



# Quantum Imaging Theory

Jonathan P. Dowling

Hearne Institute for Theoretical Physics  
Department of Physics & Astronomy  
Louisiana State University  
Baton Rouge, Louisiana USA

[quantum.phys.lsu.edu](http://quantum.phys.lsu.edu)

*Quantum Imaging MURI Program Review, 17 NOV 08, UMBC*

## Quantum Interferometric Optical Lithography: Exploiting Entanglement to Beat the Diffraction Limit

Agedi N. Boto,<sup>1</sup> Pieter Kok,<sup>2</sup> Daniel S. Abrams,<sup>1</sup> Samuel L. Braunstein,<sup>2</sup>  
Colin P. Williams,<sup>1</sup> and Jonathan P. Dowling<sup>1,\*</sup>

Classical optical lithography is diffraction limited to writing features of a size  $\lambda/2$  or greater, where  $\lambda$  is the optical wavelength. Using nonclassical photon-number states, entangled  $N$  at a time, we show that it is possible to write features of minimum size  $\lambda/(2N)$  in an  $N$ -photon absorbing substrate. This result allows one to write a factor of  $N^2$  more elements on a semiconductor chip. A factor of  $N = 2$  can be achieved easily with entangled photon pairs generated from optical parametric down-conversion. It is shown how to write arbitrary 2D patterns by using this method.

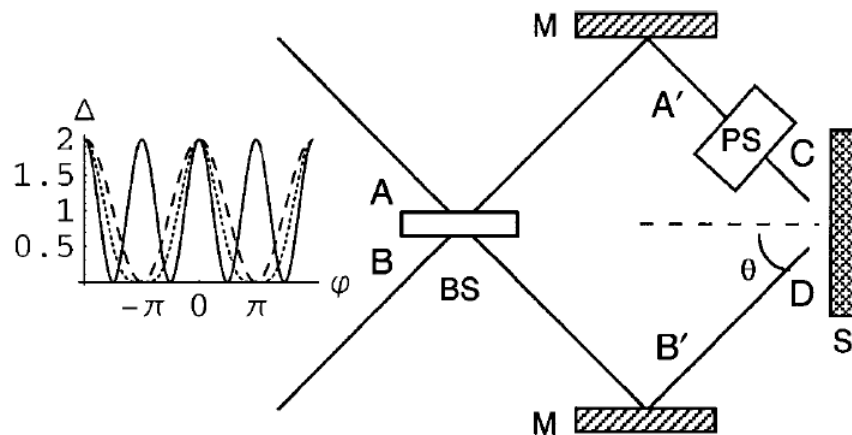


FIG. 1. Interferometric lithography set up utilizing photons entering ports  $A$  and  $B$ . The photons strike the symmetric, lossless, beam splitter (BS) and then reflect off the mirrors (M). The photon amplitude in the upper path accumulates a phase shift  $\varphi$  at the phase shifter (PS), before the two branches interfere on the substrate. Inset: The deposition rate  $\Delta$  as a function of the phase shift  $\varphi$  for uncorrelated single-photon absorption (dashed), uncorrelated two-photon absorption (dotted), and entangled two-photon absorption (solid). Note that the classical two-photon curve has narrower features than the one-photon, but that the entangled two-photon has even narrower features still, and also shows the critical halving of the peak separation.



# PROGRAMMATICS

---

## Personnel:

Faculty: JP Dowling & HW Lee

Postdocs: Hugo Cable / Petr Anisimov

Grads: Ryan Glasser and William Plick

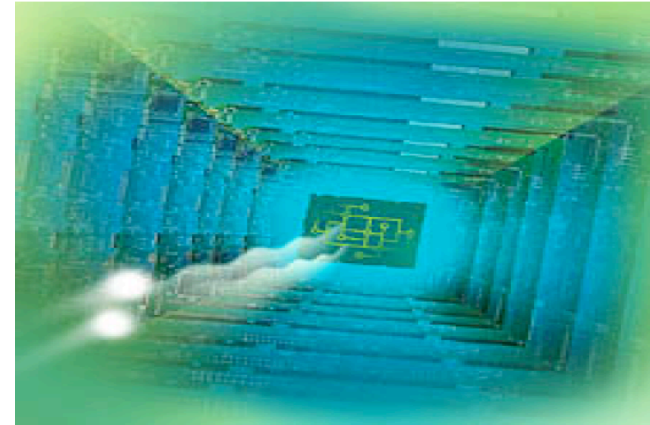
## Facilities:

4 Dell Work Stations – 2 From DURIP

## Objective:

- Entangled Photons Beat Diffraction Limit
- Lithography With Long-Wavelengths
- Dispersion Cancellation
- Masking Techniques
- N-Photon Resists

## New York Times



## Approach:

- Investigate Which States are Optimal
- Design Efficient Quantum State Generators
- Investigate Masking Systems
- Develop Theory of N-Photon Resist
- Integrate into Optical System Design

## Accomplishments FY08:

- Properties of NOON States
- Efficient NOON Generators
- Bright NOON Generators Thy/Exp
- Masking Lithography
- N-Photon Absorption
- Loss in NOON-State Imaging
- Loss in Interaction Free Imaging



