Writing a Results section

Explain your measurements and how they address the question(s) you were trying to answer / problem(s) you were trying to solve

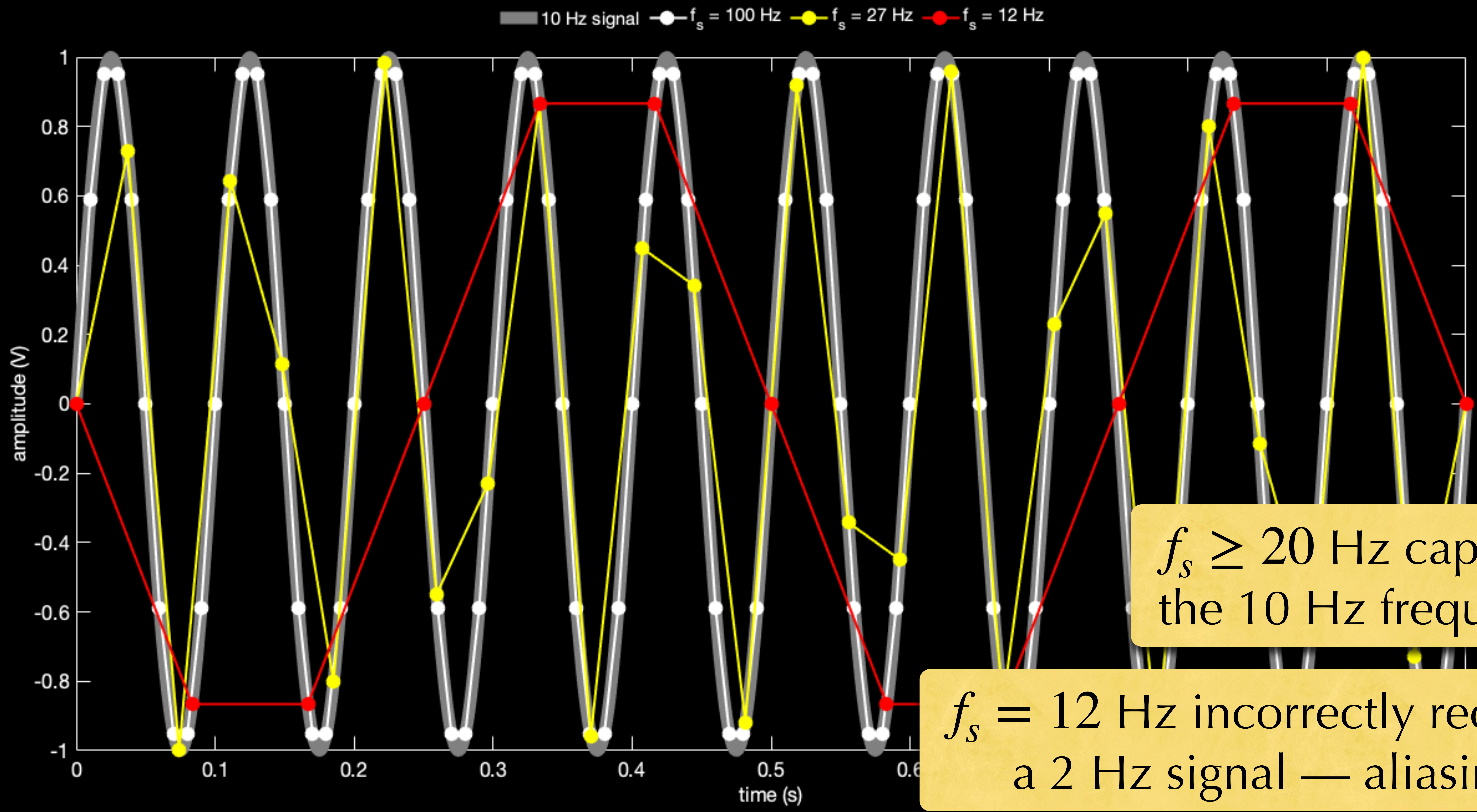
- Write a few sentences describing the implications of each plot
- Don't repeat what's in the captions; explain at a higher level
- Prevent sufficient evidence to convince other engineers that you answered the question / solved the problem — or explain why it wasn't possible

Nyquist sampling theorem

If a signal is sampled at frequency f_s and contains no frequencies exceeding $f_s/2$, then the signal can be exactly reconstructed from the samples.

