by asynchronous transmitters using non-orthogonal numerologies in adjacent bands is investigated. ISI and ACI depend on the power offset between desired and interfering users, the instantaneous channel impulse responses of interfering users and transmitter and receiver window functions. Therefore, joint and adaptive utilization of CP requires real-time calculation of ISI and ACI. Analytical expressions for expected ISI and ACI at each subcarrier of the desired user are derived to minimize their combination. Accordingly, an adaptive algorithm consisting of windowing each subcarrier at the receiver with window length that minimizes the combined interference at that subcarrier by optimally exchanging ISI and ACI is proposed. Interference reduction performances of current, outdated and average optimal window length raised cosine receiver windows are assessed and compared to fixed and no receiver windowing. Windowing reduces interference even when CP is shorter than the channel if window length is determined using the proposed design guidelines.

I. INTRODUCTION

Conventional orthogonal frequency division multiplexing (OFDM) receivers are designed assuming the cyclic prefix (CP) is longer than the maximum excess delay (MED) of the desired users channel to not experience intersymbol interference (ISI). Users in adjacent bands are assumed to cause negligible adjacent channel interference (ACI). This is achieved by avoiding channels with MEDs longer than CPs by elongating the CP durations, such as the extended-CP option in Long Term Evolution (LTE). Possible ACI due to interferers in adjacent bands are either mitigated using interference cancellation [1], avoided by increasing guard bandwidth until ACI power becomes negligible [2], or suppressed [3].

There are numerous approaches to suppress ACI, with the most prominent one being windowing due to its low computational complexity and efficacy. Windowing can be applied at the transmitter to reduce out-of-band (OOB) emission and corresponding ACI before it eventuates [4], or at the receiver [5] to reject present ACI. However, both references utilize the same window function at all subcarriers, while it is known that edge subcarriers are critical in OOB emissions and are more prone to present ACI. Motivated by this property, [6] introduces subcarrier specific window (SSW) concept at the transmitter side whereas [7] introduces optimal SSW function design for both transmitter and receiver. Addressing conventional systems, [6] assumes CP is longer than the MED of the channel to accommodate windowing and limit the window length to the guard interval that is not disturbed by multipath reception while [7] even allocates additional samples for windowing, reducing spectral efficiency.

Cellular communication standards beyond 5G are envisioned to provide diverse services with various requirements simultaneously to a myriad of devices. Increasing spectral efficiency is crucial to support the projected number of devices, especially, in lower carrier frequencies, favoring reduced guards [8]. Using CP durations shorter than users’ MEDs are proposed [9] to satisfy the lower latency required by new services in systems beyond 5G while increasing spectral efficiency. Thus, the augmented guards promoted by [7] aside, even the more than sufficient CP required by [6] becomes a luxury in current trend. These conventional approaches do not address the requirements of communication systems beyond 5G and therefore need to be extended. Asynchronous non-orthogonal waveforms with different parameterizations, referred to as numerologies, are also proposed to be used in adjacent bands to provide diverse services in future standards [8]. Although such scenario is mentioned in [7], how the ACI caused by such non-orthogonal numerologies can be determined is not provided in any of aforementioned works.

In this paper, we utilize insufficient CP optimally to jointly minimize ISI and ACI, addressing spectral efficiency requirements of systems beyond 5G and the corresponding real-time conditions adaptively. To the best of authors’ knowledge, this is the first work propos-
ing windowing in a system with insufficient CP. We first determine incident ISI caused by insufficient CP, ACI caused by different numerologies in adjacent bands, and the combined interference power for each subcarrier as it is the optimization metric to be minimized. These analyses lay out the framework for optimal SSW functions at the receiver, but we limit the discussion to raised cosine receiver window lengths. We also analyze the interference reduction performances of resulting optimal SSW and fixed length windowing compared to no receiver windowing; with window lengths determined for current and outdated channel impulse responses (CIRs) and power delay profiles (PDPs) to demonstrate the possible gains and robustness of the design example.

