Distinction of Tripartite Greenberger-Horne-Zeilinger and W States Entangled in Time-Energy and Space

Jianming Wen* and Morton H. Rubin
Physics Department, University of Maryland, Baltimore County, Baltimore, Maryland 21250, USA
(Dated: January 28, 2009)

In tripartite discrete systems, two classes of genuine tripartite entanglement have been discovered, namely, the Greenberger-Horne-Zeilinger (GHZ) class and the W class. To date, much research effort has been concentrated on the polarization entangled three-photon GHZ and W states. Most studies of continuous variable multi-particle entanglement have been focused on Gaussian states. In this paper, we examine two classes of three-photon entangled states in space and time. One class is a three-mode three-photon entangled state and the other is a two-mode triphoton state. These states show behavior similar to the GHZ and W states when one of the photons is not detected. The three-mode entangled state resembles a W state while a two-mode three-photon state resembles a GHZ state when one of the photons is traced away. We characterize the distinction between these two states by comparing the second-order correlation functions $G^{(2)}$ with the third-order correlation function $G^{!(3)}$.

PACS numbers: 42.50.Dv, 03.65.Ud, 03.67.Mn, 01.55.+b

I. INTRODUCTION

Generating entangled states is a primary task for the application of quantum information processing. The experimental preparation, manipulation and detection of multi-photon entangled states is of great interest for the implementation of quantum communication schemes, quantum cryptographic protocols, and for fundamental tests of quantum theory. Generating entangled photon pairs has been demonstrated from the processes of spontaneous parametric down conversion (SPDC) [1–3] and four-wave mixing [4]. These paired photons have proved to be key elements in many research fields such as quantum computing, quantum imaging, and quantum lithography. Although entanglement of bipartite systems is well understood, the characterization of entanglement for multipartite systems is still under intense study. In entangled three-qubit states it has been shown that there are two inequivalent classes of states, under stochastic local operations and classical communicati! ons, namely, the Greenberger-Horne-Zeilinger (GHZ) class [5] and the W class [6].

The GHZ class is a three qubit state of the form $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$, which leads to a conflict between local realism and nonstatistical predictions of quantum theory. Another three-qubit state, the W state, takes the form $|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$. It has been shown that this state is inequivalent to the GHZ state under stochastic local measurements and classical exchange of messages [7]. The entanglement in the W state is robust against the loss of one qubit, while the GHZ state is reduced to a product of two qubits. That is, tracing over one of the three qubits in the GHZ state leaves $\frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$ which is an unentangled mixture state; however, tracing out one qubit in the W state and the density matrix of the remaining qubits becomes $\frac{2}{3}|\Psi^+\rangle\langle\Psi^+| + \frac{1}{3}|00\rangle\langle 00|$ with $|\Psi^{!+}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ being a maximally entangled state of two qubits. It has been further shown that the W state allows for a generalized GHZ-like argument against the Einstein-Podolsky-Rosen type of elements of reality [8].

To date, much effort has been concentrated on the polarization entangled three-photon GHZ and W states. Experimental realizations of polarization entangled GHZ states and more recently W states have been performed in optical and trapped ion experiments [9–15]. Recently, the study of continuous-variable (CV) multipartite entanglement was initiated in [16], where a scheme was suggested to create pure CV N-party entanglement using squeezed light and $N-1$ beam splitters. In [17] a complete classification of trimode Gaussian states was with a necessary and sufficient condition for the separability to determine to which class a given state belongs. The CV analysis requires quadrature-type measurement; in this paper we shall be interested in studying three-photon states using direct photon counting detection. We here consider three-photon GHZ- and W-like states! entangled in time and space, which differ from the CV characterization of [16]. We will show that three-mode states, which we denote by $|1,1,1\rangle$, are similar to W states while two-mode states, denoted by $|1,2\rangle$, resemble GHZ-type states. The distinction between these two states has been demonstrated by looking at the second-order coherence function $G^{(2)}$. For related work with emphasis

* Electronic address: jianm1@umbc.edu
Current address: Physics Department, University of Virginia, Charlottesville, Virginia 22904, USA

on the entanglement properties of CV three particle Gaussian GHZ and W states see [18, 19]. This research is of importance, not only for testing foundations of quantum theory, but also for many promising applications based on quantum entanglement [20, 21].

