Wolf Equations for Two-Photon Light

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(Received 13 December 2004; published 7 June 2005)

The spatiotemporal two-photon probability amplitude that describes light in a two-photon entangled state obeys equations identical to the Wolf equations, which are satisfied by the mutual coherence function for light in any quantum state. Both functions therefore propagate similarly through optical systems. A generalized van Cittert–Zernike theorem explains the predicted enhancement in resolution for entangled-photon microscopy and quantum lithography. The Wolf equations provide a particularly powerful analytical tool for studying three-dimensional imaging and lithography since they describe propagation in continuous inhomogeneous media.

DOI: 10.1103/PhysRevLett.94.223601 PACS numbers: 42.50.Dv, 42.50.Ar, 42.50.St

Fifty years ago, Wolf pioneered the description of light in terms of observable quantities [1]. He showed that the second-order coherence function, which is defined at a pair of space-time points, obeys two wave equations, now known as the Wolf equations. These equations govern propagation in linear homogeneous media and underlie a correspondence between the spatial pattern of the coherence function (the pattern generated when one point is held fixed and the other is scanned) for light emitted from an *incoherent* source, and the diffraction pattern of *coherent* light transmitted through the source aperture. The van Cittert–Zernike theorem [2,3] is an example of this correspondence. A formal hierarchy of coherence functions was subsequently defined by Glauber [4] for light in *any quantum state*, as the expected values of normally ordered products of optical-field operators at multiple space-time points [4,5]. The Wolf equations are applicable to light in any quantum state and can be generalized to higher-order coherence functions.

In this Letter, we consider light in a *two-photon quantum state*. Such light is described in terms of a spatiotemporal two-photon probability amplitude whose squared modulus is the probability density of simultaneously detecting two photons at two space-time points [5,6]. Though the two-photon probability amplitude is not a coherence function, we demonstrate that it does obey the Wolf equations, and therefore exhibits propagation and diffraction phenomena analogous to those of the second-order coherence function, including the van Cittert–Zernike theorem. A duality between the two-photon probability amplitude and the second-order coherence function has previously been highlighted [7] and the Wolf equations described in this Letter provide its mathematical underpinning.

The results are used to compare two-photon imaging [8–11] and thermal-light photon-correlation imaging [12–14], a topic of high current interest. They are also used to provide a more general explanation for the origin of resolution enhancement in quantum microscopy [15] and quantum lithography [16]. Since the Wolf equations describe propagation in continuous inhomogeneous media, they provide a powerful analytical tool for studying *three-dimensional* imaging and lithography.

Coherence functions and the Wolf equations.—The hallmark of the theory of coherence is the second-order coherence function $\Gamma^{(1)}(\mathbf{x}_1, \mathbf{x}_2) = \langle\Psi|\hat{E}^-(\mathbf{x}_1)\hat{E}^+(\mathbf{x}_2)|\Psi\rangle$, where the quantities $\hat{E}^-(\mathbf{x})$ and $\hat{E}^+(\mathbf{x})$ represent the negative- and positive-frequency components of the optical-field operator at the space-time point $\mathbf{x} = (\mathbf{r}, t)$, respectively, and $|\Psi\rangle$ represents the state vector. In free space, this function satisfies the Wolf equations [1],

$$\left[\nabla_j^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t_j^2}\right]\Gamma^{(1)} = 0, \qquad j = 1, 2, \tag{1}$$

where ∇_j^2 is the Laplacian operator with respect to $\mathbf{r}_j$. For stationary light, $\langle\Psi|\hat{E}^-(\mathbf{r}_1, t)\hat{E}^+(\mathbf{r}_2, t+\tau)|\Psi\rangle \equiv \Gamma^{(1)}(\mathbf{r}_1, \mathbf{r}_2, \tau)$ is independent of the time t and is known as the mutual coherence function. In this case, the cross-spectral density $G^{(1)}(\mathbf{r}_1, \mathbf{r}_2, \omega)$ is defined as the Fourier transform of $\Gamma^{(1)}(\mathbf{r}_1, \mathbf{r}_2, \tau)$ with respect to τ. Under these conditions, the Wolf equations take the form of Helmholtz equations,

$$(\nabla_j^2 + k^2)G^{(1)} = 0, \qquad j = 1, 2, \tag{2}$$

where $k = \omega/c$ is the wave number. These equations govern the propagation and diffraction of partially coherent light.

