Quantum Imaging Technologies: Entanglement Utilizing Complex Pump Mode Patterns

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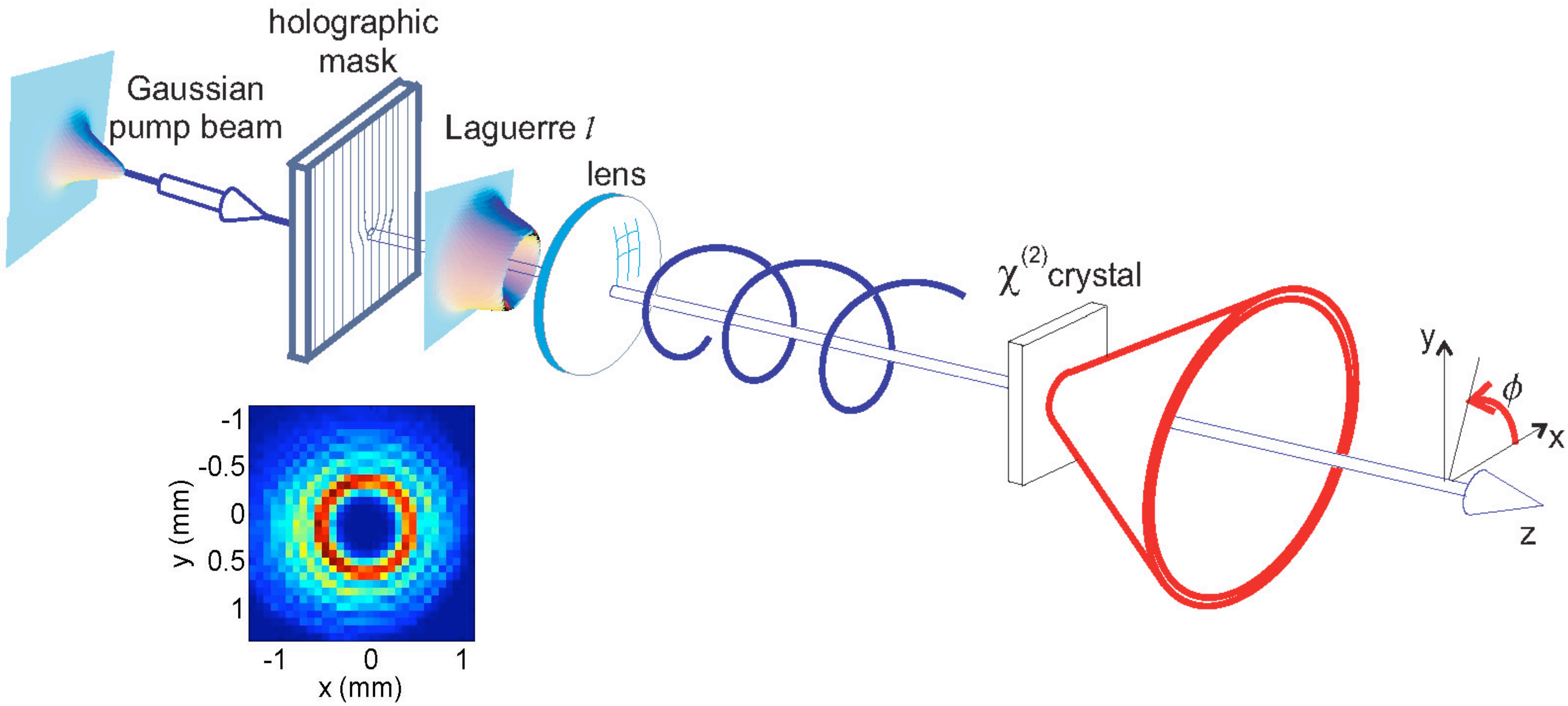
Our Motivation

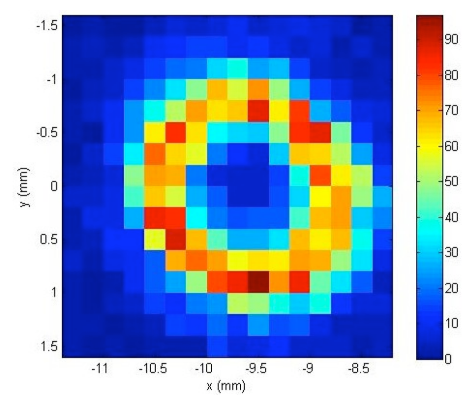
Parametric down conversion with generalized pumps

- In particular, pumps carrying orbital angular momentum
  - Arnaut and Barbosa, PRL **85**, 286 (2001)
- Manifestation of conservation of OAM in type-I parametric down conversion
  - Mair, Vaziri, Weihs, and Zeilinger, Nature **412**, 313 (2001)
- Coincident spot is predicted to be split, another manifestation of OAM
  - Barbosa, Euro Phys. Jour. D **22**, 433 (2003)
- *We demonstrated this feature for the first time in 2005*
  - Altman *et al*., PRL 2005
- What about type-II SPDC and consequences on hyper-entanglement?

