

# Entanglement-Seeded-Dual Optical Parametric Amplification: Applications to Quantum Communication, Imaging, and Metrology

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The study of optical parametric amplifiers (OPAs) has been successful in describing and creating nonclassical light for use in fields such as quantum metrology and quantum lithography [Agarwal, *et al.*, *J. Opt. Soc. Am. B*, **24**, 2 (2007)]. In this paper we present the theory of an OPA scheme utilizing an entangled state input. The scheme involves two identical OPAs seeded with the maximally path-entangled  $|N00N\rangle$  state  $(|2, 0\rangle + |0, 2\rangle)/\sqrt{2}$ . The stimulated amplification results in output state probability amplitudes that have a dependence on the number of photons in each mode, which differs greatly from two-mode squeezed vacuum. The output contains a family of entangled states directly applicable to quantum key distribution. Specific output states allow for the heralded creation of  $N = 4$   $N00N$  states, which may be used for quantum lithography, to write sub-Rayleigh fringe patterns, and for quantum interferometry, to achieve Heisenberg-limited phase measurement sensitivity.

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## I. INTRODUCTION

Nonclassical states of light have been studied in depth both experimentally and theoretically since the emergence of quantum electronics. Squeezed light, in particular, has been applied to a variety of systems, including interferometry, lithography, and cryptography which show improvement beyond limitations imposed by classical optics [1–4]. One such device that creates a type of squeezed light is an optical parametric amplifier (OPA). OPAs are typically non-centrosymmetric crystals that exhibit a nonzero  $\chi^{(2)}$  optical nonlinearity [5]. Pump, signal, and idler modes propagate through the crystal, and photons from the pump beam are down converted into lower energy photons in the signal and idler modes. Previous work focused on the case that the signal and idler modes couple to the vacuum at the input. This produces the two-mode squeezed vacuum state, which exhibits a highly nonclassical behavior [2, 3, 6]. In the present paper we analyze a scheme in which two identical OPAs are seeded by entangled photon pairs. The scheme produces a heralded source for a large family of entangled states, of interest for applications in quantum information, metrology, and imaging. These states are generated by conditioning the output on photodetection on two of the four total output modes.

A particularly useful heralded state that our scheme generates is the so-called ‘ $N00N$ ’ state with  $N = 4$ .

A  $N00N$  state is a maximally path entangled state such that, in a Fock-state basis,  $|N00N\rangle \propto |N\rangle_A |0\rangle_B + e^{iN\varphi} |0\rangle_A |N\rangle_B$ , where  $\varphi$  is the relative phase difference between the two spatial modes A and B [4]. These states allow for super-resolution by producing lithographic features with a minimum size of  $\lambda/(2N)$ , when incident on an  $N$ -photon absorbing substrate, thus allowing an  $N$ -fold enhancement over standard lithographic methods [1, 4, 7].  $N00N$  states have also been shown to exhibit super-sensitivity in interferometric applications, thus reaching the Heisenberg Limit of  $\Delta\phi = 1/N$  [1, 8–10]. Classically, in an interferometer using coherent light, precision in phase-uncertainty measurement is limited by the shot-noise limit of  $\Delta\phi = 1/\sqrt{\bar{n}}$ , where  $\bar{n}$  is the average photon number. Experimentally, up to  $N = 4$   $N00N$  states have been reported and shown to exhibit both super-sensitivity and super-resolution [11, 12]. However, implementing  $N00N$ -state generators that produce states of photon number greater than two, which simultaneously achieve high fidelities and high flux, is very challenging experimentally. Recently we proposed a scheme that scales well with  $N$  and works for an input of any superposition of  $|N, N\rangle$  photons coupled with feed-forward [13]. Our new scheme, presented here, produces heralded  $N = 4$   $N00N$  states with relatively high probability, and is experimentally accessible. Additionally, a wide variety of other useful entangled states are produced in our new scheme, which can be applied to quantum metrology and cryptography.

In section II we will review the process of optical parametric amplification and squeezing. In section III we describe the novel entanglement-seeded-dual optical parametric amplification scheme. Finally, in section IV we an-

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alyze the properties of the output state, including probabilities and applications.