Since $G^{(1)}(\mathbf{r}_1, \mathbf{r}_2, \omega)$ has Hermitian symmetry and is non-negative definite, Mercer's theorem can be used to expand it as a sum of products,

$$G^{(1)}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \sum_n \alpha_n(\omega)u_n^*(\mathbf{r}_1, \omega)u_n(\mathbf{r}_2, \omega), \tag{3}$$

where $u_n(\mathbf{r}, \omega)$ and $\alpha_n(\omega)$ are, respectively, the eigenfunctions and the eigenvalues of $G^{(1)}(\mathbf{r}_1, \mathbf{r}_2, \omega)$. The eigenfunctions form an orthonormal set and the eigenvalues are real and non-negative. Furthermore, the $u_n(\mathbf{r}, \omega)$ satisfy the Helmholtz equation [5],

$$(\nabla^2 + k^2)u_n = 0, \qquad \forall\ n. \tag{4}$$

This modal expansion is known as the *coherent-mode representation* since each of its terms represents a spatially coherent mode.

The propagation of partially coherent light between two planes is determined by using a modal expansion of $G^{(1)}(\mathbf{r}_1, \mathbf{r}_2, \omega)$ in the source plane and propagating the eigenfunctions u_n between the two planes as coherent waves; the eigenvalues remain unchanged.

If the optical system between the two planes is described by the Green's function $h(\boldsymbol{\rho}_a, \boldsymbol{\rho}, \omega)$, where $\boldsymbol{\rho} = (x, y)$ is a point in the transverse plane, then we obtain the following explicit relation between the cross-spectral densities $G_d^{(1)}(\boldsymbol{\rho}_a, \boldsymbol{\rho}_b, \omega)$ and $G_s^{(1)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega)$ in the observation plane ($z = d$) and the source plane ($z = 0$):

$$G_d^{(1)}(\boldsymbol{\rho}_a, \boldsymbol{\rho}_b, \omega) = \iint G_s^{(1)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) h^*(\boldsymbol{\rho}_a, \boldsymbol{\rho}_1, \omega) \times h(\boldsymbol{\rho}_b, \boldsymbol{\rho}_2, \omega) d\boldsymbol{\rho}_1 d\boldsymbol{\rho}_2. \tag{5}$$

Wolf equations for two-photon light.—Light in a two-photon pure quantum state is described in a Hilbert space with a continuum of spatiotemporal modes occupied by a total of exactly two photons. It is a superposition of multimode states, each of which has the two photons occupying a different pair of modes, with all other modes empty. It is assumed for simplicity that the light is linearly polarized. Two-photon light is generated, for example, by spontaneous parametric down-conversion in a type-I second-order nonlinear optical crystal [6]. Conservation of energy and momentum dictate that the state be spectrally and spatially entangled.

For light in an arbitrary quantum state, the probability of observing a photon at the space-time position $\mathbf{x}_1$, and another at $\mathbf{x}_2$, is proportional to the intensity correlation function,

$$\Gamma^{(2)}(\mathbf{x}_1, \mathbf{x}_2) = \langle \Psi | \hat{E}^-(\mathbf{x}_1) \hat{E}^-(\mathbf{x}_2) \hat{E}^+(\mathbf{x}_2) \hat{E}^+(\mathbf{x}_1) | \Psi \rangle. \tag{6}$$

In particular, for light in the two-photon state, the right-hand side of Eq. (6) factors into the form $\langle \Psi | \hat{E}^-(\mathbf{x}_1) \hat{E}^-(\mathbf{x}_2) | 0 \rangle \langle 0 | \hat{E}^+(\mathbf{x}_2) \hat{E}^+(\mathbf{x}_1) | \Psi \rangle$, so that

$$\Gamma^{(2)}(\mathbf{x}_1, \mathbf{x}_2) = |\psi(\mathbf{x}_1, \mathbf{x}_2)|^2, \tag{7}$$

where

$$\psi(\mathbf{x}_1, \mathbf{x}_2) = \langle 0 | \hat{E}^+(\mathbf{x}_1) \hat{E}^+(\mathbf{x}_2) | \Psi \rangle. \tag{8}$$

The function $\psi(\mathbf{x}_1, \mathbf{x}_2)$ can therefore be regarded as the two-photon probability amplitude.