# FY08: Quantum Imaging Publications



1. Wildfeuer, CF; Dowling, JP, Strong violations of Bell-type inequalities for Werner-like states, PHYSICAL REVIEW A, 78 (3): Art. No. 032113 SEP 2008
2. Glasser, RT; Cable, H; Dowling, JP, Entanglement-seeded, dual, optical parametric amplification: Applications to quantum imaging and metrology, PHYSICAL REVIEW A, 78 (1): Art. No. 012339 JUL 2008
3. Dowling, JP, Quantum optical metrology - the lowdown on high-NOON states, CONTEMPORARY PHYSICS, 49 (2): 125-143 2008
4. Thanvanthri, S; Kapale, KT; Dowling, JP, Arbitrary coherent superpositions of quantized vortices in Bose-Einstein condensates via orbital angular momentum of light, PHYSICAL REVIEW A, 77 (5): Art. No. 053825 Part B MAY 2008
5. Sciarrino, F; Vitelli, C; De Martini, F; et al., Experimental sub-Rayleigh resolution by an unseeded high-gain optical parametric amplifier for quantum lithography, PHYSICAL REVIEW A, 77 (1): Art. No. 012324 JAN 2008
6. VanMeter, NM; Lougovski, P; Uskov, DB; et al., General linear-optical quantum state generation scheme: Applications to maximally path-entangled states, PHYSICAL REVIEW A, 76 (6): Art. No. 063808 DEC 2007
7. Wildfeuer, CF; Lund, AP; Dowling, JP, Strong violations of Bell-type inequalities for path-entangled number states, PHYSICAL REVIEW A, 76 (5): Art. No. 052101 NOV 2007
8. Cable, H; Dowling, JP, Efficient generation of large number-path entanglement using only linear optics and feed-forward, PHYSICAL REVIEW LETTERS, 99 (16): Art. No. 163604 OCT 19 2007
9. Huver SD, Wildfeuer CF, Dowling JP, Entangled Fock States for Robust Quantum Optical Metrology, Imaging, and Sensing, arXiv:0805.0296.



# FY08: Invited Talks



“Quantum Technologies,” US Army Unified Quest 2009, Emerging Technologies Seminar, 6–9 OCT 2008, McLean.

“Quantum Technologies,” US Army Training and Doctrine Command, Disruptive/Revolutionary Tech Arenas Operation ‘Mad Scientist’ Workshop, 19–21 August 2008, Portsmouth.

“What’s New with NOON States?” SPIE Photonics West: Quantum Electronics Metrology, 19–24 JAN 2008, San Jose.

“Quantum Sensors: The Lowdown on High-NOON”, 38th Winter Colloquium on The Physics of Quantum Electronics, 6–10 January 2008, Snowbird (plenary).







# Team Visitors & Lecturers @ LSU



Robert W. Boyd, M. Parker Givens Professor of Optics and Professor of Physics, The Institute of Optics, University of Rochester, Rochester

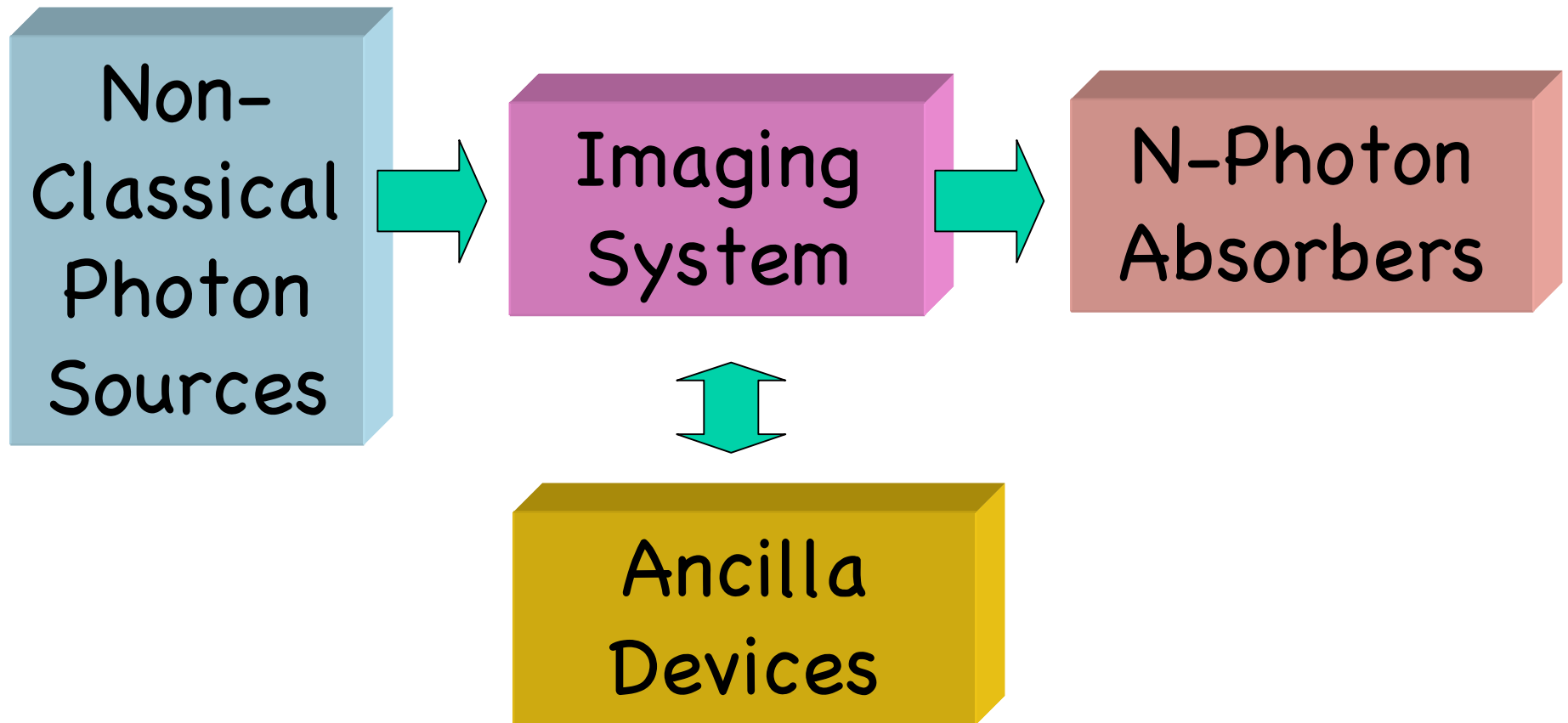
Malvin Teich, Director, Photonics Center, Boston University, Boston.