II. TRIPHOTON W STATE

To illustrate the distinction between $|1,1,1\rangle$ and $|1,2\rangle$ states, we start with the case in which the source produces three-photon entangled states in different modes. For simplicity, a monochromatic plane-wave pump beam is assumed to travel along the $\hat{z}$ direction in the medium producing a state at the output face of the medium given by

$$|\Psi_1\rangle = \int d\omega_1 d\omega_2 d\omega_3 \int d\vec{\alpha}_1 d\vec{\alpha}_2 d\vec{\alpha}_3 \Phi(L\Delta)\delta(\omega_1+\omega_2+\omega_3-\Omega)H(\vec{\alpha}_1+\vec{\alpha}_2+\vec{\alpha}_3)|1_{\vec{k}_1},1_{\vec{k}_2},1_{\vec{k}_3}\rangle, \tag{1}$$

where Ω is the pump frequency, and ω_j with $\vec{\alpha}_j$ are the frequencies and transverse wave vectors of photons in mode $\vec{k}_j$, respectively. $\delta(\omega_1+\omega_2+\omega_3-\Omega)$ is the steady-state or the frequency phase-matching condition. The integral over the finite length L of the system gives the longitudinal detuning function, $\Phi(L\Delta)$, which determines the natural spectral width of the triphoton state. The longitudinal detuning function, in the non-depleted pump approximation usually takes the form of

$$\Phi(x) = \frac{1-e^{ix}}{ix} = -\mathrm{sinc}\Big(\frac{x}{2}\Big)e^{i\frac{x}{2}}, \tag{2}$$

with $x = L\Delta$ and $\Delta = (\vec{k}_p - \vec{k}_1 - \vec{k}_2 - \vec{k}_3)\cdot\hat{z}$, and $\vec{k}_p$ is the wave vector of the input pump field. Let $\omega_j = \Omega_j + \nu_j$ with fixed frequency Ω_j. Choosing the central frequencies so that $\Omega = \Omega_1+\Omega_2+\Omega_3$, frequency phase matching now becomes $\nu_1+\nu_2+\nu_3 = 0$. Assuming $|\nu_j| << \Omega_j$, and that the crystal is cut for collinear phase matching, $k_p = K_1+K_2+K_3$, we can expand k_j in powers of ν_j, $k_j = K_j + \nu_j/u_j + \cdots$ where $1/u_j$ is the group velocity of the photon j evaluated at Ω_j. Then to leading order we may write x as

$$x = -\sum_{j=1}^{3} L\nu_j/u_j = -\nu_1 L/D_{12} - \nu_3 L/D_{32}, \tag{3}$$

where we have used frequency phase matching to eliminate ν_2, and L/D_{ij} is the time difference between the ith photon and the jth passing through a unit length material. With a slight abuse of notation, we shall write $\Phi(L\Delta) = \Phi(\nu_1,\nu_3)$. The integration over the transverse coordinates ($\vec{\rho}$) on the output surface(s) of the source gives the transverse detuning function as

$$H(\vec{\alpha}_1+\vec{\alpha}_2+\vec{\alpha}_3) = \frac{1}{A}\int d\vec{\rho} e^{i\vec{\rho}\cdot(\vec{\alpha}_1+\vec{\alpha}_2+\vec{\alpha}_3)}. \tag{4}$$