A principal contribution of this Letter is to show that the two-photon probability amplitude $\psi(\mathbf{x}_1, \mathbf{x}_2)$, although not a coherence function, does satisfy the Wolf equations,

$$\left[\nabla_j^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t_j^2}\right] \psi = 0, \qquad j = 1, 2. \tag{9}$$

This is demonstrated by writing the wave equation for the field $\hat{E}^+(\mathbf{x}_2)$, multiplying by $\hat{E}^+(\mathbf{x}_1)$, applying the bra $\langle 0|$ on the left and the ket $|\Psi\rangle$ on the right, and using the definition of ψ provided in Eq. (8).

Since the functions $\Gamma^{(1)}$ and ψ satisfy the same pair of partial differential equations, they share similar properties. Hence the behavior of two-photon light in optical systems, including its diffraction and propagation, mirrors that of partially coherent light [7]. It is important to keep in mind, however, that these two functions represent distinct physical phenomena. For example, the factorizability of $\Gamma^{(1)}$ for light in any quantum state indicates *complete coherence*, whereas the factorizability of ψ for light in a two-photon state corresponds to *nonentanglement*. Conversely, when $\Gamma^{(1)}(\mathbf{x}_1, \mathbf{x}_2) \propto \delta(\mathbf{x}_1 - \mathbf{x}_2)$, the light is *completely incoherent*, whereas $\psi(\mathbf{x}_1, \mathbf{x}_2) \propto \delta(\mathbf{x}_1 - \mathbf{x}_2)$ corresponds to a *maximally entangled* state.

In the frequency domain, the 2D Fourier transform of $\psi(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2)$ with respect to t_1 and t_2 is a function $\phi(\mathbf{r}_1, \omega_1; \mathbf{r}_2, \omega_2)$ that is analogous to the cross-spectral density $G^{(1)}(\mathbf{r}_1, \mathbf{r}_2, \omega)$ for stationary light. However, since two-photon light (e.g., nondegenerate spontaneous parametric down-conversion) often takes the form of two beams with spectra centered about two different frequencies, two frequencies are retained in the definition of ϕ.

The Wolf equations set forth in Eq. (9) imply that ϕ also satisfies the Helmholtz equations,

$$(\nabla_j^2 + k_j^2)\phi = 0, \qquad j = 1, 2, \tag{10}$$

where $k_1 = \omega_1/c$ and $k_2 = \omega_2/c$. Equations (10) are analogous to Eqs. (2) so that ϕ and $G^{(1)}$ behave similarly. It is noteworthy, however, that $G^{(1)}$ has Hermitian symmetry, whereas ϕ does not necessarily have such symmetry.

The analogy with the theory of partial coherence can be extended further, to the modal decomposition. The function $\phi(\mathbf{r}_1, \omega_1; \mathbf{r}_2, \omega_2)$ can be decomposed into a superposition of separable functions,

$$\phi(\mathbf{r}_1, \omega_1; \mathbf{r}_2, \omega_2) = \sum_n \beta_n v_n(\mathbf{r}_1, \omega_1) u_n(\mathbf{r}_2, \omega_2), \tag{11}$$

each representing a nonentangled component. Since ϕ does not necessarily have Hermitian symmetry, a Schmidt decomposition is used in place of Mercer's theorem. Symmetric kernels are constructed,

$$\Phi_1(\mathbf{r}, \omega; \mathbf{r}', \omega') = \iint \phi(\mathbf{r}, \omega; \mathbf{r}_2, \omega_2) \times \phi^*(\mathbf{r}', \omega'; \mathbf{r}_2, \omega_2) d\mathbf{r}_2 d\omega_2$$