## II. OPTICAL PARAMETRIC AMPLIFICATION

To obtain the input state for our scheme, some squeezing formalism will be reviewed. We will work in the Heisenberg picture and use a Fock (number) state basis throughout the paper. Modes are represented with capital letters, such as mode A, mode B, and so on. The creation and annihilation operators for the respective modes are  $\hat{a}^\dagger$ ,  $\hat{a}$ ,  $\hat{b}^\dagger$ , and  $\hat{b}$ . The mode labels are dropped from the kets, but proceed in alphabetical order such that  $|N\rangle_A|M\rangle_B \equiv |N, M\rangle$ .

The unitary operator describing the action of an OPA is the two-mode squeezing operator [14],

$$\hat{S}(\xi) = e^{-\xi \hat{a}^\dagger \hat{b}^\dagger + \xi^* \hat{a} \hat{b}}, \quad (1)$$

where  $\xi = r e^{i\varphi}$  is the complex squeezing parameter. Here a strong, undepleted classical pump is also assumed. Here  $r$  is the gain and  $\varphi$  is the phase associated with the OPA. As previously mentioned, the gain  $r$  depends on the pump amplitude and the length and nonlinearity of the crystal. The action of the two-mode squeezing operator on a vacuum input produces the two-mode squeezed vacuum [15],

$$\hat{S}(\xi)|0, 0\rangle = \frac{1}{\cosh r} \sum_{n=0}^{\infty} (-1)^n e^{in\varphi} \tanh^n(r) |n, n\rangle. \quad (2)$$

In the low-gain limit, an OPA acts as a spontaneous parametric down converter (SPDC) [5]. The output is then well approximated as vacuum and a stream of  $|1, 1\rangle$  states. We assume spontaneous parametric downconversion to initially produce the state  $|1, 1\rangle$ , then input this two-mode state on a 50:50 beam splitter that takes modes  $\hat{a}^\dagger \rightarrow (\hat{a}^\dagger + i\hat{b}^\dagger)/\sqrt{2}$  and  $\hat{b}^\dagger \rightarrow (i\hat{a}^\dagger + \hat{b}^\dagger)/\sqrt{2}$ . Due to the Hong-Ou-Mandel effect, two, single, indistinguishable photons that are incident simultaneously on a beam splitter evolve to a superposition in which only one mode is occupied by both photons at the output. We then obtain our desired entangled low-N00N input state  $(|2, 0\rangle + |0, 2\rangle)/\sqrt{2}$  [16].

The action of the unitary operator describing an OPA transforms input modes  $A$  and  $B$  as [2],

$$\hat{S}(\xi)\hat{a}^\dagger\hat{S}^\dagger(\xi) = \hat{a}^\dagger \cosh r + \hat{b}e^{-i\varphi} \sinh r, \quad (3a)$$

$$\hat{S}(\xi)\hat{b}^\dagger\hat{S}^\dagger(\xi) = \hat{b}^\dagger \cosh r + \hat{a}e^{-i\varphi} \sinh r. \quad (3b)$$

In any type of parametric amplification, energy and momentum must be conserved. Momentum conservation provides a phase-matching condition between the pump and the signal and idler modes, which generates path entanglement. Conservation of energy requires the frequencies of the signal and idler to add up to the frequency of

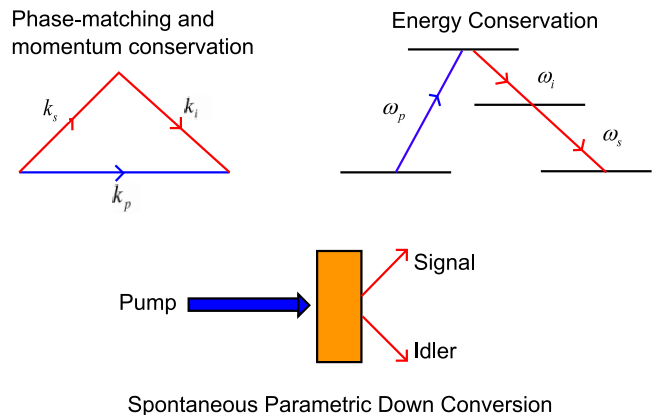


FIG. 1: Schematic of optical parametric amplification and spontaneous parametric downconversion. The bottom diagram shows the crystal being pumped by a laser, along with the signal and idler output modes. The top left diagram depicts momentum conservation between the pump, signal and idler modes, otherwise known as the phase-matching condition. The top right picture diagrammatically shows energy conservation of the system, such that the frequency of the pump is equal to the signal and idler frequencies added together. These restrictions lead to the path entanglement we desire in our scheme.

the pump beam. We will be considering the case of degenerate parametric amplification, such that the signal  $\omega_s$  and idler  $\omega_i$  frequencies are identical and half of the pump frequency  $\omega_p$ ; that is  $\frac{1}{2}\omega_p = \omega_s = \omega_i = \omega$  [5]. Schematically, energy and momentum conservation can be understood from Fig. 1.