Jeffrey H. Shapiro, Director Research Laboratory of Electronics and Julius A. Stratton Professor of Electrical Engineering, Department of Electrical Engineering and Computer Science, MIT.

Claude Fabre, Laboratoire Kastler-Brossel Ecole Normale Supérieure and University Pierre et Marie Curie, Paris.

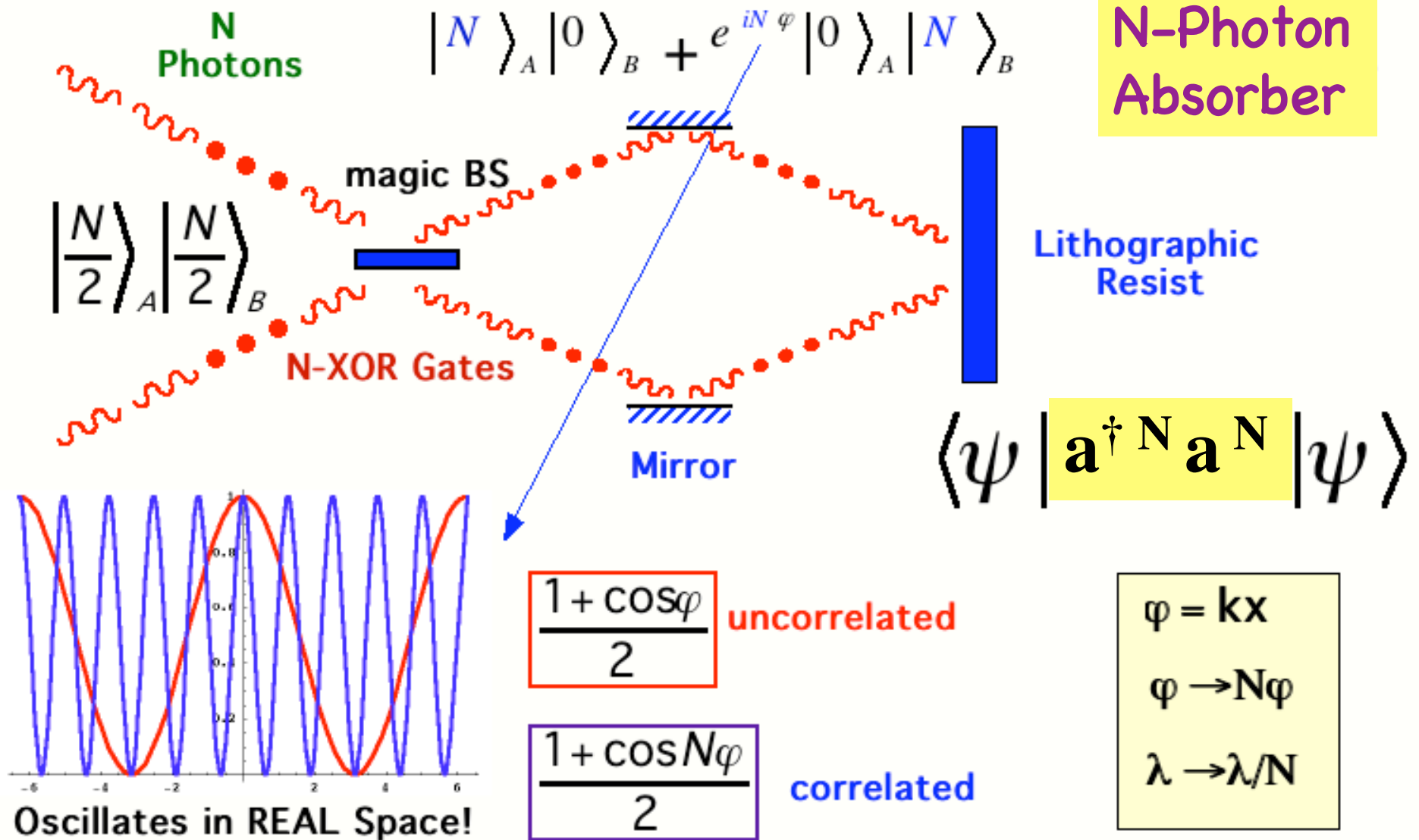
Hans Bachor, Research Director and Federation Fellow, Australian National Centre of Excellence for Quantum-Atom Optics, Australian National University, Canberra.







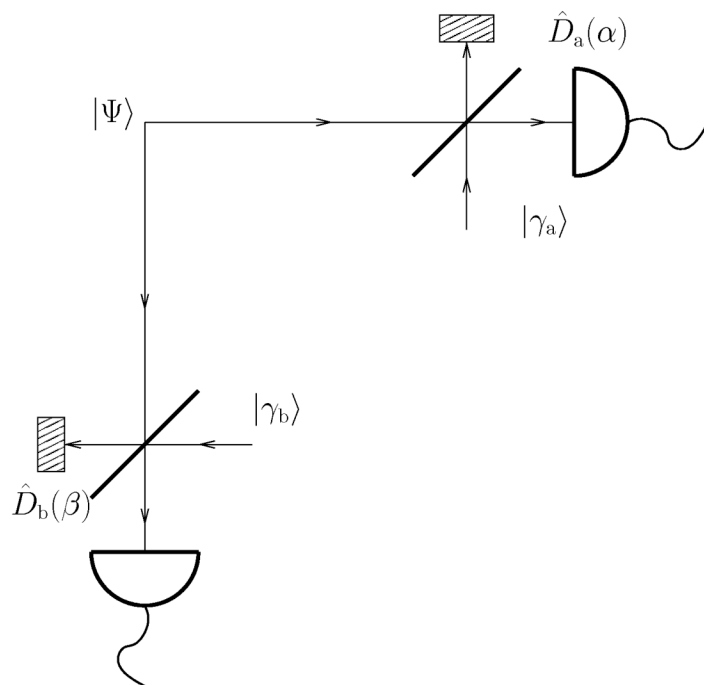
# FROM QUANTUM INTERFEROMETRY TO QUANTUM LITHOGRAPHY