II. SYSTEM MODEL

A 1-indexed algebra is used where \( \mathbf{I}_N \) is the \( N \times N \) identity matrix, \( \mathbf{O}_{N \times M} \) is the \( N \times M \) zero and \( \mathbf{I}_{N \times M} \) is the \( N \times M \) ones matrix. Conjugate, transpose and Hermitian operations are denoted by \((\cdot)^*, (\cdot)^T\) and \((\cdot)^H\), respectively. \( \mathbf{A} \odot \mathbf{B} \) is the Hadamard product of matrices \( \mathbf{A} \) and \( \mathbf{B} \) and \( \mathbf{A} \otimes \mathbf{B} \) denotes the Hadamard division of \( \mathbf{A} \) to \( \mathbf{B} \). \( \mathcal{X}^{\circ \otimes} \) is the Hadamard product of matrix \( \mathbf{X} \) with itself. \( \mathcal{E}_\mu \{ \cdot \} \) is the expectation operator over variable \( a \). \( \text{diag}(c_1, c_2, \ldots, c_N) \) represents the \( N \times N \) diagonal matrix with diagonal elements \( c_1, c_2, \ldots, c_N \). \( \text{toep} \) \( (A, B) \) denotes the Toeplitz matrix of which first column is \( A \) and first row is \( B \). \( \delta(\cdot) \) is the Dirac delta function, \( \mathcal{N}(\mu; \sigma^2) \) is the normal distribution with mean \( \mu \) and variance \( \sigma^2 \), and \( \text{flipr} (\cdot) \) is the function that flips a matrix from left to right, i.e., \( \mathbf{X}_{M \times N} = \text{flipr} (\mathbf{X}_{M \times N-1}) \). All properties existing with subscripts \( _u \) denote that the given matrix or vector is associated with the \( u \)th user.

Let \( s_u \in \mathbb{C}^{M_u \times l_u} \) denote the modulated data symbols, where \( M_u \) is number of \( u \)th user’s data subcarriers and \( l_u \) is the number of \( u \)th user’s OFDM symbols in a frame. \( \mathbf{Q}_u \in \mathbb{R}^{N_u \times M_u} \) is \( u \)th user’s subcarrier mapping matrix. \( \mathbf{A}_u \in \mathbb{R}^{N_u \times K_u \times N_u} \) is \( u \)th user’s CP insertion matrix , consisting of

\[
\mathbf{A}_u = \begin{bmatrix}
\mathbf{0}_{K_u \times (N_u-K_u)} & \mathbf{I}_{K_u} \\
\mathbf{I}_{N_u} & \mathbf{0}_{N_u \times K_u}
\end{bmatrix}
\]

in case of no transmitter windowing where \( K_u \) is the number of CP samples. The CP removal and windowing matrix \( \mathbf{B}_{u}^{\text{w},d} \in \mathbb{R}^{N_u \times K_u \times N_u} \) is shown in (2), where \( \mathbf{L}_{n,i,u}^{\text{w},d} \in \mathbb{C}^{1 \times l_u} \) is the taper length of either side of the window in number of samples, used for the reception of the \( n \)th subcarrier of \( \text{ith} \) OFDM symbol of \( u \)th user and \( \mathbb{W}_{n,i,u}^{\text{w},d} = \mathbb{R}^{1 \times l_u} \), the receiver window coefficients, are calculated using \( \mathbb{W} \left( k; \mathbf{L}_{n,i,u}^{\text{w},d} \right) = 0.5 \left( 1 + \cos \left( \frac{\pi k}{\mathbf{L}_{n,i,u}^{\text{w},d}} \right) \right) \), \( k = 1, 2, \ldots, \mathbf{L}_{n,i,u}^{\text{w},d} \), which generates raised cosine window coefficients using taper length instead of roll-off. Note that for \( \mathbf{L}_{n,i,u}^{\text{w},d} = 0 \), (2) simplifies to \( \mathbf{B}_0 = \begin{bmatrix}
\mathbf{0}_{N_u \times K_u} & \mathbf{I}_{N_u}
\end{bmatrix} \), which is the CP removal matrix without windowing.