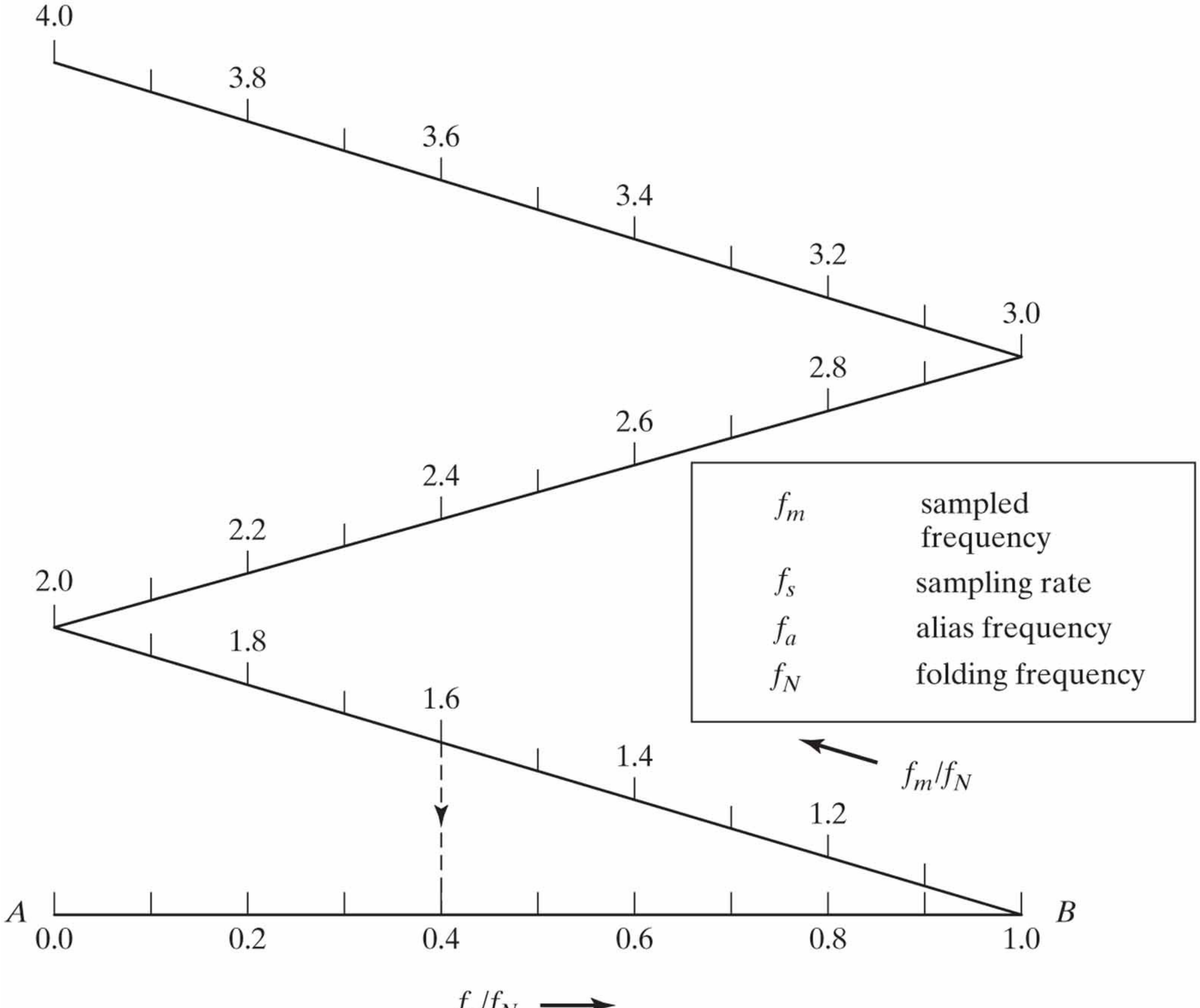
- … assuming there's no quantization error.
- $f_s/2$ is the "Nyquist frequency", aka f_N
- If the signal does contain frequencies exceeding $f_s/2$, sampling causes them to appear incorrectly as frequencies below $f_s/2$; this is called "aliasing".