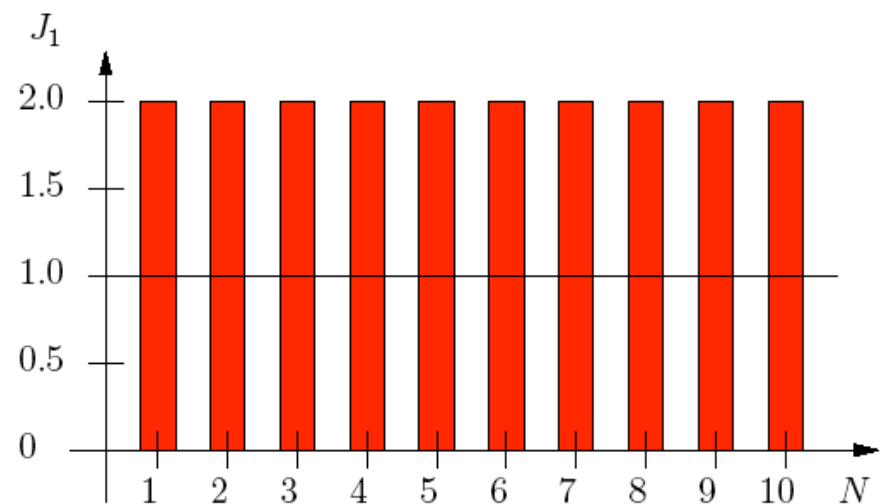
# Strong violations of Bell-type inequalities for path-entangled number states

Christoph F. Wildfeuer,<sup>1,\*</sup> Austin P. Lund,<sup>2</sup> and Jonathan P. Dowling<sup>1</sup>

We show that nonlocal correlation experiments on the two spatially separated modes of a maximally path-entangled number state may be performed. They lead to a violation of a Clauser-Horne Bell inequality for any finite photon number  $N$ . We also present an analytical expression for the two-mode Wigner function of a maximally path-entangled number state and investigate a Clauser-Horne-Shimony-Holt Bell inequality for such a state. We test other Bell-type inequalities. Some are violated by a constant amount for any  $N$ .



$$J_1 = Q(\alpha) + Q(\beta) + Q(\gamma) + Q(\delta) - Q(\alpha, \beta) - Q(\alpha, \gamma) - Q(\alpha, \delta) - Q(\beta, \gamma) - Q(\beta, \delta) - Q(\gamma, \delta),$$



## Efficient Generation of Large Number-Path Entanglement Using Only Linear Optics and Feed-Forward

Hugo Cable\* and Jonathan P. Dowling

*Hearne Institute for Theoretical Physics, Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803, USA*

(Received 30 April 2007; published 18 October 2007)

We show how an idealized measurement procedure can condense photons from two modes into one and how, by feeding forward the results of the measurement, it is possible to generate efficiently superposition states commonly called  $N00N$  states. For the basic procedure sources of number states leak onto a beam splitter, and the output ports are monitored by photodetectors. We find that detecting a fixed fraction of the input at one output port suffices to direct the remainder to the same port, with high probability, however large the initial state. When instead photons are detected at both ports, macroscopic quantum superposition states are produced. We describe a linear-optical circuit for making the components of such a state orthogonal, and another to convert the output to a  $N00N$  state. Our approach scales exponentially better than existing proposals. Important applications include quantum imaging and metrology.

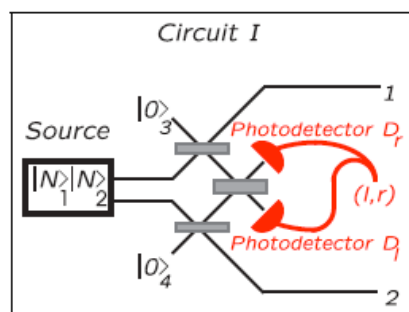


FIG. 1: (color online) Circuit I. Beam splitters couple some fraction  $f$  of the population from the principal modes one and two into ancillae modes three and four, initially the vacuum. The ancillae are combined at a 50:50 beam splitter, and subjected to number-resolving photodetection.