\( \hat{h}_{i,u} \in \mathbb{C}^{1 \times l_u} \) denotes the CIR invariant during reception of the corresponding OFDM symbol where \( L_u \) is the MED \( u \)th user experiences in number of samples, which is obtained by \( \hat{h}_{i,u} (k) = \sqrt{P_u} \frac{\alpha_u}{\alpha_u + \alpha_d} \alpha_d \nu^k \) where \( P_u \) is the received power of \( u \)th user’s signal, \( \alpha_u \) is the exponential decay rate of \( u \)th user’s channel and \( \nu (k) \in \mathbb{C}^{1 \times l_u} \sim \mathcal{CN}(0,1) \forall k \in \{0,1,\ldots,l_u-1\} \) [10]. Then, \( \mathbf{h}_{u}^{\text{conv}} \in \mathbb{C}^{N_u \times K_u \times N_u \times K_u} \) is the linear channel convolution matrix bounded to one symbol duration, where \( \mathbf{h}_{u}^{\text{conv}} = \text{toep} \left( \left[ \hat{h}_{i,u}^{-} 0_{1 \times N_u+K_u-l_u} \right]^T, \left[ \hat{h}_{i,u}^{-} (0) 0_{1 \times N_u+K_u-l_u-1} \right] \right) \).

\( \hat{H}_{i,u} \in \mathbb{C}^{N_u \times 1} \) is the channel frequency response (CFR) of \( u \)th user’s \( i \) th OFDM symbol, which can be calculated as \( \hat{H}_{i,u} = \sqrt{N_u} \mathbf{F}_u \hat{h}_{i,u}^{-} 0_{1 \times N_u-l_u} \). Let us define the ISI free condition as

\[
K_u - L_{n,i,u}^{w} \geq L_u, \forall n \in \{1, 2, \ldots, N_u\}
\]

Assume desired OFDM symbol is the \( d \)th OFDM symbol of \( 0 \)th user. Let us first assume the absence of the interfering users and (3) is satisfied. In this case the product \( \mathbf{B}_{u}^{\text{w},d} \mathbf{h}_{u}^{\text{conv}} \mathbf{B}_{0}^{H} \) results in the perfect circular channel convolution matrix \( \mathbf{h}_{u}^{\text{pre}} \in \mathbb{C}^{N_u \times N_u} \) shown in (4). Furthermore, \( \mathbf{F}_u \mathbf{B}_{i,w}^{\text{w},d} \mathbf{h}_{u}^{\text{conv}} \mathbf{F}_0^{H} \) results in \( \text{diag} \left( \hat{H}_{d,0} \right) \), where \( \mathbf{F}_u \in \mathbb{C}^{N_u \times N_u} \) denotes the normalized fast Fourier transformation (FFT) matrix \( u \)th user uses in the generation and reception of OFDM symbols. Hence, ignoring the noise, the received symbols \( y_{d,0} \in \mathbb{C}^{N_u \times 1} \), where \( y_{d,0} = \mathbf{F}_0 \mathbf{b}_{d,0} \mathbf{h}_{u}^{\text{conv}} \mathbf{A}_u \mathbf{F}_0^{H} \mathbf{Q}_u \mathbf{s}_{d,0} = \text{diag} \left( \hat{H}_{d,0} \right) \mathbf{s}_{d,0} = \hat{H}_{d,0} \mathbf{s}_{d,0} \), \( y_{d,0} \) is equalized using zero forcing (ZF) equalization [11] via a similar Hadamard division by CFR to obtain the symbol estimates \( \hat{s}_{d,0} \in \mathbb{C}^{N_u \times 1} \):

\[
\hat{s}_{d,0} = \mathbb{Q}_u^{H} y_{d,0} \otimes \hat{H}_{d,0}
\]

where \( \hat{H}_{d,0} \) is the desired OFDM symbol’s CFR estimated at the receiver.

In this work, although there would be residual ISI as (3) is invalid, equalization will still be performed as in (5) and no interference cancellation technique other than receiver windowing is applied to reduce the ACI and residual ISI. In the scenario of interest, the received signal consists of the distorted desired signal and interference from other signals, including ISI from the previous symbol, and ACI from signals in adjacent bands. The aim of this study is to minimize the aggregation of the distortion of desired signal and interference.