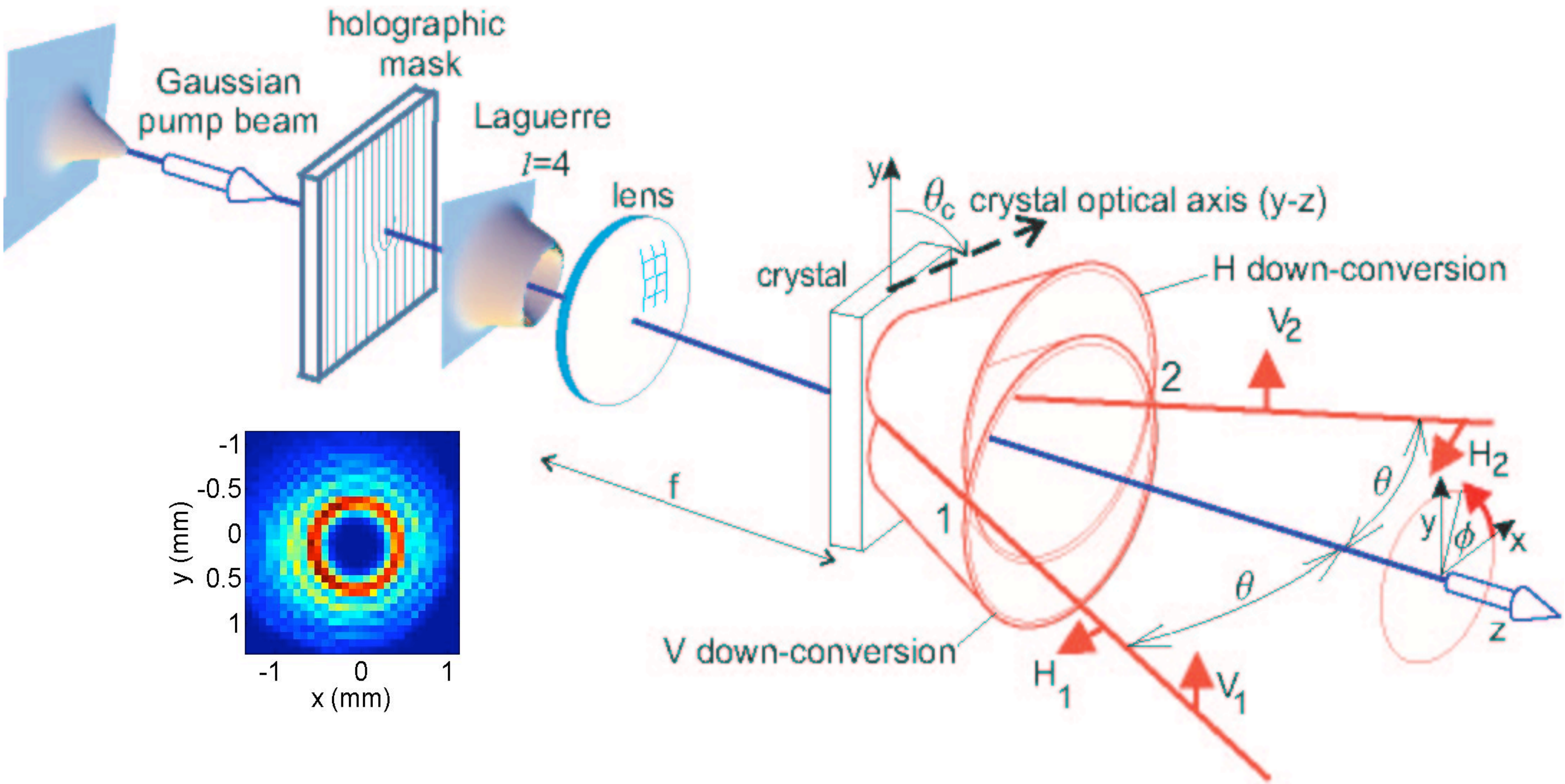
Type-I SPDC: OAM Conserved



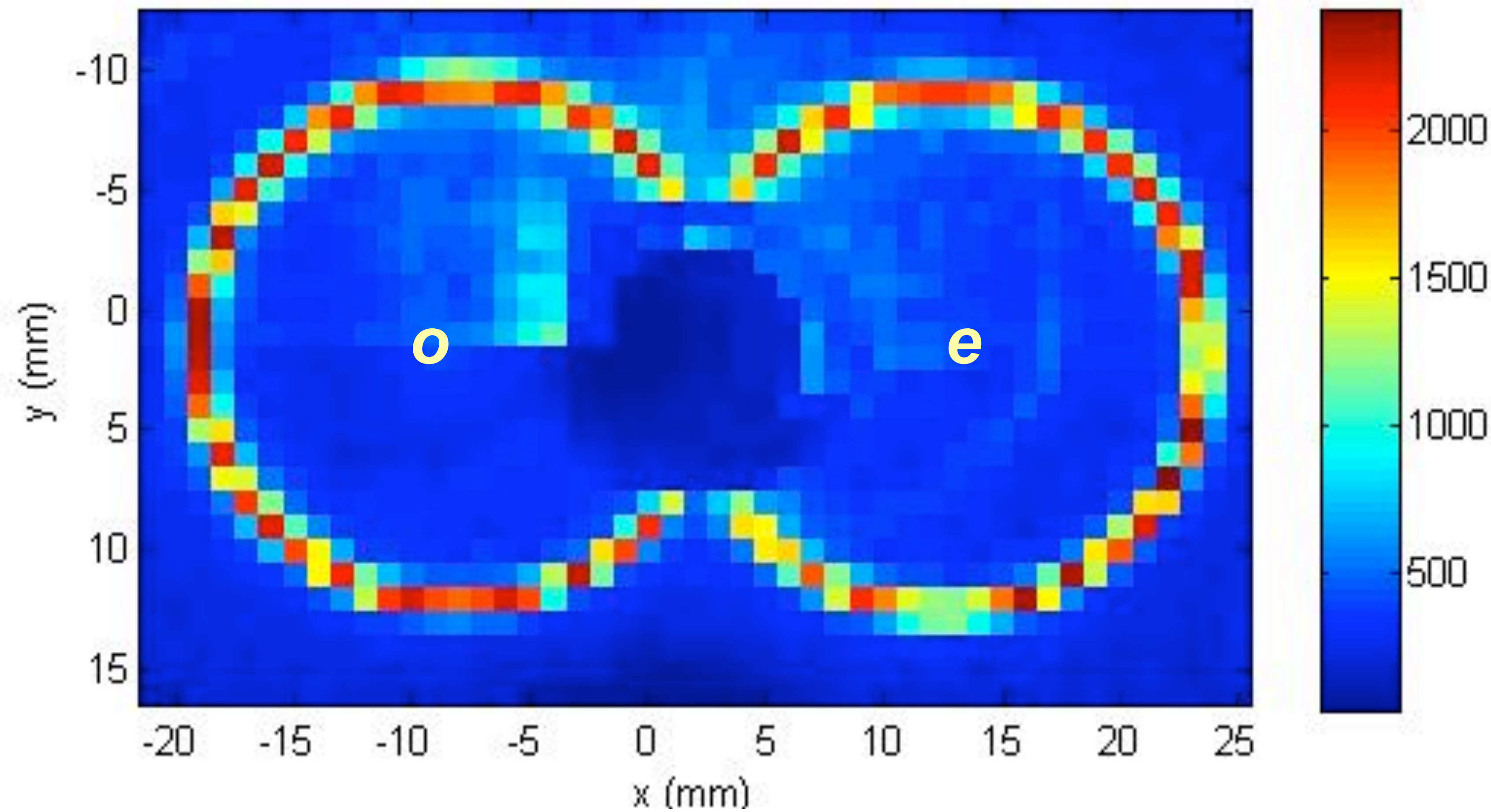
Transverse profile of pump beam
= Biphoton detection amplitude