Much of the research involving OPAs and two-mode squeezing assumed vacuum input modes [1, 18–20]. This results in the two-mode-squeezed vacuum state previously mentioned. Some theoretical and experimental work has assumed non-vacuum inputs, typically with coherent light input in one mode [3, 21–24]. Indeed, arguably the most useful limit is the low-gain limit of an OPA, which produces (to a good approximation) spontaneous parametric downconversion, that is, the  $|1, 1\rangle$  state. This state, and more generally two-mode squeezed vacuum, have been used to help beat the shot-noise limit in interferometric applications [25]. De Martini's group has recently demonstrated the idea of seeding OPAs with nonclassical light, namely number states. They showed that entanglement was preserved between two of the output modes and one input mode of an OPA. Importantly, the second input mode is detected as a trigger for the experiment [18]. They described this process as quantum injection of an OPA, where one of the input modes was seeded with one photon from a down-converted pair produced by SPDC.

### III. ENTANGLEMENT SEEDED OPTICAL PARAMETRIC AMPLIFICATION

While optical parametric amplification of vacuum input states produces interesting squeezed vacuum states, we consider a scheme in which highly nonclassical states of light are amplified. Rather than seeding an OPA setup with either vacuum modes or number states, we assume an entangled number state input. Our scheme involves two OPAs, for a total of four input modes, which are seeded in two of the modes with the state  $(|2, 0\rangle + |0, 2\rangle)/\sqrt{2}$ . These two modes are then fed into the dual OPA scheme as modes B and C, leaving vacuum input in modes A and D, as seen in Fig. 2. With this notation it is transparent that the inner two modes contain the entangled-state input. Thus, the total input state may be written as

$$|\text{input}\rangle \propto |0, 2, 0, 0\rangle + |0, 0, 2, 0\rangle, \quad (4)$$

where we drop the consecutive mode labels A, B, C, and D. By assuming an entangled input we are naturally led to various questions about the output state. First and foremost, is the output state entangled? Due to amplification, has the degree of entanglement from the input state deteriorated, or has the path entanglement been retained? Also, what are the applications of the output state and with what probabilities does a given state occur?

Armed with the total input state and the squeezing operator transformations, we calculate the output of the scheme. We carry out the calculation by rewriting the input state in terms of the creation operators corresponding to the appropriate modes, which initially contain photons.

The state is then subject to the two OPA transformations  $\hat{S}_1(\xi)$  and  $\hat{S}_2(\xi)$ . It should be clear that both OPAs are assumed to have the same complex squeezing parameter  $\xi$ , which experimentally means they have the same  $\chi^{(2)}$  nonlinearity, are the same length, and cut to have the same phase matching condition (more simply, they are identical). Due to the unitarity of the two-mode squeezing operator we are able to resolve the identity and apply the operators to the input state, thus resulting in the output state:

$$|\text{output}\rangle = \frac{1}{2}(\hat{S}_1\hat{b}^\dagger\hat{S}_1^\dagger\hat{S}_1\hat{b}^\dagger\hat{S}_1^\dagger\hat{S}_1\hat{S}_2|0, 0, 0, 0\rangle + \hat{S}_2\hat{c}^\dagger\hat{S}_2^\dagger\hat{S}_2\hat{c}^\dagger\hat{S}_2^\dagger\hat{S}_2\hat{S}_1|0, 0, 0, 0\rangle). \quad (5)$$

Each of the two-mode squeezing operators transforms only two of the input modes.  $\hat{S}_1$  acts only on modes A and B while  $\hat{S}_2$  acts on modes C and D. Additionally, the two unitary operators commute with one another due to the fact that the different mode operators commute. The total output state is then:

$$|\text{output}\rangle = C(r) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (-e^{i\varphi} \tanh r)^{n+m} \times (\kappa(n)|n, n+2, m, m\rangle + \kappa(m)|n, n, m+2, m\rangle). \quad (6)$$