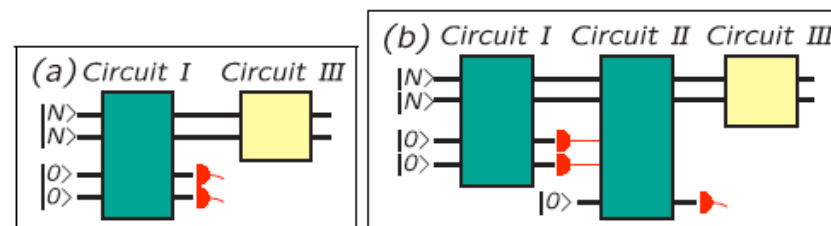


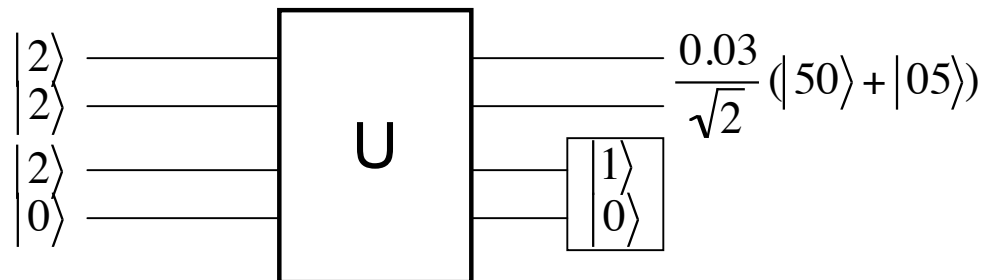
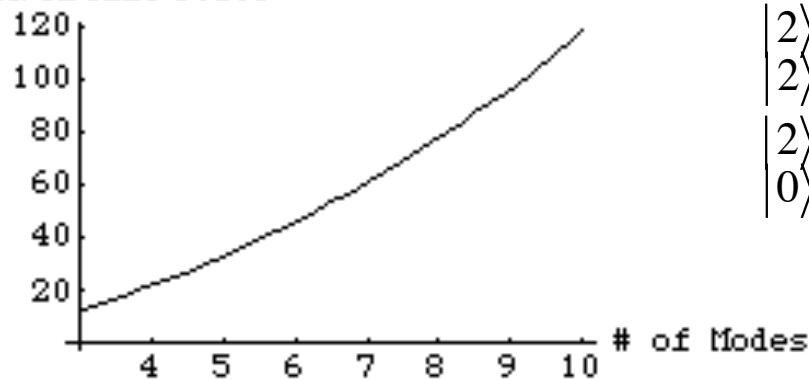
FIG. 2: (a) and (b) illustrate complete  $N00N$ -state generators in outline. Circuit I produces macroscopic superposition states non-deterministically. Circuit III consists of a  $\pi/2$  phase shifter and a 50:50 beam splitter, and performs final conversion to a  $N00N$  state. In (b) the Circuit I detection outcomes are feed forward to Circuit II, which implements a correction step using a beam splitter of variable transmittance and an ancillary mode.

## General linear-optical quantum state generation scheme: Applications to maximally path-entangled states

N. M. VanMeter,<sup>1</sup> P. Lougovski,<sup>1</sup> D. B. Uskov,<sup>1,2</sup> K. Kieling,<sup>3,4</sup> J. Eisert,<sup>3,4</sup> and Jonathan P. Dowling<sup>1</sup>

We introduce a notion of a linear-optical quantum state generator. This is a device that prepares a desired quantum state using product inputs from photon sources, linear-optical networks, and post-selection using photon counters. We show that this device can be concisely described in terms of polynomial equations and unitary constraints. We illustrate the power of this language by applying the Gröbner-basis technique along with the notion of vacuum extensions to solve the problem of how to construct a quantum state generator analytically for any desired state, and use methods of convex optimization to identify success probabilities. In particular, we disprove a conjecture concerning the preparation of the maximally path-entangled NOON-state by providing a counterexample using these methods, and we derive a new upper bound on the resources required for NOON-state generation.

Max NOON of Size State



# Quantum states of light produced by a high-gain optical parametric amplifier for use in quantum lithography

Girish S. Agarwal,<sup>1</sup> Kam Wai Chan,<sup>2</sup> Robert W.

Boyd,<sup>2,3</sup> Hugo Cable,<sup>4</sup> and Jonathan P. Dowling<sup>4,5</sup>

270 J. Opt. Soc. Am. B/Vol. 24, No. 2/February 2007

We present a theoretical analysis of the properties of an unseeded optical parametric amplifier (OPA) used as the source of entangled photons for applications in quantum lithography. We first study the dependence of the excitation rate of a two-photon absorber on the intensity of the light leaving the OPA. We find that the rate depends linearly on intensity only for output beams so weak that they contain fewer than one photon per mode. We also study the use of an  $N$ -photon absorber for arbitrary  $N$  as the recording medium to be used with such a light source. We find that the contrast of the interference pattern and the sharpness of the fringe maxima tend to increase with increasing values of  $N$ , but that the density of fringes and thus the limiting resolution does not increase with  $N$ . We conclude that the output of an unseeded OPA exciting an  $N$ -photon absorber provides an attractive system in which to perform quantum lithography. © 2007 Optical Society of America

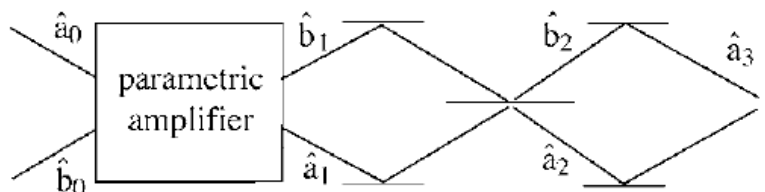


Fig. 1. Schematic of the quantum lithography architecture.

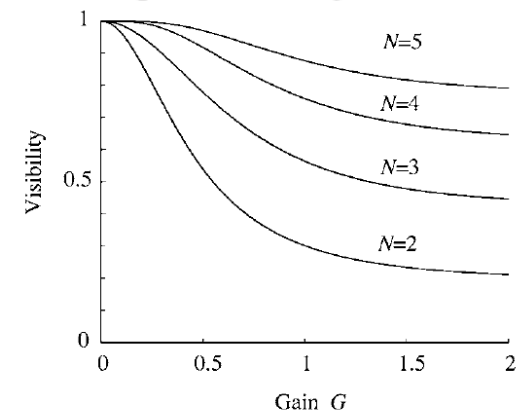


Fig. 3. Fringe visibility  $V(N)$  plotted as a function of the gain  $G$  for various values of the order  $N$  of the multiphoton absorption process.