Type-II Experimental Setup

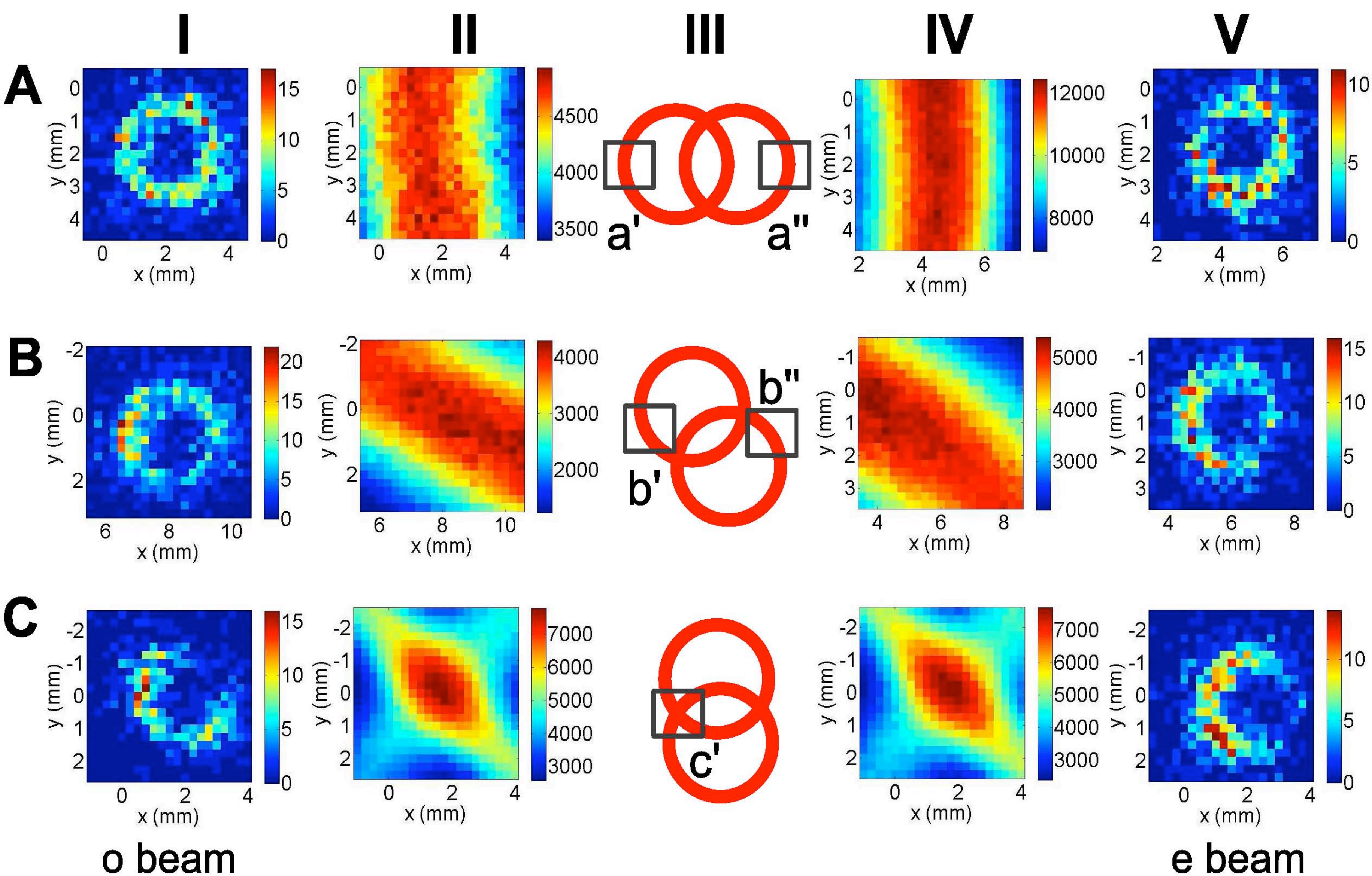


Singles-Count Image in Type-II SPDC



One-Photon Wave Function

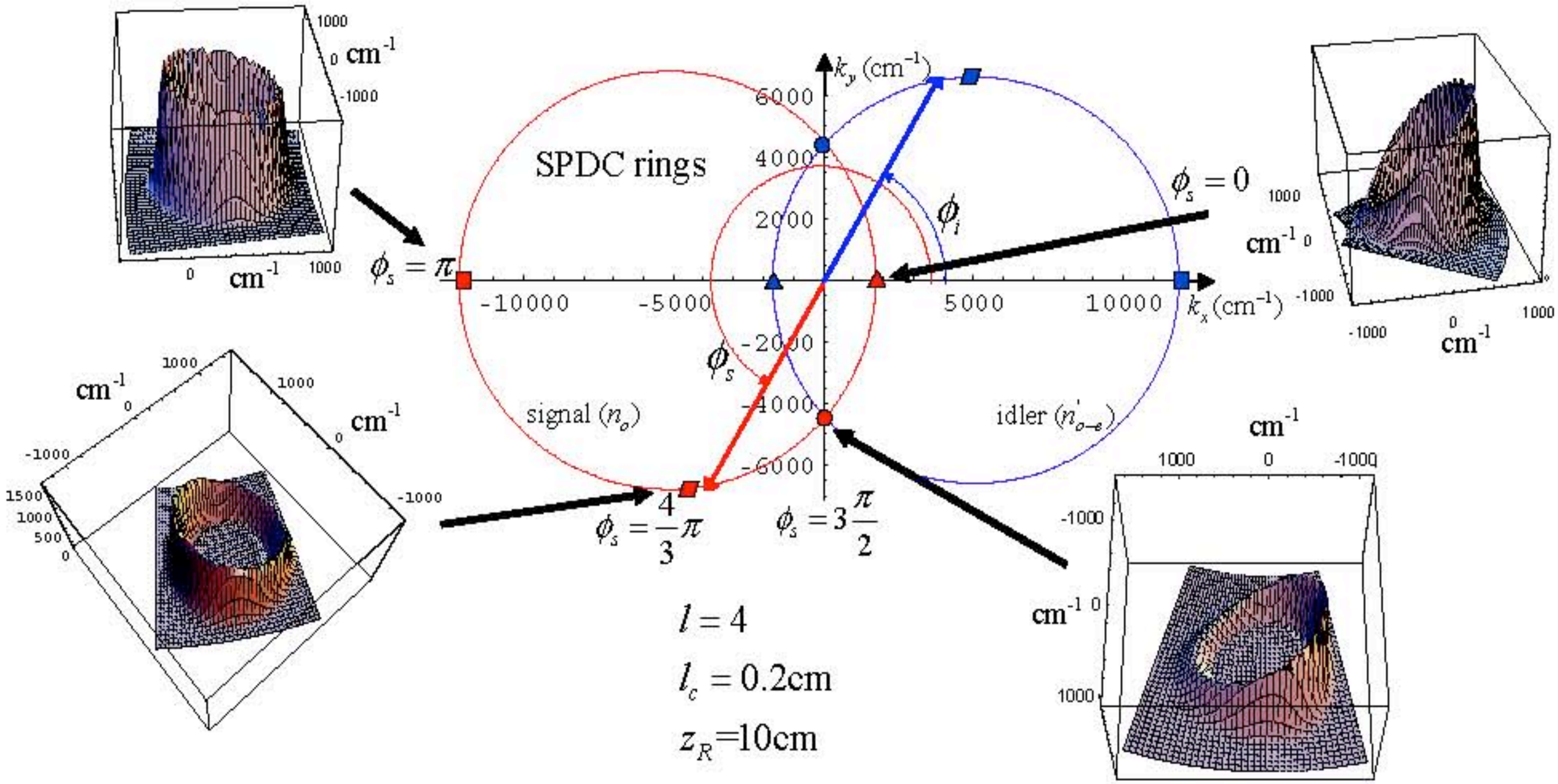
Experimental Results



Calculated Structures in Type-II SPDC

Obtained by phase matching considerations: Barbosa, PRA 76, 033821 (2007)