$$\Phi_2(\mathbf{r}, \omega; \mathbf{r}', \omega') = \iint \phi(\mathbf{r}_1, \omega_1; \mathbf{r}, \omega) \times \phi^*(\mathbf{r}_1, \omega_1; \mathbf{r}', \omega') d\mathbf{r}_1 d\omega_1,$$

with common eigenvalues λ_n and with eigenfunctions $v_n(\mathbf{r}, \omega)$ and $u_n(\mathbf{r}, \omega)$, respectively. The expansion coefficients in Eq. (11) are $\beta_n = \sqrt{\lambda_n}$. By use of the Helmholtz equations set forth in Eqs. (10), it can be readily shown that the eigenfunctions $v_n(\mathbf{r}, \omega)$ and $u_n(\mathbf{r}, \omega)$ themselves satisfy

Helmholtz equations,

$$(\nabla^2 + k_1^2)v_n = 0, \qquad (\nabla^2 + k_2^2)u_n = 0, \qquad \forall\ n, \quad (12)$$

so that the modal expansion is in terms of nonentangled contributions with spatial dependence in the form of coherent waves. The Schmidt decomposition has been previously applied to two-photon light [17,18].

The propagation of two-photon light between two planes may be determined by using a modal expansion of $\phi(\mathbf{r}_1, \omega_1; \mathbf{r}_2, \omega_2)$ in the first plane and propagating the eigenfunctions v_n and u_n to the second plane as coherent waves with frequencies ω_1 and ω_2, respectively; the eigenvalues remain unchanged. A measure of the degree of entanglement is the Schmidt number $\sum_n 1/\beta_n^2 = \sum_n 1/\lambda_n$ [18], which is invariant to propagation, as it should be.

If the optical system between the two planes is described by the Green's function $h(\boldsymbol{\rho}_a, \boldsymbol{\rho}, \omega)$, then we obtain the following explicit relation between $\phi_d(\boldsymbol{\rho}_a, \omega_1; \boldsymbol{\rho}_b, \omega_2)$ and $\phi_s(\boldsymbol{\rho}_1, \omega_1; \boldsymbol{\rho}_2, \omega_2)$ in the observation plane ($z = d$) and the source plane ($z = 0$), respectively:

$$\phi_d(\boldsymbol{\rho}_a, \omega_1; \boldsymbol{\rho}_b, \omega_2) = \iint \phi_s(\boldsymbol{\rho}_1, \omega_1; \boldsymbol{\rho}_2, \omega_2) h(\boldsymbol{\rho}_a, \boldsymbol{\rho}_1, \omega_1) \times h(\boldsymbol{\rho}_b, \boldsymbol{\rho}_2, \omega_2) d\boldsymbol{\rho}_1 d\boldsymbol{\rho}_2. \quad (13)$$

Coherence and two-photon entanglement.—The analogy between the two-photon probability amplitude for two-photon light and the mutual coherence function for arbitrary light has its mathematical origin in their similar definitions—the former is the off-diagonal element $\psi(\mathbf{x}_1, \mathbf{x}_2) = \langle 0|\hat{E}^+(\mathbf{x}_1)\hat{E}^+(\mathbf{x}_2)|\Psi\rangle$ and the latter is the diagonal element $\Gamma^{(1)}(\mathbf{x}_1, \mathbf{x}_2) = \langle \Psi|\hat{E}^-(\mathbf{x}_1)\hat{E}^+(\mathbf{x}_2)|\Psi\rangle$. Both obey the Wolf equations and their Fourier transforms obey Helmholtz equations. One difference is the presence of only positive-frequency components in the former and normally ordered negative- and positive-frequency field operators in the latter. This distinction leads to the appearance of the conjugate operation in Eqs. (3) and (5) but not in Eqs. (11) and (13). A ramification of this distinction is that the van Cittert–Zernike theorem differs for partially coherent and two-photon light, as shown below [7].