In the ideal case, H becomes a δ-function, $\delta(\vec{\alpha}_1+\vec{\alpha}_2+\vec{\alpha}_3)$. In Eq. (1) we use the paraxial approximation, which is a good approximation for quantum imaging and lithography [22, 23]. With the quasi-monochromatic assumption $|\nu_j| << \Omega_j$ this his leads to the factoring of the state into longitudinal and the transverse degrees of freedom n the quasi-monochromatic approximation. We are interested in examining the temporal and spatial correlation between two subsystems by tracing the third in the free-propagation geometry. The second-order $[G^{(2)}]$ and third-order $[G^{(3)}]$ correlation functions are defined, respectively, as

$$G^{(2)} = \sum_{\vec{k}_3} |\langle 0|a_{\vec{k}_3} E_2^{(+)} E_1^{(+)}|\Psi_1\rangle|^2, \tag{5}$$

$$G^{(3)} = |\langle 0|E_3^{(+)} E_2^{(+)} E_1^{(+)}|\Psi_1\rangle|^2, \tag{6}$$

with freely propagating electric fields given by

$$E_j^{(+)}(\vec{\rho}_j, z_j, t_j) = \int d\omega_j \int d\vec{\alpha}_j E_j f_j(\omega_j) e^{-i\omega_j t_j} e^{i(k_j z_j + \vec{\alpha}_j\cdot\vec{\rho}_j)} a_{\vec{k}_j}, \tag{7}$$

where $E_j = \sqrt{\hbar\omega_j/2\epsilon_0}$, $k_j = \omega_j/c$ is the wave number, z_j and $\vec{\rho}_j$ are spatial coordinates of the jth detector, and $a_{\vec{k}_j}$ is a photon annihilation operator at the output surface of the source and obeys $[a_{\vec{k}}, a^\dagger_{\vec{k}'}] = \delta(\vec{\alpha}-\vec{\alpha}')\delta(\omega-\omega')$,

respectively. The function $f_j(\omega)$ is a narrow bandwidth filter function which is assumed to be peaked at Ω_j. In Eq. (7) we have decomposed $\vec{k}_j$ into $k_j\hat{z} + \vec{\alpha}_j$.

Substituting Eqs. (1) and (7) into (5) gives

$$G^{(2)} = C_0 G_l^{(2)}(\tau_1 - \tau_2) \times G_t^{(2)}(\vec{\rho}_1 - \vec{\rho}_2), \tag{8}$$

where C_0 is a slowly varying constant, and the temporal and spatial correlations, respectively, are

$$G_l^{(2)}(\tau_1 - \tau_2) = \int d\nu_3 \left| \int d\nu_1 f_1(\nu_1) f_2(\nu_1 + \nu_3) \Phi(\nu_1, \nu_3) e^{-i\nu_1(\tau_1 - \tau_2)} \right|^2, \tag{9}$$

$$G_t^{(2)}(\vec{\rho}_1 - \vec{\rho}_2) = \int d\vec{\alpha}_3 \left| \int d\vec{\alpha}_1 e^{i\vec{\alpha}_1 \cdot (\vec{\rho}_1 - \vec{\rho}_2)} \right|^2, \tag{10}$$

where $\tau_j = t_j - z_j/c$ and $\omega_j = \Omega_j + \nu_j$. Similarly, plugging Eqs. (1) and (7) into (6) yields

$$G^{(3)} = C_1 G_l^{(3)}(\tau_1 - \tau_2, \tau_3 - \tau_2) \times G_t^{(3)}(\vec{\rho}_1 - \vec{\rho}_2, \vec{\rho}_3 - \vec{\rho}_2), \tag{11}$$

where the third-order temporal and spatial correlations are

$$G_l^{(3)}(\tau_1 - \tau_2, \tau_3 - \tau_2) = \left| \int d\nu_1 d\nu_3 f_1(\nu_1) f_2(\nu_1 + \nu_3) f_3(\nu_3) \Phi(\nu_1, \nu_3) e^{-i\nu_1(\tau_1 - \tau_2)} e^{-i\nu_3(\tau_3 - \tau_2)} \right|^2, \tag{12}$$

$$G_t^{(3)}(\vec{\rho}_1 - \vec{\rho}_2, \vec{\rho}_3 - \vec{\rho}_2) = \left| \int d\vec{\alpha}_1 d\vec{\alpha}_3 e^{i\vec{\alpha}_1 \cdot (\vec{\rho}_1 - \vec{\rho}_2)} e^{i\vec{\alpha}_3 \cdot (\vec{\rho}_3 - \vec{\rho}_2)} \right|^2, \tag{13}$$