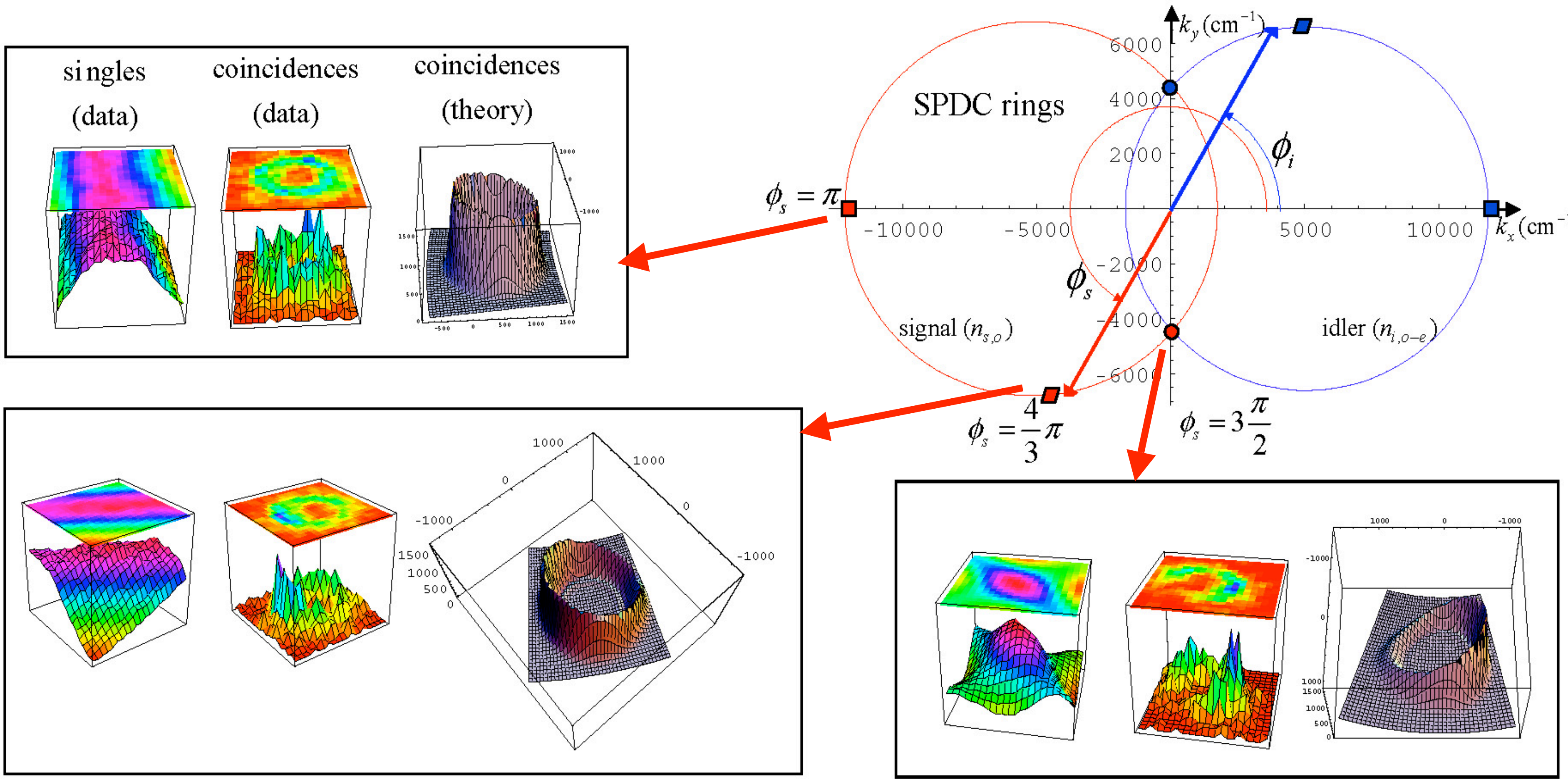
$$P_{scatt} \sim \frac{1}{|A_{\mathbf{k},s;\mathbf{k}',s'}|^2} \times \left| \frac{d^3k\, d^3k'}{d\omega\, d\omega'\, d\theta\, d\theta' d\phi\, d\phi'} F_{s,s'}(\mathbf{k},\mathbf{k}') \right|^2$$



Deformed donut-like structures,

a signature of azimuthal breaking symmetry in SPDC

Comparison with Experiments



OAM in One and Two Beam Cases

- **OAM of one-beam light:** (Arnaut and Barbosa, PRL 2000)

$$\left|\psi_1(t)\right\rangle = \sum_{\mathbf{k}} g(\mathbf{k},t) e^{i l \phi_{\mathbf{k}}} a^{+}(\mathbf{k}) \left|0\right\rangle$$

$\phi_{\mathbf{k}}$ is the azimuthal angle of the wave vector $\mathbf{k}$

$\mathrm{g}(\mathbf{k},t) = \mathrm{g}(p_\rho, k_z, t)$ is independent of $\phi_{\mathbf{k}}$, $p_\rho = \sqrt{\mathbf{k}^2 - k_z^2}$

l gives the value of OAM $\alpha = l\hbar$

The one-photon detection amplitude is measurable:

$$\varphi_1^l(\mathbf{q}_\rho, t) \equiv \left\langle 0 \right| \hat{E}^{(+)}(\mathbf{r}) \left|\psi_1(t)\right\rangle = \sum_{\mathbf{p}_\rho} h(p_\rho, t) e^{i l \phi_p} e^{i \mathbf{p}_\rho \cdot \mathbf{q}_\rho}$$

- **OAM of two-beam light:**

$$\left|\psi_2(t)\right\rangle = \sum_{\mathbf{k}_s \mathbf{k}_i} g_s(\mathbf{k}_s, t) g_i(\mathbf{k}_i, t) e^{i(l_s\phi_s + l_i\phi_i)} a_s^{+}(\mathbf{k}_s) a_i^{+}(\mathbf{k}_i) \left|0\right\rangle$$