# Experimental sub-Rayleigh resolution by an unseeded high-gain optical parametric amplifier for quantum lithography

Fabio Sciarrino,<sup>1,2</sup> Chiara Vitelli,<sup>1</sup> Francesco De Martini,<sup>1</sup> Ryan Glasser,<sup>3</sup> Hugo Cable,<sup>3</sup> and Jonathan P. Dowling<sup>3</sup>

Quantum lithography proposes to adopt entangled quantum states in order to increase resolution in interferometry. In the present paper we experimentally demonstrate that the output of a high-gain optical parametric amplifier can be intense yet exhibits quantum features, namely, sub-Rayleigh fringes, as proposed by Agarwal et al. (Phys. Rev. Lett. **86**, 1389 (2001)). We investigate multiphoton states generated by a high-gain optical parametric amplifier operating with a quantum vacuum input for a gain values up to 2.5. The visibility has then been increased by means of three-photon absorption. The present article opens interesting perspectives for the implementation of such an advanced interferometrical setup.

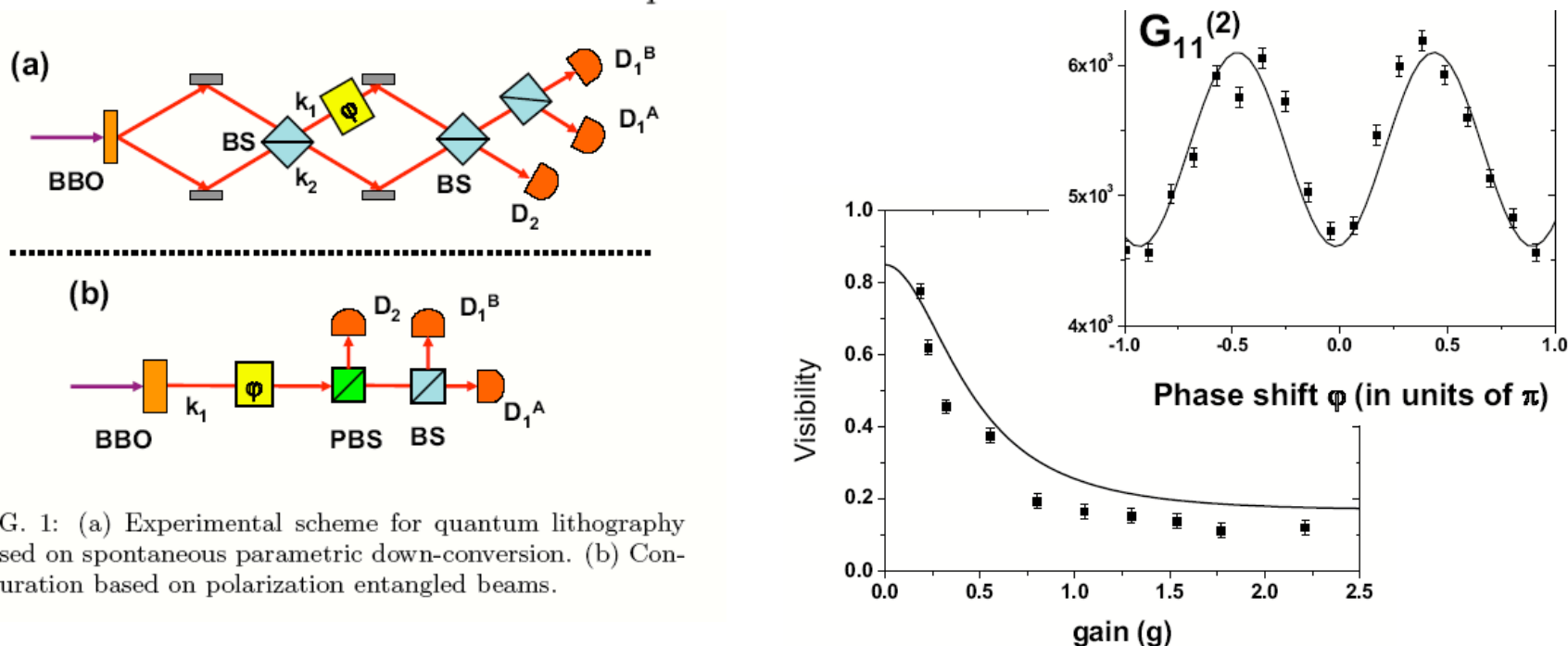
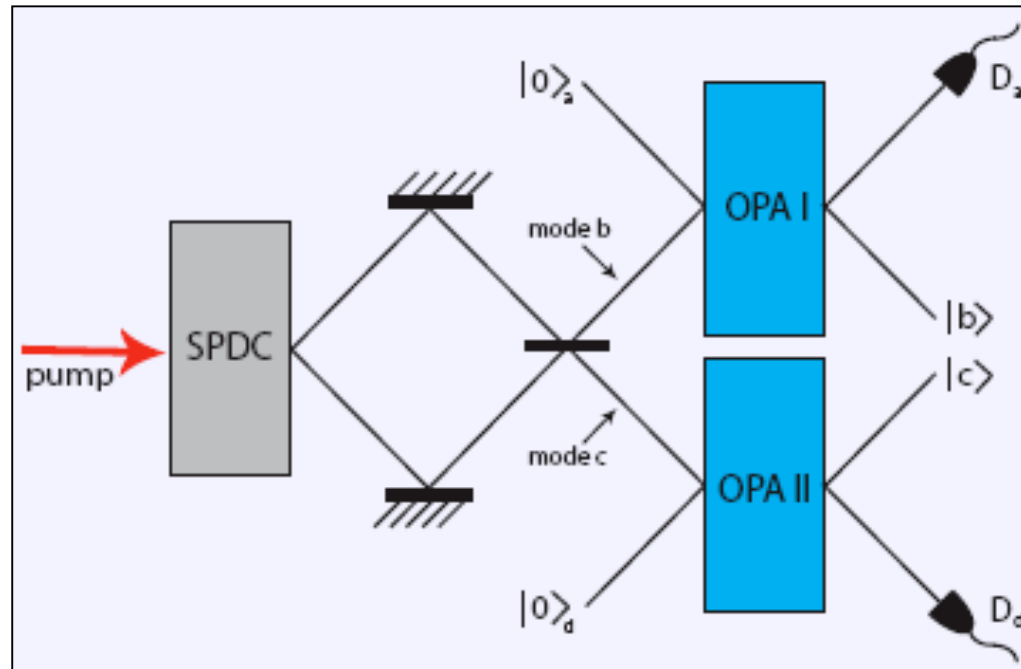


FIG. 1: (a) Experimental scheme for quantum lithography based on spontaneous parametric down-conversion. (b) Configuration based on polarization entangled beams.