If the source cross-spectral density $G_s^{(1)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = I_s(\boldsymbol{\rho}_1)\delta(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)$, i.e., if the light is completely spatially incoherent over the support of the source function $I_s(\boldsymbol{\rho})$, then Eq. (5) yields

$$G_d^{(1)}(\boldsymbol{\rho}_a, \boldsymbol{\rho}_b, \omega) = \int I_s(\boldsymbol{\rho}) h^*(\boldsymbol{\rho}_a, \boldsymbol{\rho}, \omega) h(\boldsymbol{\rho}_b, \boldsymbol{\rho}, \omega) d\boldsymbol{\rho}. \quad (14)$$

As an example, consider an optical system comprising a lens of focal length f imaging a distant source onto its own focal plane. The Green's function is then $h(\boldsymbol{\rho}_a, \boldsymbol{\rho}, \omega) \propto \exp(-ik\boldsymbol{\rho}_a \cdot \boldsymbol{\rho}/f)$, whereupon

$$G_d^{(1)}(\boldsymbol{\rho}_a, \boldsymbol{\rho}_b, \omega) = \int I_s(\boldsymbol{\rho}) \exp[-ik(\boldsymbol{\rho}_b - \boldsymbol{\rho}_a) \cdot \boldsymbol{\rho}/f] d\boldsymbol{\rho}. \quad (15)$$

Thus the cross-spectral density at the $z = d$ plane, plotted as a function of the coordinate difference $\boldsymbol{\rho}_b - \boldsymbol{\rho}_a$, is the Fourier transform of the source function $I_s(\boldsymbol{\rho})$; this is the van Cittert–Zernike theorem [2,3].

The analogous relation for two-photon light is obtained by substituting $\phi_s(\boldsymbol{\rho}_1, \omega_1; \boldsymbol{\rho}_2, \omega_2) = \zeta_s(\boldsymbol{\rho}_1)\delta(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)$ into Eq. (13) to obtain

$$\phi_d(\boldsymbol{\rho}_a, \omega_1; \boldsymbol{\rho}_b, \omega_2) = \int \zeta_s(\boldsymbol{\rho}) h(\boldsymbol{\rho}_a, \boldsymbol{\rho}, \omega_1) h(\boldsymbol{\rho}_b, \boldsymbol{\rho}, \omega_2) d\boldsymbol{\rho}. \quad (16)$$

For an optical system composed of a lens of focal length f imaging a distant object onto its own focal plane, the result is now

$$\phi_d(\boldsymbol{\rho}_a, \omega_1; \boldsymbol{\rho}_b, \omega_2) = \int \zeta_s(\boldsymbol{\rho}) \exp[-i(k_1\boldsymbol{\rho}_a + k_2\boldsymbol{\rho}_b) \cdot \boldsymbol{\rho}/f] d\boldsymbol{\rho}. \quad (17)$$

This means that the two-photon probability amplitude in the focal plane, when plotted as a function of the *sum* $\omega_1\boldsymbol{\rho}_a + \omega_2\boldsymbol{\rho}_b$, is the Fourier transform of the source function $\zeta_s(\boldsymbol{\rho})$, which provides an example of the duality between partial coherence and partial entanglement [7]. If $\zeta_s(\boldsymbol{\rho})$ is uniform over the entire plane, then the two-photon probability amplitude in the observation plane is proportional to $\delta(\omega_1\boldsymbol{\rho}_a + \omega_2\boldsymbol{\rho}_b)$, indicating that the positions at which the two photons are observed must satisfy the relation $\omega_1\boldsymbol{\rho}_a = -\omega_2\boldsymbol{\rho}_b$. Since the observation plane is the focal plane, this relation corresponds to wave vectors with equal and opposite transverse components, i.e., to exact conservation of momentum. This is, of course, expected for an infinitely large source for which the two photons are always emitted from the same point.

Thermal light and two-photon light.—We have seen so far that the two-photon coincidence probability for two-photon light is $|\psi(\mathbf{x}_1, \mathbf{x}_2)|^2$, where $\psi(\mathbf{x}_1, \mathbf{x}_2)$ obeys the Wolf equations, which also govern the propagation of the second-order coherence function $\Gamma^{(1)}(\mathbf{x}_1, \mathbf{x}_2)$. For light in an arbitrary state, the two-photon coincidence probability is proportional to the intensity correlation function $\Gamma^{(2)}(\mathbf{x}_1, \mathbf{x}_2)$ defined in Eq. (6). This function does *not* obey the Wolf equations, but it is a special value of the four-point fourth-order coherence function [4], which does satisfy four Wolf equations. There is therefore no direct general analogy between the functions $|\psi(\mathbf{x}_1, \mathbf{x}_2)|^2$ and $\Gamma^{(2)}(\mathbf{x}_1, \mathbf{x}_2)$ for light in an arbitrary state.