The two-photon detection amplitude (**TPDA**):

$$\varphi_2^l(\mathbf{q}_s, \mathbf{q}_i, t) = \sum_{\mathbf{p}_s, \mathbf{p}_i} h_s(p_s, t) h_i(p_i, t) e^{i(l_s\phi_s + l_i\phi_i)} e^{i(\mathbf{p}_s \cdot \mathbf{q}_s + \mathbf{p}_i \cdot \mathbf{q}_i)}$$

$l = l_s + l_i$ gives the value of total OAM $\alpha = l\hbar$

Intrinsic and Extrinsic OAM

Feng *et al.* *PRL* **101**, 163602 (2008)

- **IOAM and EOAM in two-beam systems**

TPDA in terms of joint variables: $\mathbf{p}_\pm = \mathbf{p}_s \pm \mathbf{p}_i$, $\mathbf{q}_\pm = \mathbf{q}_s \pm \mathbf{q}_i$

$$\varphi_2^l(\mathbf{q}_+,\mathbf{q}_-,t)[\equiv \varphi_2^l(\mathbf{q}_s,\mathbf{q}_i,t)]$$

$$= \sum_{n_s,n_i}^{l_s,l_i}\sum_m 2^{-(l_s+l_i)}(-1)^{l_i-n_i}\binom{l_s}{n_s}\binom{l_i}{n_i}\times\varphi_2^{(l,m,n_s,n_i)}(\mathbf{q}_+,\mathbf{q}_-,t)$$

where

$$\varphi_2^{(l,m,n_s,n_i)}(\mathbf{q}_+,\mathbf{q}_-,t) = \sum_{\mathbf{p}_+,\mathbf{p}_-} g_2(p_+,p_-,t)e^{-i(l_+\phi_+ + l_-\phi_-)}e^{-i(\mathbf{p}_+\mathbf{q}_+ + \mathbf{p}_-\mathbf{q}_-)/2}$$

$$l_+ = m + n_s + n_i\ , \quad l_- = (l_s + l_i) - (m + n_s + n_i)$$

$$l_+ + l_- = l_s + l_i$$

- **Only IOAM studied so far in SPDC processes**

Extrinsic OAM in SPDC

Feng *et al.* *PRL* **101**, 163602 (2008)