Aliasing when sampling rate fails the Nyquist criterion



$f_s \geq 20$ Hz captures the 10 Hz frequency

$f_s = 12$ Hz incorrectly records a 2 Hz signal — aliasing


4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
A
B
0.0
0.2
0.4
0.6
0.8
1.0
f_a/f_N
f_m/f_N
f_m sampled frequency
f_s sampling rate
f_a alias frequency
f_N folding frequency

$$f_N = f_s/2$$

How to avoid aliasing?

- Choose $f_s \geq 2f_{m0}$. No aliasing, but noise remains and digital file (or signal) is unnecessarily large.
- Better: low-pass filter to eliminate noise before sampling.
    - Cutoff frequency? Sampling frequency?

How to avoid aliasing?

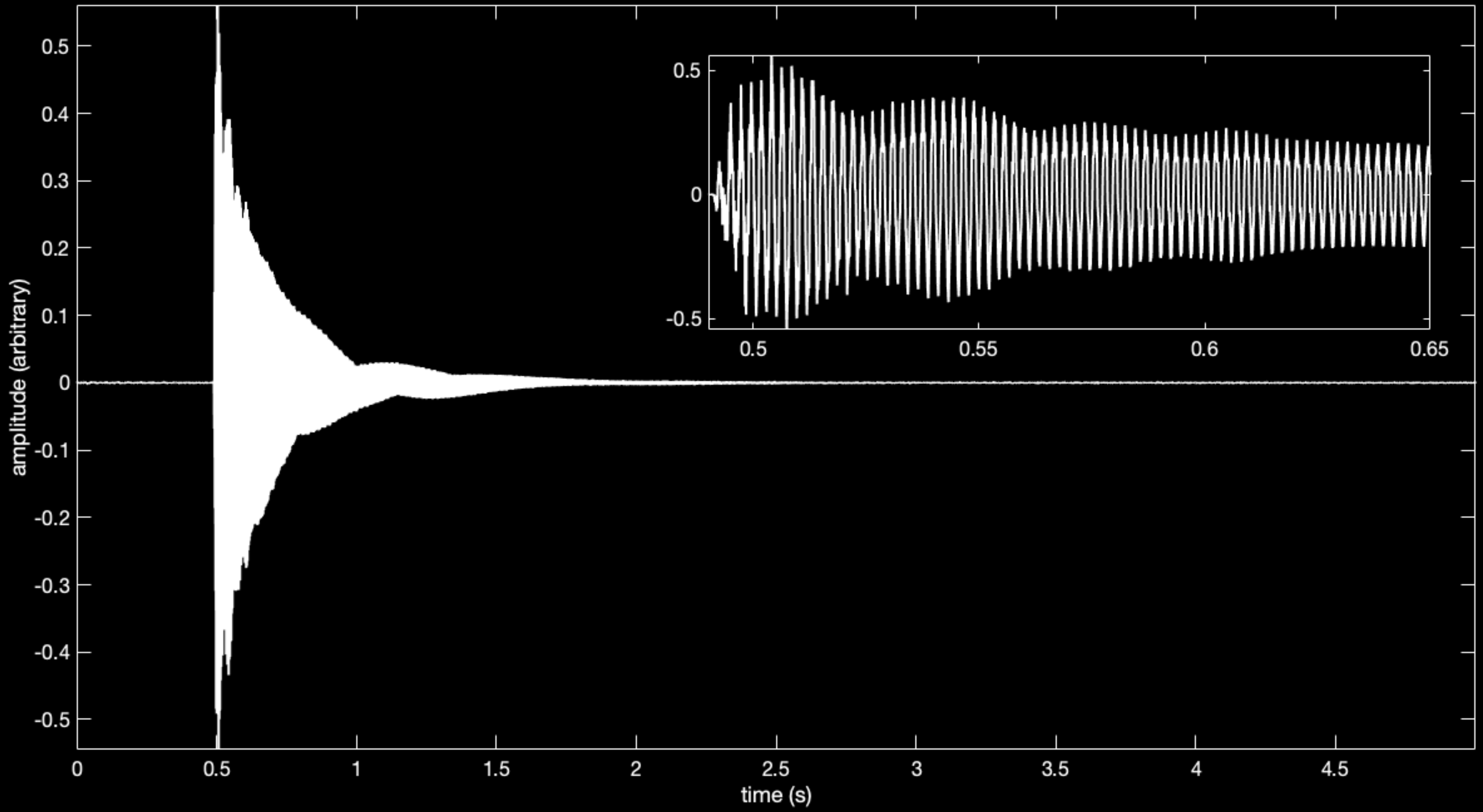
- Choose filter cutoff at f_c and $f_s \geq 2f_{mf}$, where $f_{mf} = f_c 2^{N_{\text{oct}}}$ is the frequency at which signal power drops below the least significant bit of the analog-to-digital converter,