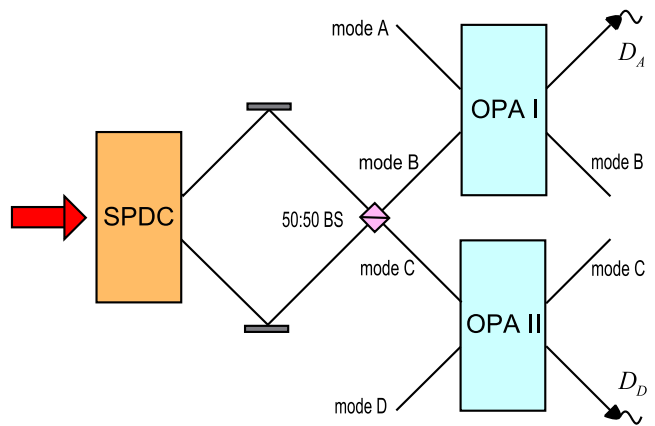


FIG. 2: Diagram of our entanglement-seeded-dual optical amplification scheme. On the far left is the relatively weak pump beam pumping a nonlinear crystal in the low-gain regime in order to produce spontaneous parametric downconversion. This output state, taken initially to be  $|1, 1\rangle$ , is then incident on a beamsplitter, which leads to the maximally spatially entangled state  $(|2, 0\rangle + |0, 2\rangle)/\sqrt{2}$ . This state is then incident into one mode of each of the two OPAs. The other two modes are left as vacuum inputs. We then assume that OPA I and OPA II are pumped by the same high gain laser in order to achieve parametric amplification. Note that the pumps for all three of the nonlinear crystals are to be phase locked. All four modes are amplified, resulting in entanglement between modes B and C. We have drawn photodetectors at modes A and D which are to be used in heralded production of specific states.

Here,  $\varphi$  is the phase associated with the two OPAs,  $n$  is the index resulting from the two-mode squeezing due to  $\hat{S}_1$  and  $m$  the index corresponding to the squeezing induced by  $\hat{S}_2$ . The constant  $C(r)$  depends only on the gain of the OPAs as  $C(r) = \frac{1}{2}(\cosh r)^{-4}$ . Also,  $\kappa(n) = \sqrt{n+1}\sqrt{n+2}$ .

We see immediately that the inner two modes, B and C, are path entangled just as the input state was. Modes A and D will prove to be particularly valuable when detected, thus giving us information about the inner two modes. The output of the scheme is similar to a two two-mode squeezed vacuum state, with the entangled input state being amplified in the inner two modes.

### IV. DISCUSSION

The output state is particularly useful when we consider placing photodetectors  $D_A$  and  $D_D$  at the outputs of the transformed modes A and D. If we assume perfect number-resolving photodetectors which implement projective measurements on modes A and D, we are able to determine with certainty which state the inner two entangled modes are in. This gives us a specific heralded entangled state depending on what photon numbers we measure at  $D_A$  and  $D_D$ . The entangled state after detecting  $n$  photons at detector  $D_A$  and  $m$  photons at de-

tector  $D_D$  will then be

$$|\text{heralded}\rangle \propto |n+2, m\rangle + |n, m+2\rangle \quad (7)$$

in modes B and C. The probability of detecting these  $n$  and  $m$  photons at their respective detectors is given by

$$\text{Prob}(n, m) = C(r)^2 \tanh^{2(n+m)} r \times [(n+1)(n+2) + (m+1)(m+2)] \quad (8)$$

We can see that the parametric amplification results in an output state that is dependent on the number of photons in the four modes. For low values of gain, in which we expect spontaneous parametric downconversion, the vacuum  $n = m = 0$  term dominates, due to the exponential dependence on  $n$  and  $m$  of the hyperbolic tangent. However, for higher values of gain, in which we obtain parametric amplification, the amplified vacuum term is no longer the most probable outcome, as seen in Fig. 3. The maximum shifts towards states with higher photon numbers. Additionally, the photon number difference between the two inner modes is a defining characteristic, which makes our heralded scheme nontrivial.

The immediate consequence of the photon number difference in the two inner modes of the output state applies to quantum cryptography. We imagine detecting  $n$  photons at  $D_A$  and  $m$  photons at  $D_D$ . If we have perfect number resolving detectors, any time we measure  $n = m$  we have the inner mode entangled state  $(|n+2, n\rangle + |n, n+2\rangle)/\sqrt{2}$ . To begin the QKD protocol, photodetector measurements at  $D_A$  and  $D_D$  are announced publicly, while photon number measurements afterwards on modes B and C by two parties (Alice and Bob) will be perfectly correlated. The time-energy entanglement of the two modes results in a violation of the classical separability bound of the joint time and energy uncertainties  $(\Delta E_{B,C})^2 (\Delta t_{B,C})^2 \geq \hbar^2$  [26, 27]. This type of entanglement is exploited to create a one-time pad. A setup analogous to the experiment carried out by Howell's group can then be implemented [26]. In their scheme arrival times of photon pairs created from SPDC, which are highly correlated, are used to create a cryptographic key. A time-bin setup is used in order to ensure that detections at both Alice and Bob's positions are due to the same SPDC pair. This discretization of continuous-variable entanglement has been implemented experimentally [26].