- **EOAM existence is determined by symmetry**

Two-photon detection amplitude in transverse planes:

$$\varphi_2^l(\mathbf{q}_+,\mathbf{q}_-) = R_+(\mathbf{q}_+)\,R_-(\mathbf{q}_-)$$

The part of OAM in *center-of-momentum* movement: IOAM

$$R_+(\mathbf{q}_+) = \int d^2 p_+\, e^{i(\mathbf{p}_+\cdot\mathbf{q}_+/2 - p_+^2 c z_0/4\bar{\omega})} F_+(\mathbf{p}_+)$$

$$F_+(\mathbf{p}_+) = \left[B^{(lp)} p_+^l L_p^l\left(z_R p_+^2/k_P\right) e^{-z_R p_+^2/2k_P}\right] e^{il\phi_+}$$

$$R_-(\mathbf{q}_-) = \int d^2 p_-\, e^{i(\mathbf{p}_-\cdot\mathbf{q}_-/2 - p_-^2 c z_0/4\bar{\omega})} F_-(\mathbf{p}_-)$$

$$F_-(\mathbf{p}_-) = [V^2 t_{\text{int}}/4(2\pi)^6] \int d\omega D(\omega) W[\Delta k_z(\omega,\mathbf{p}_-)]$$

EOAM

EOAM in SPDC

- **EOAM existence is determined by symmetry**

$$F_+(\mathbf{p}_+) = [B^{(lp)} p_+^l L_p^l (z_R p_+^2 / k_P) e^{-z_R p_+^2 / 2k_P}] e^{il\phi_+}$$

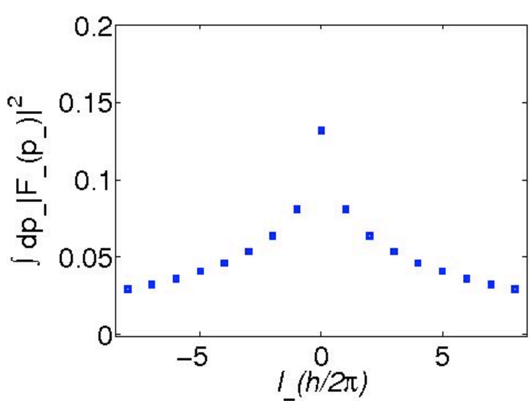
→ **IOAM equal to pump OAM**

$$F_-(\mathbf{p}_-) = [V^2 t_{\text{int}} / 4(2\pi)^6] \int d\omega D(\omega) W[\Delta k_z(\omega, \mathbf{p}_-)]$$

$$= \sum_m F_-^{(m)}(p_-) e^{im\phi_-}$$

→ **The part of OAM in *relative* movement: EOAM**

TOAM = IOAM + EOAM



Conclusions

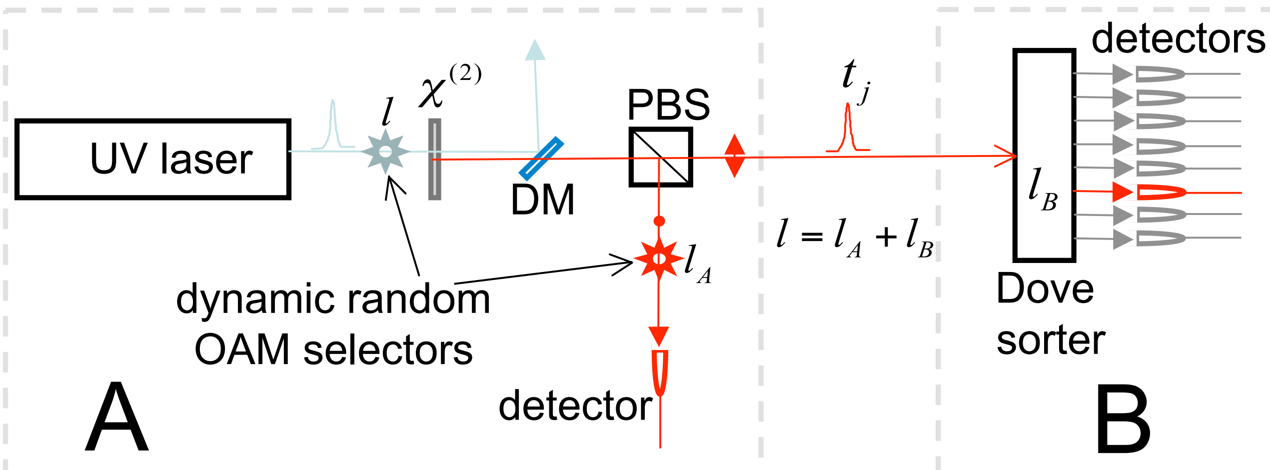
Feng *et al.* *PRL* **101**, 163602 (2008)

- Extrinsic OAM is a non-negligible part of OAM when a two-beam system is considered.
- EOAM exists in type-II SPDC processes.
- EOAM is the key to understand the OAM conservation rule in SPDC processes.
- EOAM is an unexplored freedom degree as OAM is studied for practical applications.
- New techniques are needed to measure EOAM.