The distortion of the desired signal can be calculated by calculating the difference between the signal that
where the transmit samples are left multiplied by the circular convolution matrices when CP is added, and then signal is calculated by projecting samples of each received OFDM symbol to the corresponding samples of the desired OFDM symbol to find the desired OFDM symbol is found as $x_{n,d}^\text{dist} = F_0 h_{d,n} \circ \text{vec} n_{d,n}$. The ISI and ACI from all other signals are calculated by projecting samples of each received OFDM symbol to the corresponding samples of the desired OFDM symbol in this asynchronous scenario. Each received OFDM symbol affects a total of $N_u + K_u + (L_u - 1)$ time samples. The channel output, including the CIR filter tail, is calculated by left multiplying the transmit samples

$$
\mathbf{h}_{t,u}^\text{full} = \mathbb{E}[\mathbf{h}_{t,u}^\text{full} | \mathbf{h}_{t,u}] = \mathbb{E}[\mathbf{h}_{t,u}^\text{dist} | \mathbf{h}_{t,u}] = \mathbb{E}[\mathbf{h}_{t,u}^\text{dist} | \mathbf{h}_{t,u}] = \mathbb{E}[\mathbf{h}_{t,u}^\text{dist} | \mathbf{h}_{t,u}]
$$

where $\mathbf{R} \in \mathbb{C}^{\frac{K_u}{\Delta_f}} (N_u + K_u + (L_u - 1)) \times N_u + K_u + (L_u - 1)$ is an resampling transform. Let $\tau_{i,u}^\text{dist} \in \mathbb{R}^{\frac{K_u}{\Delta_f}} (N_u + K_u + (L_u - 1)) \times 1$ denote the time indices of the received samples that contains energy from the samples of the $i$th OFDM symbol of $u$th user. Then, a projection matrix $\mathbf{I}_{i,u,d} \in \mathbb{R}^{N_u + K_u + (L_u - 1)}$ is formed such that the misaligned, asynchronous samples are projected onto the received symbol:

$$
\mathbf{I}_{i,u,d} (g, j) = \begin{cases} 1, & \tau_{i,u}^\text{dist} (g) = \tau_{i,u}^\text{dist} (j) \\ 0, & \text{o.w.} \end{cases}
$$

Thus, the aggregate interference on the $n$th subcarrier of the desired symbol is found as:

$$
y_{n,d} = y_{n,d}^\text{dist} + \sum_u \sum_i F_0 B_i^w \mathbf{I}_{i,u,d} \mathbf{h}_{t,u}^\text{full} A_u F_u^H Q_u x_{n,d}^\text{dist}
$$

Using this formulation, the instantaneous interference power is calculated easily if all parameters are known.

However, practically, information symbols of all users are unknown at the time of reception, and an estimate of the expected interference power is needed. To calculate this value, the following statistical conjecture is used:

**Conjecture 1.** The symbols transmitted using any subcarrier of any OFDM symbol of any user are independent from each other and the used modulation is uniform average power, i.e., $\mathbb{E}[s_{i,u}^d s_{i',u'}^d] = \delta (n - n') \delta (i - i') \delta (u - u')$.

Conjecture 1 implies that, for practical number of subcarriers, the variance of their sum is the sum of their variances by the law of large numbers [12]. Each column of $F^H$ contains the phase rotation of a normal random variable and the sum of variances of all columns yields the total interference power contributed to the symbol. Thus, the expected aggregate interference to the $n$th received subcarrier of the desired user is given in the $n$th column of

$$
\mathbb{E}_s \left\{ \mathbf{p}_{n,d} \right\} = 1_{1 \times N} F_0 h_{d,n}^\text{dist} F_0^H T + \sum_u \sum_{\{i,u\} \neq \{d,u\}} 1_{1 \times N} F_0 B_i^w \mathbf{I}_{i,u,d} \mathbf{h}_{t,u}^\text{full} A_u F_u^H Q_u X^u T
$$

where the $n$th column of $1_{1 \times N} X^u T$ contains the sum of all elements in the $n$th row of $X$.