Thermal light is an exception since the intensity correlation function $\Gamma^{(2)}(\mathbf{x}_1, \mathbf{x}_2)$ is related to $|\Gamma^{(1)}(\mathbf{x}_1, \mathbf{x}_2)|^2$ via the Siegert relation [19],

$$\Gamma^{(2)}(\mathbf{x}_1, \mathbf{x}_2) = \Gamma^{(1)}(\mathbf{x}_1, \mathbf{x}_1)\Gamma^{(1)}(\mathbf{x}_2, \mathbf{x}_2) + |\Gamma^{(1)}(\mathbf{x}_1, \mathbf{x}_2)|^2. \quad (18)$$

The Hanbury Brown–Twiss interferometer [20] operates on the basis of this formula. It measures the dependence of the intensity correlation, or the photon coincidence rate, on the separation of a pair of detectors, and makes use of the van Cittert–Zernike relation to estimate the angular diameter of thermal-light sources such as stars. The first term in the Siegert relation is the product of the intensities, i.e., the single-photon rates. The second term is the absolute square of the second-order coherence function, $|\Gamma^{(1)}(\mathbf{x}_1, \mathbf{x}_2)|^2$. Since $\Gamma^{(1)}(\mathbf{x}_1, \mathbf{x}_2)$ for any source of light is analogous to $\psi(\mathbf{x}_1, \mathbf{x}_2)$ for two-photon light, it follows that there is a correspondence between the intensity cross-covariance function $\Gamma^{(2)}(\mathbf{x}_1, \mathbf{x}_2) - \Gamma^{(1)}(\mathbf{x}_1, \mathbf{x}_1)\Gamma^{(1)}(\mathbf{x}_2, \mathbf{x}_2) = |\Gamma^{(1)}(\mathbf{x}_1, \mathbf{x}_2)|^2$ for thermal light and $|\psi(\mathbf{x}_1, \mathbf{x}_2)|^2$ for two-photon light.

Simple optical systems that rely on measurements of two-photon coincidences for thermal and two-photon light sources thus share some similarities. This has been exploited in a number of recent experiments that have demonstrated ghost imaging using thermal light and photon-correlation measurement [12–14]; these results accord fully with the theory developed in Ref. [7]. However, a principal advantage of using two-photon light in such configurations is the absence of the first (background) term in the Siegert relation set forth in Eq. (18), as initially pointed out in Ref. [7]. The second-term in the Siegert relation, which is responsible for the photon-bunching effect, is significantly reduced if the detection time is greater than the coherence time and/or the detection area is greater than the coherence area [19]; this leads to the undesirable result that the background term dominates.

Quantum microscopy and quantum lithography.—A fundamental difference between photon-correlation imaging with thermal and two-photon light lies in the conjugate operation that is present in Eq. (14) but absent in Eq. (16). This operation converts a diverging wave into a converging wave, an effect that can be achieved with a lens. However, when the positions of the two detectors coincide, the absence of the conjugation offers an imaging paradigm that cannot be achieved using thermal light: entangled-photon microscopy [15] and quantum lithography [16].

The probability of two-photon coincidence at a point $\mathbf{x}$, measured by a two-photon absorber, is then proportional to $\Gamma^{(2)}(\mathbf{x}, \mathbf{x})$. For two-photon light, Eqs. (7) and (16) yield a rate of two-photon absorption proportional to

$$\Gamma^{(2)}(\boldsymbol{\rho}_a, \boldsymbol{\rho}_a) = \left| \int \zeta_s(\boldsymbol{\rho}) h^2(\boldsymbol{\rho}_a, \boldsymbol{\rho}, \omega) d\boldsymbol{\rho} \right|^2, \qquad (19)$$

where we have assumed that a monochromatic filter is used to select $\omega_1 = \omega_2 = \omega$. For thermal light, in contrast, Eqs. (18) and (14) yield

$$\Gamma^{(2)}(\boldsymbol{\rho}_a, \boldsymbol{\rho}_a) = [\Gamma^{(1)}(\boldsymbol{\rho}_a, \boldsymbol{\rho}_a)]^2 + \left| \int I_s(\boldsymbol{\rho}) |h(\boldsymbol{\rho}_a, \boldsymbol{\rho}, \omega)|^2 d\boldsymbol{\rho} \right|^2. \qquad (20)$$