and C_1 is constant. By comparing Eq. (9) with (12), it is clear that although one photon is not detected (traced away) in the two-photon detection, there remains a correlation between the remaining two photons. The width of the two-photon temporal correlation depends on the three photon bandwidth. The comparison between Eqs. (10) and (13) indicates that the spatial correlation between two photons is limited by the bandwidth of the transverse modes. Ideally, point-to-point correlation is achieved by assuming infinite transverse bandwidth. Combining the temporal and spatial properties together show that the $|1,1,1\rangle$ state (1) is a W state entangled in time and space, which is robust against one photon loss.

There are several schemes which might produce such a state. One scheme is three-photon cascade emission whose spectral properties have been analyzed in [24]. Another configuration utilizes two parametric down conversions and one up-conversion to create a triphoton state, as proposed by Keller *et al* [25]. The transverse properties of triphotons generated from such a case have been studied in [26] by considering quantum imaging experiments. It was shown that by implementing two-photon imaging, the quality of the images is limited by the bandwidth of the transverse modes of the non-detected third photon.

In Fig. 1 we have compared the temporal correlations between the third-order correlation function $G_l^{(3)}(\tau_{12}, \tau_{32})$ and the second-order $G_l^{(2)}(\tau_{12})$ with Gaussian filters in Eqs. (9) and (12). The filters were taken to the same bandwidth which is large compared to the width of the $\Phi(L\Delta)$ function. The plots have been normalized with respect to their maximum value. In generating the figure D_{12} has been taken equal to D_{32} and they have both been taken to be negative. Because of this the plot of $G^{(3)}$is symmetric around the line $\tau_{12} = \tau_{32}$, and only positive values of the τ_{ij} are physically allowed. The length of $G^{(3)}$ is determined by the phase matching function Φ as illustrated in Fig. 1(a); Fig. 1(b) shows the conditional measurement of $G_l^{(3)}(\tau_{12})$ obtained by setting $\tau_{32} = -\tau_{12} + |L/D_{12}|$. The width of $G^{(3)}$ is determined by the filters. ! In Fig. 1(c) the second-order temporal correlation $G_l^{(2)}(\tau_{12})$ is plotted. The width of $G_l^{(2)}(\tau_{12})$ is larger than that of the conditional $G_l^{(3)}(\tau_{12})$ reflecting the lack of cut off of the bandwidth of for the non-detected third photon.

III. TRIPHOTON GHZ STATE

After analyzing the properties of the $|1,1,1\rangle$ state, we now consider the case in which the source produces three-photon entangled states with a pair of degenerate photons of the form [27]

$$|\Psi_2\rangle = \int d\omega_1 d\omega_2 \int d\vec{\alpha}_1 d\vec{\alpha}_2 \Phi(x) \delta(2\omega_1 + \omega_2 - \Omega) \delta(2\vec{\alpha}_1 + \vec{\alpha}_2) |2_{\vec{k}_1}, 1_{\vec{k}_2}\rangle, \tag{14}$$