In our scheme, we exploit the number difference between the two modes, as well as the time-energy entanglement, in order to create a key. After the values measured at  $D_A$  and  $D_D$  are publicly announced, one of each of the remaining modes is sent to Alice, and the other to Bob. Each of them then makes a photon number measurement on the mode they have received. The analogy to Howell's experiment is that Alice and Bob must implement a time-bin system in order to ensure that the measurements they are making are on modes produced from the same event. Additionally, they must establish beforehand, via an open channel, that if one of

them measures the mode with the two additional photons, it will correspond to a certain bit. For example, if Alice measures  $n+2$  photons (implying Bob measures  $n$ ), the bit will be a zero. Correspondingly, if Alice measures  $n$  photons and Bob measures  $n+2$ , the bit will be a one. The measurement outcomes of which mode contains  $n$  or  $n+2$  photons are completely random run-to-run. Repeating this process will result in a perfectly correlated string of bits between Alice and Bob, thus establishing a key for use as a one-time pad. Noise-free photon number-resolving detectors with up to 88% efficiency have been experimentally demonstrated at NIST [28]. However, imperfect photodetectors have been shown to provide useful reconstruction of photon-number distributions as well [29].

The security of the system is established in a manner completely analogous to Howell's experiment; namely, Alice and Bob's measurement devices must consist of a Franson interferometer [30]. This detection scheme requires that Alice and Bob each use an unbalanced Michelson interferometer, resulting in interference fringes due to the path mismatch between the two modes they are measuring. It has been shown that the Franson fringe visibility corresponds to a Bell-type inequality, which allows for detection of an eavesdropper if there is a reduction in the fringe visibility [26, 30].

Another straightforward application of the output state is to quantum metrology and quantum lithography. If we obtain a detection of exactly one photon at each detector  $D_a$  and  $D_{\bar{a}}$ , thus telling us that  $n = m = 1$ , we know with certainty the entangled inner modes are in the state  $|\text{inner}\rangle = (|3, 1\rangle + |1, 3\rangle)/\sqrt{2}$ . If this state (in modes B and C) is then incident on a beam splitter, using the transformations [31],

$$\hat{b}^\dagger \rightarrow \frac{\hat{b}^\dagger + e^{i\theta} \hat{c}^\dagger}{\sqrt{2}}, \quad \hat{c}^\dagger \rightarrow \frac{\hat{b}^\dagger - e^{i\theta} \hat{c}^\dagger}{\sqrt{2}}, \quad (9)$$

and for  $\theta = \pi$  we obtain the  $N = 4$   $N00N$  state,  $(|4, 0\rangle + |0, 4\rangle)/\sqrt{2}$ . As discussed earlier, if this state is used to measure a path-length difference in a Mach-Zehnder interferometer, it achieves a doubling in sensitivity compared to the standard shot-noise limit. Regarding use as a source for quantum lithography, proposed by Boto et al, 4-photon  $N00N$  states are predicted to achieve interference patterns of the form  $1 + \cos(4\phi)$ , where the phase  $\phi$  corresponds to translation along the substrate [4]. This corresponds to a four-fold improvement in resolution compared to the classical case, for which the pattern is of the form  $1 + \cos(\phi)$ .

Optimizing the gain  $r$  such that we obtain the highest probability of obtaining a measured output state of  $n = m = 1$  gives a quantitative prediction for how often the desired state for quantum lithography will be heralded. The optimal value is  $r = 0.66$ . However, we are also able to find values of gain such that the  $n = m = 1$  output state is more likely to occur than the vacuum or any other  $n = m$  output. This is due to the output state dependence on the number of photons in the modes,