OAM Modes for Cryptography

OPTICS LETTERS 18, 2119 (2008), Barbosa

Consider cryptographic setup where l is the key to be shared.
Alice's system generates l and randomly detects l_A while Bob detects l_B.
Over a public channel, A informs B the value l_A.
B obtains $l_A + l_B$ $(= l$, in principle).
l is then shared by A and B?



If detection geometries are not carefully designed to avoid severe restrictions collected wave vectors l may not be obtained (= a large number of errors will be made).

WHY?
The l index is NOT attached to a single wave vector but it is a mode-property.

Problems Under Study

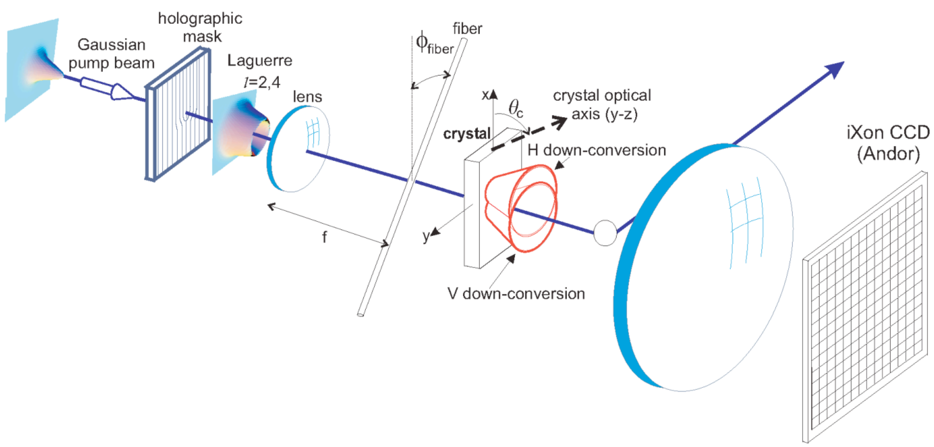
1. Rotation sensor

intends to develop a method using OAM modes to increase resolution in rotation measurements according to: $\Delta\phi\Delta L \geq \hbar / 2$

Use of l modifies conventional Standard Quantum limit to $\Delta\phi = \frac{1}{l\sqrt{N}}$, or Heisenberg's limit to $\Delta\phi = \frac{1}{lN}$

S. M. Barnett, C. Fabre, and A. Maitre, Eur. Phys. J. D **22**, 513-519 (2003).
S. M. Barnett, R. Zambrini, J. of Modern Optics **53**, 613-625 (2006).

A possible setup:



The need for multi-pixel correlation with single photon-sensitivity cameras has been problematic: Electronic noise – including from electrons dragged in the readout processes– has been a killer.

2. Quantum Cryptography with OAM modes (large alphabets)

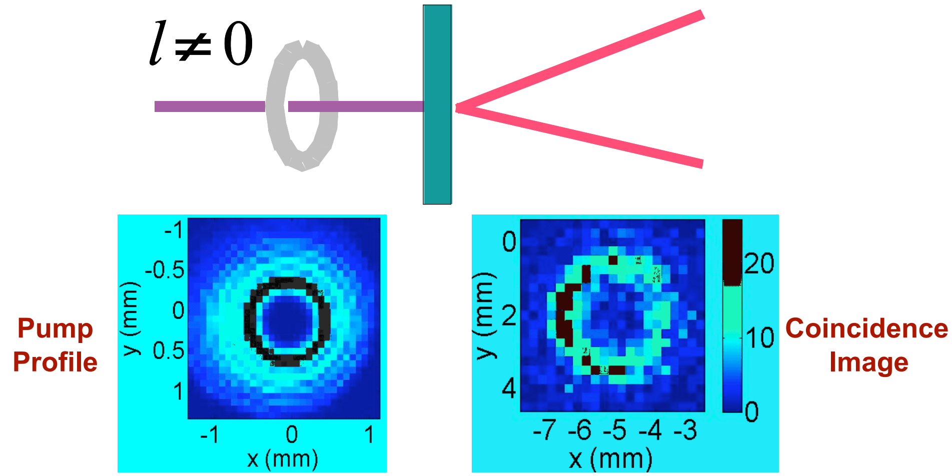
Study details for specific geometries aiming to increase detection efficiencies

Entanglement Utilizing Complex Pump Mode Patterns

Objectives

- Develop a source of hyper-entangled photons based on orbital angular momentum (OAM) of light
- Measure and quantify the degree of spatial correlation of the hyper-entangled state of light
- Investigate potential applications for photons with OAM

SPDC with OAM -Carrying Pump



Approach

- Apply coincidence imaging methods to investigate OAM transfer from pump to down-converted photons
- Investigate implications on generation of hyper entanglement with SPDC
- Quantum half-adder as a test platform for linear optics quantum computation with hyper-entangled photons
- OAM modes for cryptography

Accomplishments

- Demonstrated quantum image transfer in type-I SPDC – pumps with OAM and resolution test patterns
- Demonstrated violation of OAM conservation in type-II SPDC
- Developed theory to explain azimuthal-asymmetry induced non-conservation of OAM in type-II SPDC. Transverse coincidence structures calculated. PRA 76, 033821 (2007)
- Proposed and tested a method to measure the degree of violation of OAM non-conservation OPTICS LETTERS 18, 2119 (2008)
- Developed theory emphasizing geometrical problems to be avoided when using OAM modes for cryptography:
- Proposed Quantum Half-Adder: PRA 73, 052321 (2006)