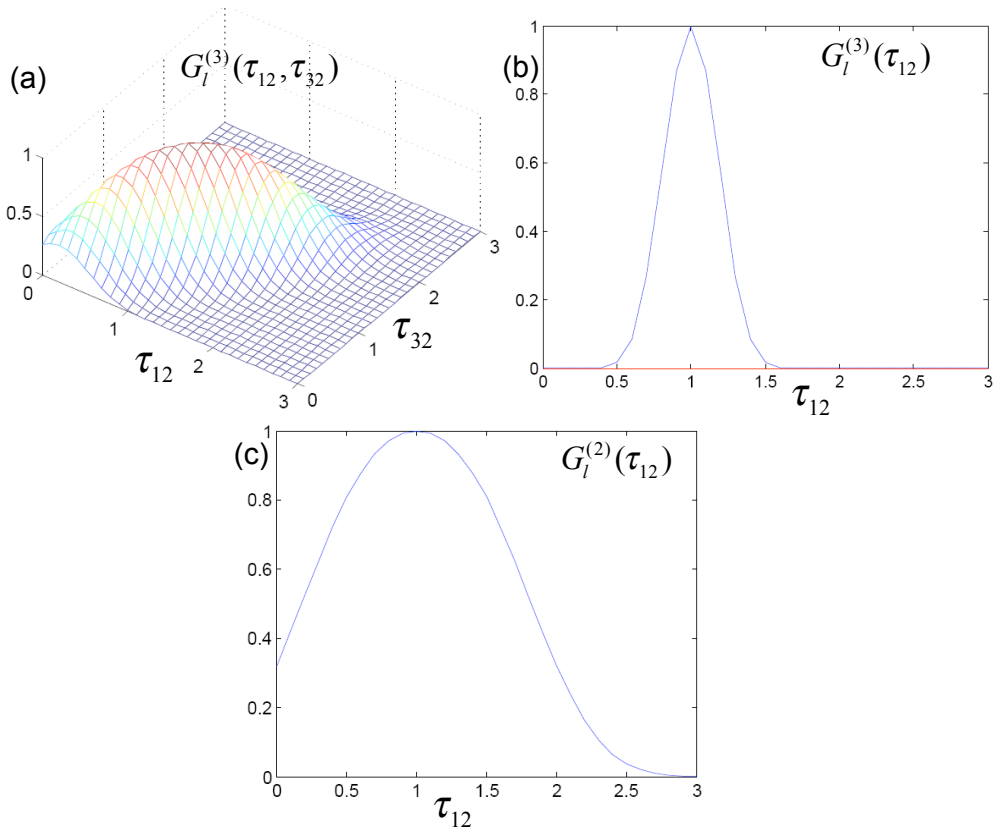

FIG. 1: (color online) Temporal correlations of $G_l^{(3)}$ and $G_l^{(2)}$ for the $|1,1,1\rangle$ state normalized to unity at their origin. The units of $\tau_{ij} = \tau_i - \tau_j$ are 10 ps. (a) Third-order temporal correlation $G_l^{(3)}(\tau_{12},\tau_{32})$. (b) Conditional third-order correlation $G_l^{(3)}(\tau_{12})$ obtained by setting $\tau_{32} = -\tau_{12} + |L/D_{12}|$. (c) Second-order temporal correlation $G_l^{(2)}(\tau_{12})$. The corresponding parameters are chosen as $L/2D_{ij} = 10$ ps and all the filters are Gaussian with the same bandwidth of 0.4 THZ.

where Φ characterizes the natural bandwidth of triphotons and has the same form as Eq. (2), with $x = -2\nu_1/D_{12}$, $\omega_j = \Omega_j + \nu_j$, and $\vec{\alpha}_j$ are the frequencies and transverse wave vectors of the degenerate ($j = 1$) and non-degenerate ($j = 2$) photons. In [27] we show that by sending two degenerate photons to the target while keeping the non-degenerate one traversing the imaging lens, a factor of two of spatial resolution improvement can be obtained, beyond the Rayleigh diffraction limit. Before proceeding the discussion, we note that the major difference between the $|1,2\rangle$ state [Eq. (14)] and $|1,1,1\rangle$ [Eq. (1)] is that the $|1,1,1\rangle$ state has more degrees of freedom than the $|1,2\rangle$ state. This is the source of the difference between two states when performing two-photon detection, as we shall see. Physically, because two of the photons are degenerate, the measurement of one of them separately uniquely determines the state of the other one and the two photon state becomes a product state. This is true even if ! the photon is not measured but can be measured separately in principle. The effect of this is that the state generated is a mixed state. Note that for the completely degenerate case, a similar argument implies that tracing away one of the photons gives a mixed two photon state.