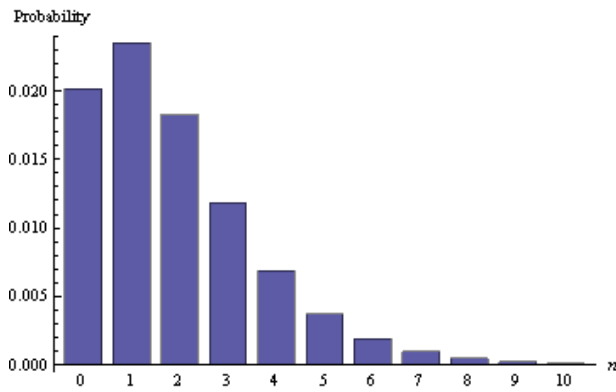


FIG. 3: Probability of obtaining output states with  $n = m$  for a fixed gain of  $r = 1.08$ . A joint detection of equal photon number at modes  $a$  and  $d$  results in the inner two modes being in the state  $|n + 2, n\rangle + |n, n + 2\rangle$ . We see that for this experimentally feasible gain that the vacuum term  $n = 0$  is no longer the most probable outcome, whereas the desirable  $n = 1$  output is.

which is different from standard two-mode squeezed vacuum, as previously mentioned. Fig. 4 shows the probabilities of obtaining a measurement of  $n$  and  $m$  at the two detectors. The diagonal values are where  $n = m$ . The inability to see the  $n = m = 0$  term is due to the entanglement-seeding of the two OPAs. The value of gain in this plot is  $r = 1.08$ . This value is easily obtainable [7, 32]. Comparing the probabilities of obtaining the  $N = 4$   $N00N$  state to that of a typical linear optics based scheme [33], we find that the dual OPA scheme produces the desired state more frequently. In Reference [33] the  $N = 4$   $N00N$  state is probabilistically produced  $3/64$  of the time. Our state produces the same state at approximately 5 times that rate. This is due to the fact that the linear optical scheme relies on an input state of  $|3, 3\rangle$ , whereas our scheme requires that each crystal produces the state  $|1, 1\rangle$ , which is much more likely for OPAs. Also, our scheme is able to minimize vacuum contributions and shift the maximum probability to higher photon number, as mentioned before, and as seen in Fig. 3 and Fig. 4.

## V. CONCLUSION

In conclusion, we propose a scheme that involves seeding two optical parametric amplifiers with an entangled state input. The amplification of this entangled input state results in a four-mode entangled output state, which is useful in a variety of applications. Two of the modes may be measured, thus providing insight into what entangled state the other two modes are in. The heralded

output state is a perfect fit for quantum cryptographic purposes; analogous to polarization entangled quantum key distribution as envisioned by Ekert. Additionally, due to parametric amplification, the output state probabilities depend on the number of photons in the modes.

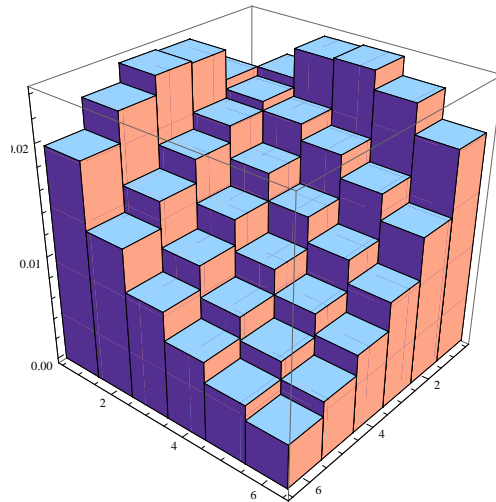


FIG. 4: Probabilities of obtaining  $n$  and  $m$  photon states for fixed  $r = 1.08$ . The most likely joint photodetection at detectors  $D_a$  and  $D_d$  is when each mode contains only one photon. The vacuum  $n = m = 0$  term is the top diagonal term, and is not visible because the  $n = m = 1$  term is more probable. A joint photodetection of  $n = m = 1$  leads to the entangled state  $|3, 1\rangle + |1, 3\rangle$  between modes  $b$  and  $c$ , which when incident on a beam splitter leads to the  $N = 4$   $N00N$  state  $|4, 0\rangle + |0, 4\rangle$ .

For experimentally realistic values of gain, this allows for a high probability of obtaining specific outputs, assuming photon number resolving detectors [28, 29]. Specifically, the scheme produces heralded  $N = 4$   $N00N$  states when a triggered output is incident on a 50:50 beam splitter. This state can then be used for quantum interferometry and lithography. The setup employs three nonlinear crystals, all of which can be identical, save that the seeding crystal needs to be pumped in the low-gain regime. The other optical tools needed are beam splitters and photodetectors.

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