  where $N_{\text{oct}} = \dfrac{20 \log_{10} 2^N}{\text{filter attenuation rate (dB/octave)}}$ and N is the bit depth.

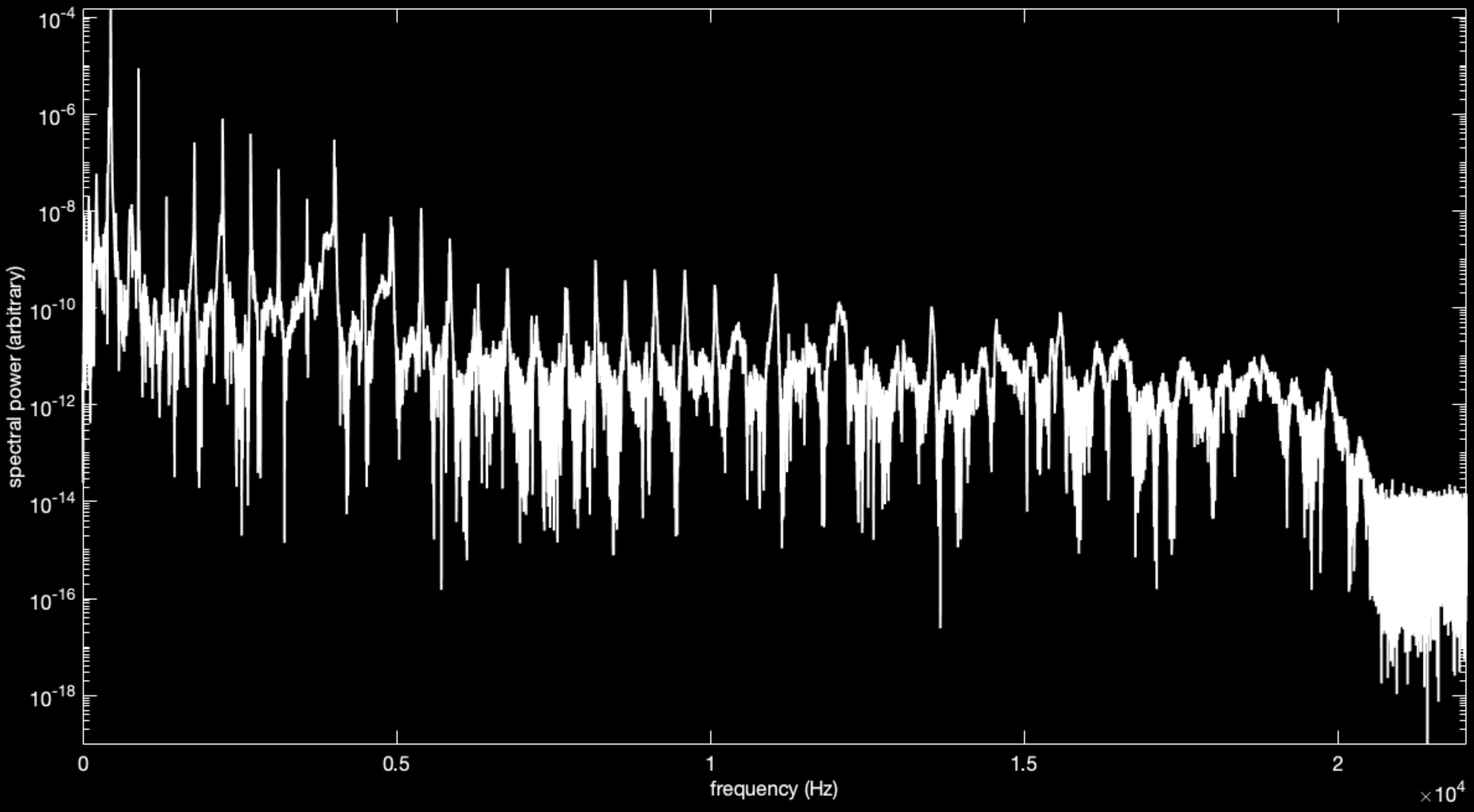