- Two identical OPAs pumped with the same laser are seeded with the entangled input:  $\frac{1}{\sqrt{2}}(|2,0\rangle + |0,2\rangle)$
- Input state created from a spontaneous parametric downconverter and the Hong-Ou-Mandel effect.

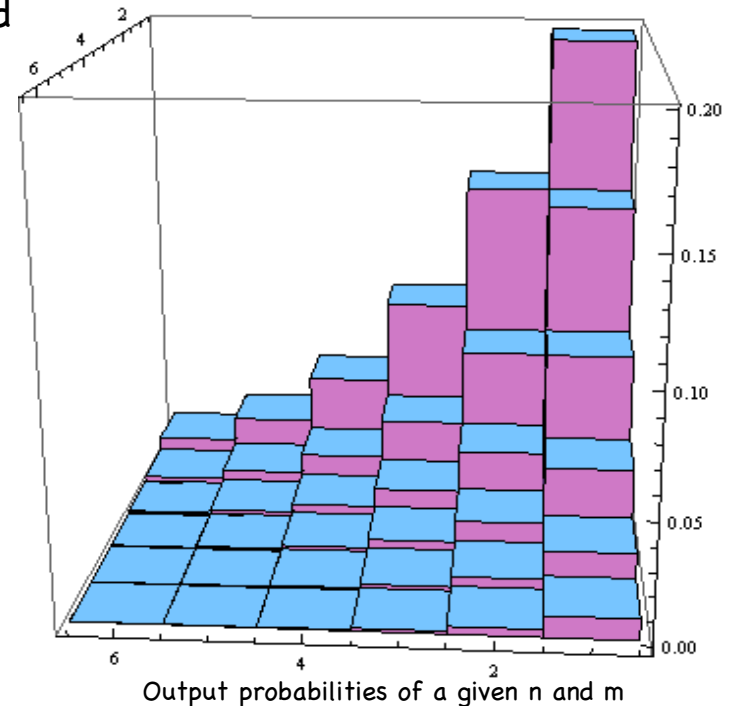
• Output state is: 
$$\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} C_{n,m} \left( \sqrt{n+1}\sqrt{n+2} |n, n+2, m, m\rangle + \sqrt{m+1}\sqrt{m+2} |n, n, m+2, m\rangle \right)$$

- The factor  $C_{n,m}$  depends on the phase of the OPAs, the gain ( $r$ ) and the values of  $n$  and  $m$ .
- Inner two modes  $b$  and  $c$  are highly path entangled.
- Detecting  $n$  and  $m$  photons at  $D_a$  and  $D_d$  allows with certainty knowledge of the state the inner two modes are in.

- Probability of obtaining an output state with a given  $n$  and  $m$  is:

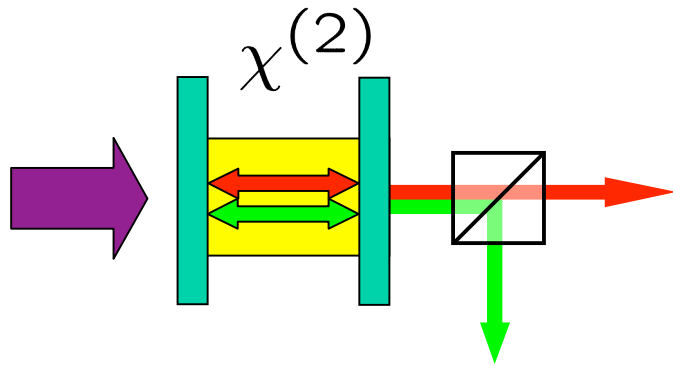
$$\left(\frac{1-2\tanh^2 r + \tanh^4 r}{2}\right)^2 \tanh^{2(n+m)} r [(n+1)(n+2) + (m+1)(m+2)]$$

- Most probable joint detection outcome is  $n=m=1$ , which occurs at an easily experimentally obtainable gain of  $r=0.66$
- This results in the state:  $\frac{1}{\sqrt{2}}(|3,1\rangle + |1,3\rangle)$
- This state input on a 50/50 beamsplitter results in the  $N=4$  NOON state:  $\frac{1}{\sqrt{2}}(|4,0\rangle + |0,4\rangle)$
- If perfect number resolving detectors exist at  $D_a$  and  $D_d$ , and we detect those modes out, we can use the entangled modes  $b$  and  $c$  for a quantum cryptography protocol.



## What's Next???

- Recently finished calculation of output state including vacuum input.
- Need to quantify amount of entanglement in output state.
- Does output state beat the shot-noise limit?
- Viable source for noiseless amplification (one quadrature)?
- How do phase sensitive versus phase insensitive parametric amplifiers affect the scheme?
- Is degenerate parametric amplification required for the input?
- Effect of imperfect detectors on phase resolution.



Frequency-degenerate modes.

Non-degenerate in polarization/direction.

Broadband source, easier to collimate.

Fluxes high, even below threshold.