Continuation:
OAM modes for cryptography
some problems to be avoided

The l index is NOT attached to a single wave vector but it is a property of the **mode** :

$$\hat{\mathbf{E}}^{(+)}(\boldsymbol{r},t) = \sum_{\boldsymbol{k}} l_E(\boldsymbol{k}) a_{\boldsymbol{k}} e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega_k t)} \mathbf{e}_{\boldsymbol{k}} \quad \text{(electric field operator)}$$

$$U_l(\boldsymbol{r},t) = \sum_{\boldsymbol{k}} U_{\boldsymbol{k},l} e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega_k t)} \quad \text{(spatial OAM mode decomposed in plane waves)}$$

$$\sum_l U_{\boldsymbol{k},l} U^*_{l,\boldsymbol{k}} = \delta_{\boldsymbol{k},\boldsymbol{k}'}$$

$$\Downarrow$$

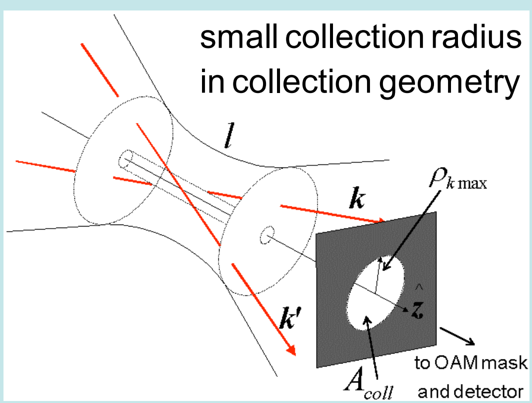
$$\hat{\mathbf{E}}^{(+)}(\boldsymbol{r},t) = \sum_l \left[\sum_{\boldsymbol{k}'} U^*_{l,\boldsymbol{k}'} l_E(\boldsymbol{k}') a_{\boldsymbol{k}'} \mathbf{e}_{\boldsymbol{k}'} \right] \left[\sum_{\boldsymbol{k}} U_{\boldsymbol{k},l} e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega_k t)} \right] = \sum_l \boldsymbol{c}_l U_l(\boldsymbol{r},t)$$

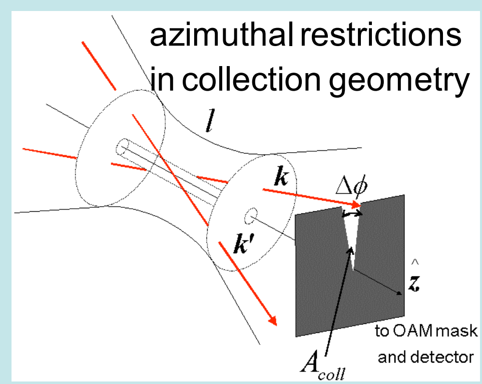
$\boldsymbol{c}_l$ → annihilation operator of a photon in the l mode

$\boldsymbol{c}_l$ depends on a set of wave vectors to define a mode

Therefore, geometric restrictions in the detection setup leads to a poor distinguishability of the mode

EXAMPLES :





Continuation:
OAM modes for cryptography
some problems to be avoided

How to make (simplified) estimates of poor detection without relying on specific geometries?

Use normalized wave state's amplitude probability written as

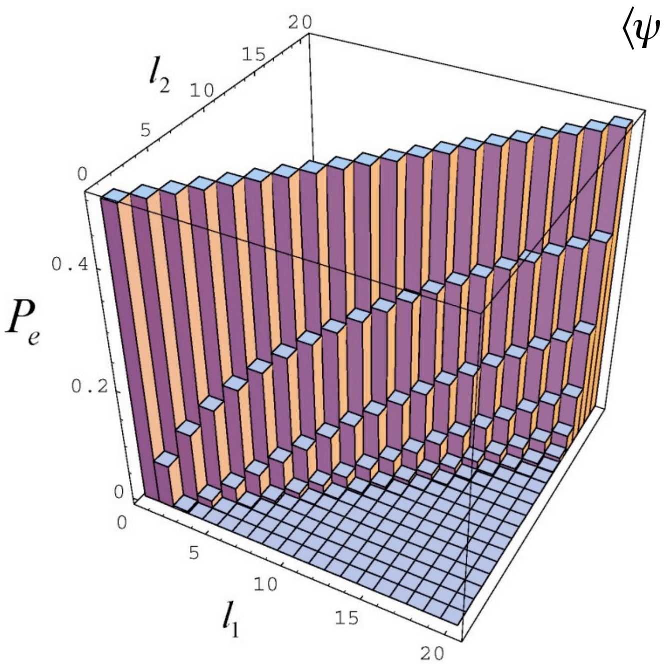
$$\psi_{lp}(\xi) = i(-1)^l \frac{e^{-\xi/2}\xi^{l/2}L_l^p(\xi)}{\sqrt{\int_0^\infty e^{-\xi}\xi^l L_l^p(\xi)^2}} \;, \qquad \left(\rho_k = \sqrt{\Delta k_x^2 + \Delta k_y^2}\;, \quad \Delta\boldsymbol{k} = \boldsymbol{k} + \boldsymbol{k}' - \boldsymbol{k}_P\right)$$

where two-point variable $\xi = \frac{z_R}{k_P}\rho_k^2$ depends on signal and idler wave vectors.

Use Helstrom's bound for binary decisions between two pure states $|\psi_{l_j p_j}\rangle$ and $|\psi_{l_k p_k}\rangle$:

$$P_e = \frac{1}{2}\left(1 - \sqrt{1 - |\langle\psi_{l_j p_j}|\psi_{l_k p_k}\rangle|^2}\right)$$

$$\langle\psi_{l_j p_j}|\psi_{l_k p_k}\rangle \to \int_0^\infty \psi^*_{l_j p_j}(\xi)\psi_{l_k p_k}(\xi)d\xi$$



Geometries that creates wave vector restrictions may lead to a large fraction of errors by A and B and may render the cryptographic systems useless.

Calculation for specific geometries have to be done with variables ***k*** and ***k'*** instead. The simplified mapping above is not sufficient.