### III. PROPOSED METHOD

The receiver is to solve either of

$$
L_{n,i,u}^\text{Sw} = \arg \min_{L_{w,n,i,u}} \mathbb{E}_t \left\{ \mathbb{E}_s \left\{ \mathbf{p}_{n,d,u} \right\} \right\}
$$

$$
L_{n,i,u}^\text{Sw} = \arg \min_{L_{w,n,i,u}} \mathbb{E}_t \left\{ \mathbb{E}_s \left\{ \mathbb{E}_r \left\{ \mathbf{p}_{n,d,u} \right\} \right\} \right\}
$$

$$
L_{n,i,u}^\text{fix} = \arg \min_{L_{w,n,i,u}} \mathbb{E}_t \left\{ \mathbb{E}_s \left\{ \mathbb{E}_r \left\{ \mathbf{p}_{n,d,u} \right\} \right\} \right\}
$$

$$
L_{n,i,u}^\text{avg} = \arg \min_{L_{w,n,i,u}} \mathbb{E}_t \left\{ \mathbb{E}_s \left\{ \mathbb{E}_r \left\{ \mathbf{p}_{n,d,u} \right\} \right\} \right\}
$$

subject to $L_{w,n,i,u} \in \{0, 1, \ldots, K_u\}$

1) optimal SSW lengths for known CIRs
2) average SSW lengths depending on users’ PDPs
3) optimal window length for conventional “fixed” receiver windowing using the same window lengths for all subcarriers for known CIRs
4) average fixed length depending on users’ PDPs where required computational complexity decreases along with performance as we get to the bottom of the options. The solutions to window length calculations are not provided but performance gain will be shown.

Provided the solutions are known, SSW requires additional $\sum_{L^w \in \text{SSW}} \left(4L^w + \frac{2}{3} \log_2 N\right)$ multiplications and $\sum_{L^w \in \text{SSW}} \left(2L^w + N \log_2 N\right)$ additions on top of fixed windowing, due to additional overlapping (first terms) and FFT operations (second terms).

IV. Numerical Verification

A system with the following parameters was simulated to demonstrate the gains of the proposed algorithm: $\alpha$, CIRs and time offset between users are randomized at each run. $H_{i,u} = H_{i,u}Vi,u$ and $E \{ h_{i,u}h^*_{i,u-\Delta i,u} \} = P_{u,0} (\Delta i)$. $P_{-1} = P_1$ always, and are equal to $2P_0$ in the remaining figures except Fig. 4. There is no guard band between any user, first subcarrier of the user with narrower bandwidth is located at the first null of the adjacent user’s edge-most subcarrier. $2 \Delta f_{-1} = \Delta f_0 = \Delta f_1 / 2$, where user indices distinguishes their order in the spectrum. The rest of the variables are given in the sampling rate of user 0. $N_{[-1,0,1]} = \{512, 256, 128\}$, $M_{[-1,0,1]} = \{123, 127, 31\}$, and $K_{[-1,0,1]} = \{36, 18, 9\}$ whereas $L_{[-1,0,1]} = \{64, 32, 16\}$.

The post-equalization expected aggregate interference for unknown signals and the actual interference for known signals for a single realization of the aforementioned setup is shown in Fig. 1. The expected interference calculations are accurate in determining the actual interference, but a slight mismatch occurs due to dependance of ACI on interfering users’ signals.

The ISI power (consisting of both the distortion of the symbol in interest and the leakage from preceding symbol of desired), ACI power and the combined interference power at the subcarriers of the desired signal are shown in Fig. 2. In case of a single realization shown in Fig. 2a, the dependency to the instantaneous channels of interfering users can be observed by the power offset at edge subcarriers although both interferers have the same transmit powers. As the results are averaged over many realizations as shown in Fig. 2b, ISI becomes uniform throughout the subcarriers and ACI becomes stronger at edges and weaker in inner subcarriers.

The results of the grid search for optimal SSW length satisfying (9) are shown in Fig. 3 for the same realization depicted in Fig. 2a, which agrees with channel dependency of optimal SSW lengths. As shown in Fig. 3b, longer window lengths are required at edge subcarriers.

The SIR gains of seven different receivers over many power offsets are calculated, and the gain over no windowing is presented in Fig. 4. SSW guarantees higher
We have determined expected and instantaneous interference powers. Interference power is used to determine subcarrier specific window lengths minimizing the interference. We laid down numerous guidelines with various computational complexities to determine optimal window lengths under insufficient CP. The proposed subcarrier specific windowing scheme improves SIR even when CP is insufficient. Average optimal window lengths depend only on PDPs, and although instantaneous optimal window lengths depend on users’ CIRs, fluctuation is little. Therefore, subcarrier specific windowing outperforms fixed windowing even with outdated window lengths in case of powerful interferers.

V. CONCLUSIONS

REFERENCES