For the two-photon measurement here, we first assume that one of the degenerate photons is not detected. The

second-order $G^{(2)}$ and third-order $G^{(3)}$ correlation functions now become

$$G^{(2)} = \sum_{\vec{k}_1} |\langle 0|a_{\vec{k}_1} E_2^{(+)} E_1^{(+)}|\Psi_2\rangle|^2, \tag{15}$$

$$G^{(3)} = |\langle 0|E_2^{(+)}[E_1^{(+)}]^2|\Psi_2\rangle|^2, \tag{16}$$

where $E_j^{(+)}$ is the free-space electric field given in Eq. (7). Note that because of the degeneracy, a two-photon detector is necessary for three-photon joint detection [27]. Following the same procedure for the $|1,1,1\rangle$ calculation, it is easy to show that the second-order and third-order correlation functions are

$$G_l^{(2)} = \int d\nu_1 \left| f_1(\nu_1) f_2(\nu_1) \Phi(-2\nu_1/D_{12}) \right|^2, \tag{17}$$

$$G_l^{(3)}(\tau_{12}) = \left| \int d\nu_1 f_1^2(\nu_1) f_2(\nu_1) \Phi(-2\nu_1/D_{12}) e^{-2i\nu_1\tau_{12}} \right|^2, \tag{18}$$

in the temporal domain, and

$$G_t^{(2)} = \int d\vec{\alpha}_1, \tag{19}$$

$$G_t^{(3)}(\vec{\rho}_1 - \vec{\rho}_2) = \left| \int d\vec{\alpha}_1 e^{2i\vec{\alpha}_1\cdot(\vec{\rho}_1 - \vec{\rho}_2)} \right|^2, \tag{20}$$

in the spatial space. Comparing Eqs. (17) and (19) with (18) and (20) shows that if one of the degenerate photons is traced away, there will be no correlation between the remaining photons, which is the property of tripartite GHZ state. Indeed, one can easily show that the $|1,2\rangle$ state (14) always reduces to a product state, if one photon is not measured. The reason for this is that if one photon is traced away, then the remaining photons is put into a definite mode because of our assumption of perfect phase matching and the resulting state is a mixed state of the form

$$\rho = \sum_{\vec{k}} |F(\vec{k})|^2 |\vec{k}_p - \vec{k}, \vec{k}\rangle\langle \vec{k}_p - \vec{k}, \vec{k}|. \tag{21}$$

Recently, we have found that to some extent, the $|1,1,1\rangle$ state can mimic some properties of the $|1,2\rangle$ state, e.g., by sending two nearly degenerate photons in the $|1,1,1\rangle$ state to the object while propagating the third one through the imaging lens in the quantum imaging configuration, a factor of two on spatial resolution enhancement is achievable in the coincidence counting measurement. However, the Gaussian lens equation is not the same as that with the $|1,2\rangle$ state and more importantly, the physics behind these two imaging processes is quite different.

IV. CONCLUSION

In summary, we have shown that the triphoton $|1,1,1\rangle$ state is analogous to a W state while the $|1,2\rangle$ state is analogus to a GHZ state by comparing the third-order and second-order correlation functions in both temporal and spatial domains. Our analysis on these state properties may be important to not only the understanding of multipartite systems but also the technologies based on quantum entanglement. For example, in Refs. [26] and [27] we have discussed quantum imaging using these two classes of states and have found different spatial resolutions in application. The essential difference between these two states is that $|1,1,1\rangle$ has a larger Hilbert spaces than $|1,2\rangle$. Specifically, measurement of one of the degenerate photons in the GHZ-like state allow for the possibility of a separate measurement of the degenerate photon state. This reduces the two photon state to a mixed state. For the W-like state, only partial information ! can in principle be obtained and so some entanglement remains.

The authors wish to thank their colleague Kevin McCann for help with the numerical computations for the figure. We thank one of the referee's for pointing out reference [19] to us. We acknowledge the financial support in part by U.S. ARO MURI Grant W911NF-05-1-0197.

[1] M. H. Rubin, D. N. Klyshko, Y.-H. Shih, and A. V. Sergienko, Phys. Rev. A **50**, 5122 (1994).