18 kHz tone

440 Hz tone

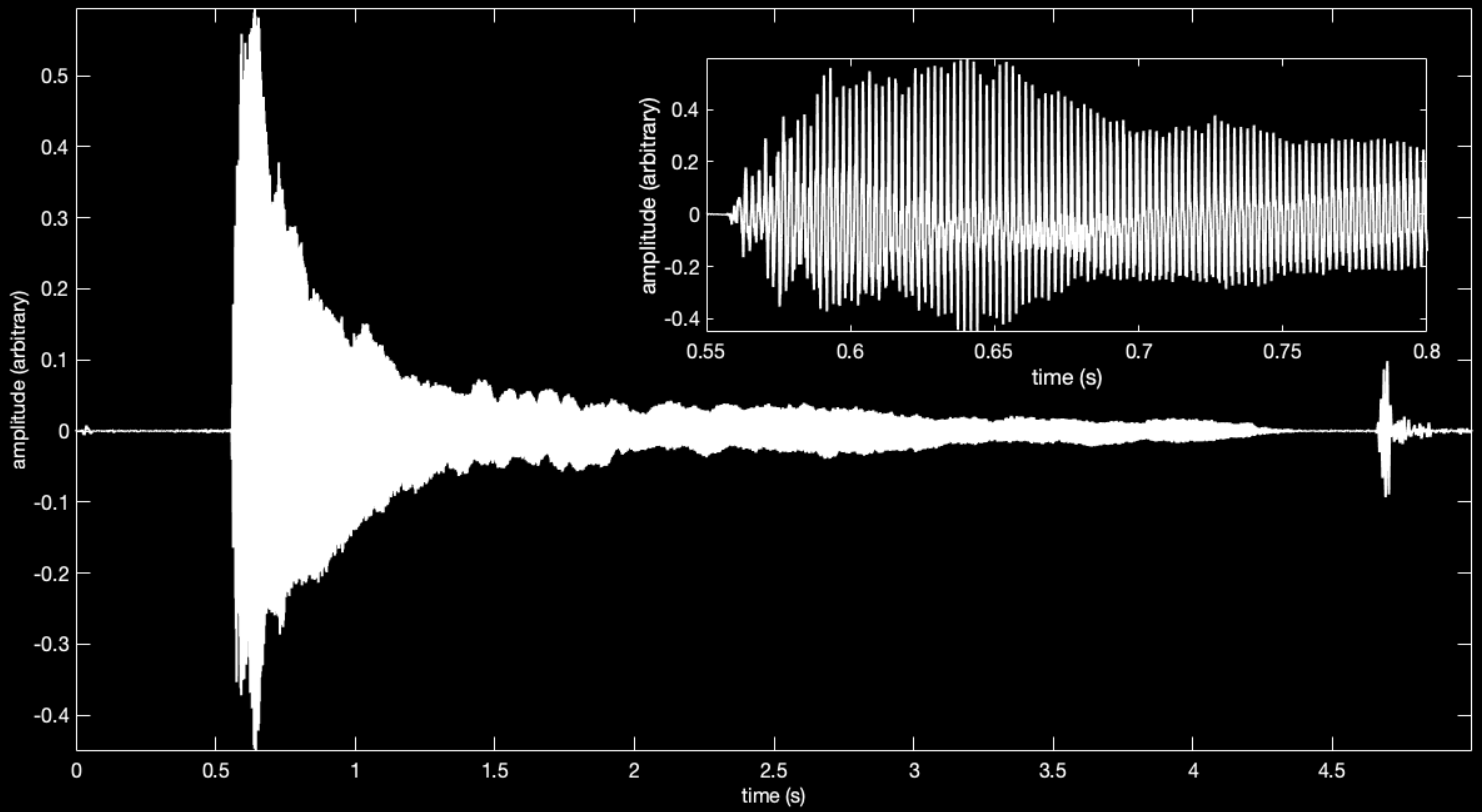
“A” note on ukelele



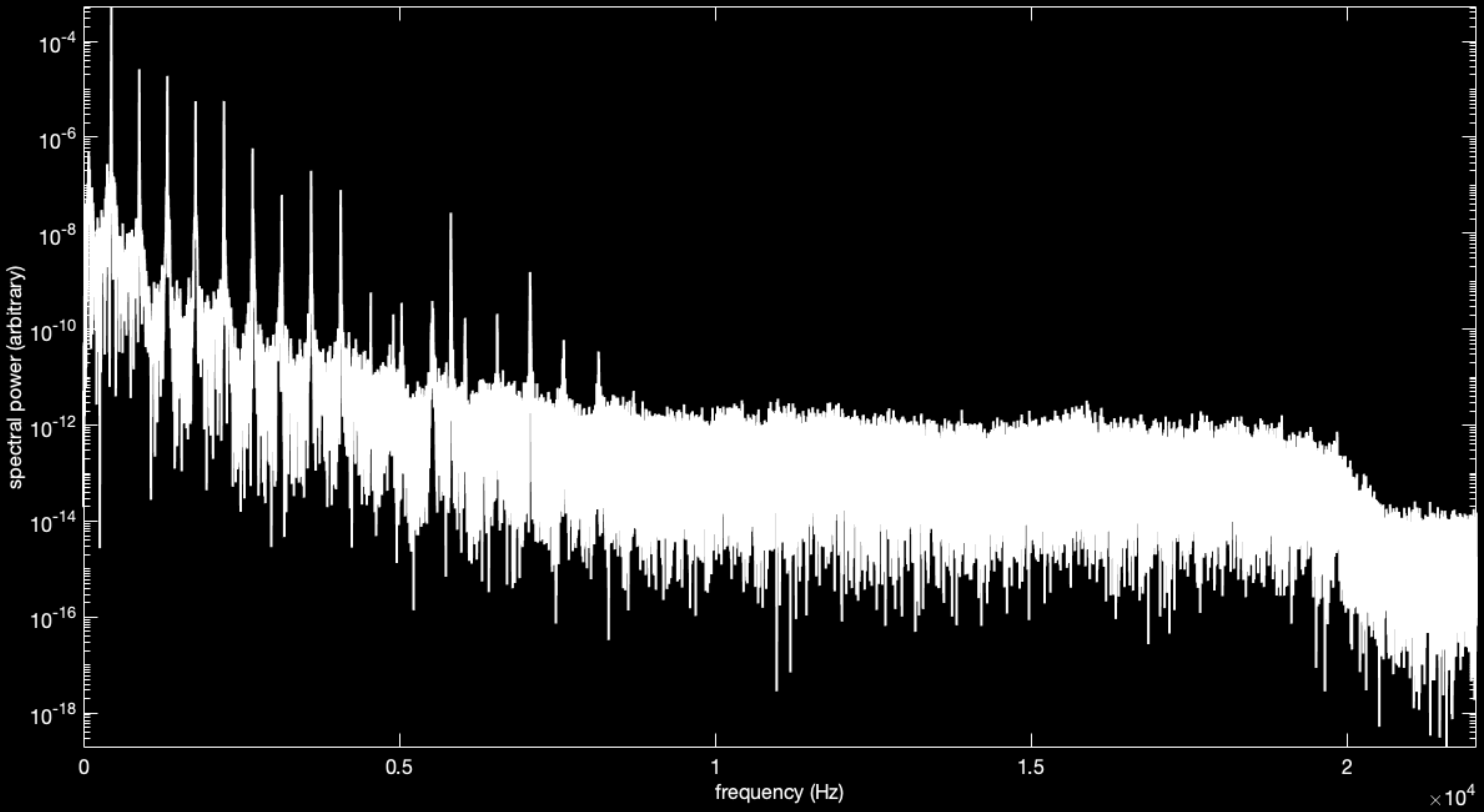
“A” note on ukelele



“A” note on piano



"A" note on piano



Ex. 5.2: Determine the amplitudes of the first, second, and third harmonic components of the signal plotted below.