Consider now a simple lens system with the Green's function $h(\boldsymbol{\rho}_a, \boldsymbol{\rho}, \omega) \propto \exp(-ik\boldsymbol{\rho}_a \cdot \boldsymbol{\rho}/f)$. For two-photon light the spatial distribution of light detected by the two-photon absorber is

$$\Gamma^{(2)}(\boldsymbol{\rho}_a, \boldsymbol{\rho}_a) = \left| \int \zeta_s(\boldsymbol{\rho}) \exp[-i2k\boldsymbol{\rho}_a \cdot \boldsymbol{\rho}/f] d\boldsymbol{\rho} \right|^2, \qquad (21)$$

whereas for thermal light it is totally insensitive to the source distribution. The quadratic form of h in Eq. (19) provides the factor of 2 enhancement in imaging resolution. Equation (21) reveals that a double-slit source provides a sinusoidal distribution at twice the spatial frequency, an advantage promised by quantum lithography that cannot be realized by using thermal light. However, Eq. (19) establishes the origin of the resolution enhancement in the squared Green's function, which has a reduced width. That more general result is applicable to masks with *arbitrary* spatial distribution.

This work was supported by the NSF Center for Subsurface Sensing and Imaging Systems (CenSSIS), the Defense Advanced Research Projects Agency (DARPA), the David and Lucile Packard Foundation, and a U.S. Army Research Office (ARO) Multidisciplinary University Research Initiative (MURI) Grant.

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[1] Emil Wolf, Nuovo Cimento **12**, 884 (1954).
[2] P. H. van Cittert, Physica (Utrecht) **1**, 201 (1934).
[3] F. Zernike, Physica (Utrecht) **5**, 785 (1938).
[4] R. J. Glauber, Phys. Rev. **130**, 2529 (1963).
[5] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, UK, 1995).
[6] D. N. Klyshko, *Photons and Nonlinear Optics* (Gordon and Breach, New York, 1988).
[7] B. E. A. Saleh *et al.*, Phys. Rev. A **62**, 043816 (2000).
[8] A. V. Belinskii and D. N. Klyshko, Zh. Eksp. Teor. Fiz. **105**, 487 (1994) [Sov. Phys. JETP **78**, 259 (1994)].
[9] T. B. Pittman *et al.*, Phys. Rev. A **52**, R3429 (1995).
[10] A. F. Abouraddy *et al.*, Phys. Rev. Lett. **87**, 123602 (2001).
[11] A. F. Abouraddy *et al.*, J. Opt. Soc. Am. B **19**, 1174 (2002).
[12] R. S. Bennink, S. J. Bentley, and R. W. Boyd, Phys. Rev. Lett. **89**, 113601 (2002); **92**, 033601 (2004).
[13] F. Ferri *et al.*, Phys. Rev. Lett. **94**, 183602 (2005).
[14] J. Xiong *et al.*, Phys. Rev. Lett. **94**, 173601 (2005).
[15] M. C. Teich and B. E. A. Saleh, Cesk. Cas. Fyz. **47**, 3 (1997); U.S. Patent No. 5 796 477 (1998).
[16] A. N. Boto *et al.*, Phys. Rev. Lett. **85**, 2733 (2000).
[17] C. K. Law, I. A. Walmsley, and J. H. Eberly, Phys. Rev. Lett. **84**, 5304 (2000).
[18] C. K. Law and J. H. Eberly, Phys. Rev. Lett. **92**, 127903 (2004).
[19] B. E. A. Saleh, *Photoelectron Statistics* (Springer, New York, 1978).
[20] R. Hanbury Brown, *The Intensity Interferomenter* (Taylor and Francis, London, 1974).