[2] Y.-H. Shih, Rep. Prog. Phys. **66**, 1009 (2003).
[3] D. N. Klyshko, *Photons and Nonlinear Optics* (Gordon and Breach Science, New York, 1988).
[4] S. Du, J.-M. Wen, and M. H. Rubin, J. Opt. Soc. Am. B **25**, C98 (2008); J.-M. Wen and M. H. Rubin, Phys. Rev. A **74**, 023808 (2006); **74**, 023809 (2006); J.-M. Wen, S. Du, and M. H. Rubin, *ibid.* **75**, 033809 (2007); **76**, 013825 (2007); J.-M. Wen, S. Du, Y. P. Zhang, M. Xiao, and M. H. Rubin, *ibid.* **77**, 033816 (2008).
[5] D. M. Greenberger, M. A. Horne, and A. Zeilinger, in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, edited by M. Kafatos (Dordrecht, Kluwer, 1989).
[6] A. Zeilinger, M. A. Horne, and D. M. Greenberger, NASA conf. Publ. No. 3135 (National Aeronautics and Space Administration, Code NTT, Washington D.C., 1997).
[7] W. Dür, G. Vidal, and J. I. Cirac, Phys. Rev. A **62**, 062314 (2000).
[8] A. Cabello, Phys. Rev. A **65**, 032108 (2002).
[9] D. Bouwmeester, J.-W. Pan, M. Daniell, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **82**, 1345 (1999).
[10] J.-W. Pan, D. Bouwmeester, M. Daniell, H. Weinfurter, and A. Zeilinger, Nature (London) **403**, 515 (2000).
[11] K. J. Resch, P. Walther, and A. Zeilinger, Phys. Rev. Lett. **94**, 070402 (2005).
[12] Y.-A. Chen, T. Yang, A.-N. Zhang, Z. Zhao, A. Cabello, and J.-W. Pan, Phys. Rev. Lett. **97**, 170408 (2006).
[13] M. Eibl, N. Kiesel, M. Bourennane, C. Kurtsiefer, and H. Weinfurter, Phys. Rev. Lett. **92**, 077901 (2004).
[14] C. F. Roos, M. Riebe, H. Häffner, W. Hänsel, J. Benhelm, G. P. T. Lancaster, C. Becher, F. Schmidt-Kaler, and R. Blatt, Sience **304**, 1478 (2004).
[15] H. Mikami, Y. Li, K. Fukuoka, and T. Kobayashi, Phys. Rev. Lett. **95**, 150404 (2005).
[16] P. van Loock and S. L. Braunstein, Phys. Rev. Lett. **84**, 3482 (2000); Phys. Rev. A **63**, 022106 (2001).
[17] G. Giedke, B. Kraus, M. Lewenstein, and J. I. Cirac, Phys. Rev. A **64**, 052303 (2001).
[18] P. van Loock and A. Furusawa, Phys. Rev. A **67**, 052315 (2003).
[19] G. Adesso and F. Illluminati, New J. Phys. **8**, 15 (2006).
[20] S. L. Braunstein and P. van Loock, Rev. Mod. Phys. **77**, 513 (2005).
[21] Y.-H. Shih, IEEE J. Sel. Top. Quantum Electron. **13**, 1016 (2007).
[22] M. H. Rubin, Phys. Rev. A **54**, 5349 (1996).
[23] J.-M. Wen, M. H. Rubin, and Y.-H. Shih, Phys. Rev. A **76**, 045802 (2007).
[24] M. V. Chekhova, O. A. Ivanova, V. Berardi, and A. Garuccio, Phys. Rev. A **72**, 023818 (2005).
[25] T. E. Keller, M. H. Rubin, Y.-H. Shih, and L.-A. Wu, Phys. Rev. A **57**, 2076 (1998).
[26] J.-M. Wen, P. Xu, M. H. Rubin, and Y.-H. Shih, Phys. Rev. A **76**, 023828 (2007).
[27] J.-M. Wen, M. H. Rubin, and Y.-H. Shih, submitted to Phys. Rev. A (2008), quant-ph/arXiv:0812.2032