Better immunity to technical noise.

- Photon pairs created by down-conversion.

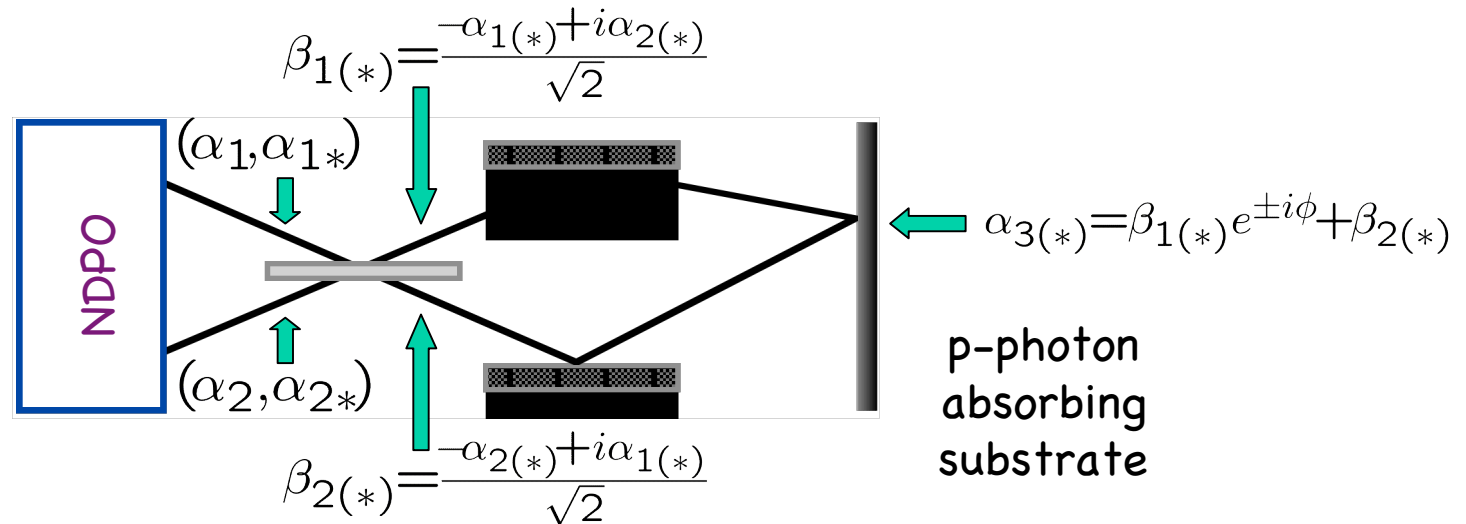
$$\hat{H} = i\hbar (\kappa\epsilon\hat{a}_1^\dagger\hat{a}_2^\dagger - \kappa\epsilon^*\hat{a}_1\hat{a}_2) + \hat{H}_{\text{loss}}$$

- Photons evolve into external modes independently.

- Strong noise reduction of difference intensity.

- Broadband source.

Work with  $c$ -numbers  $\alpha_i, \alpha_{i*}$  using +P representation.



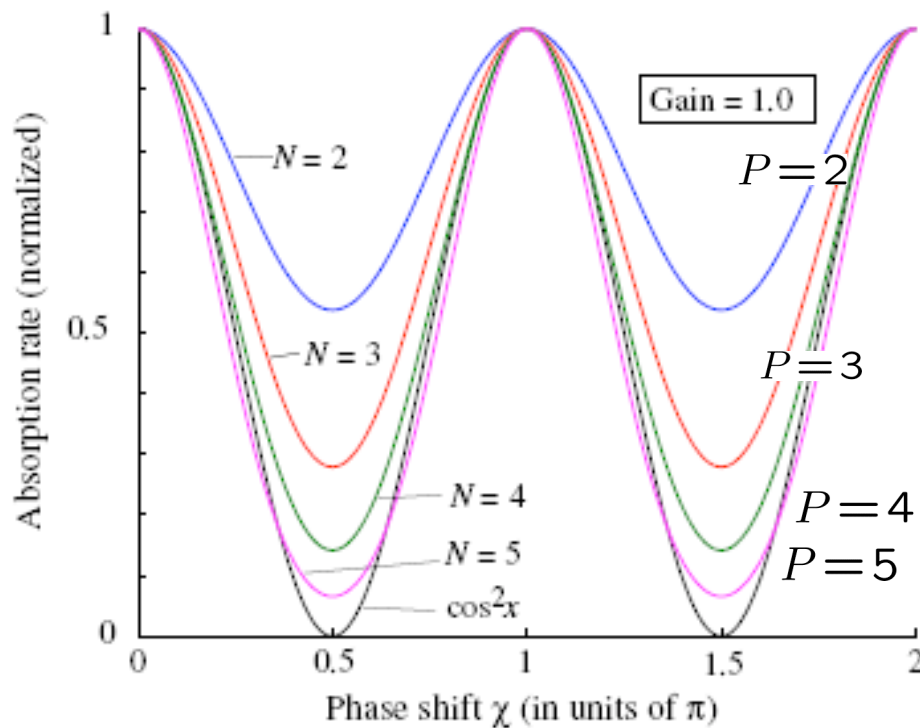
$$\alpha_{i(*)}(\phi, u_1, u_2, u_3, u_4)$$

$u_i$  real independent Gaussian variables (below threshold)

$$\langle \alpha_{3*}^p \alpha_3^p \rangle = \left( \frac{1}{8c^2 \sqrt{2n_0}} \right)^p \frac{1}{(1-r^2)^p} p! \sum_{k=0}^p \frac{(2k)!(2p-2k)!}{k!^2 (p-k)!^2} [r + \cos(\phi)]^k [r - \cos(\phi)]^{(p-k)}$$

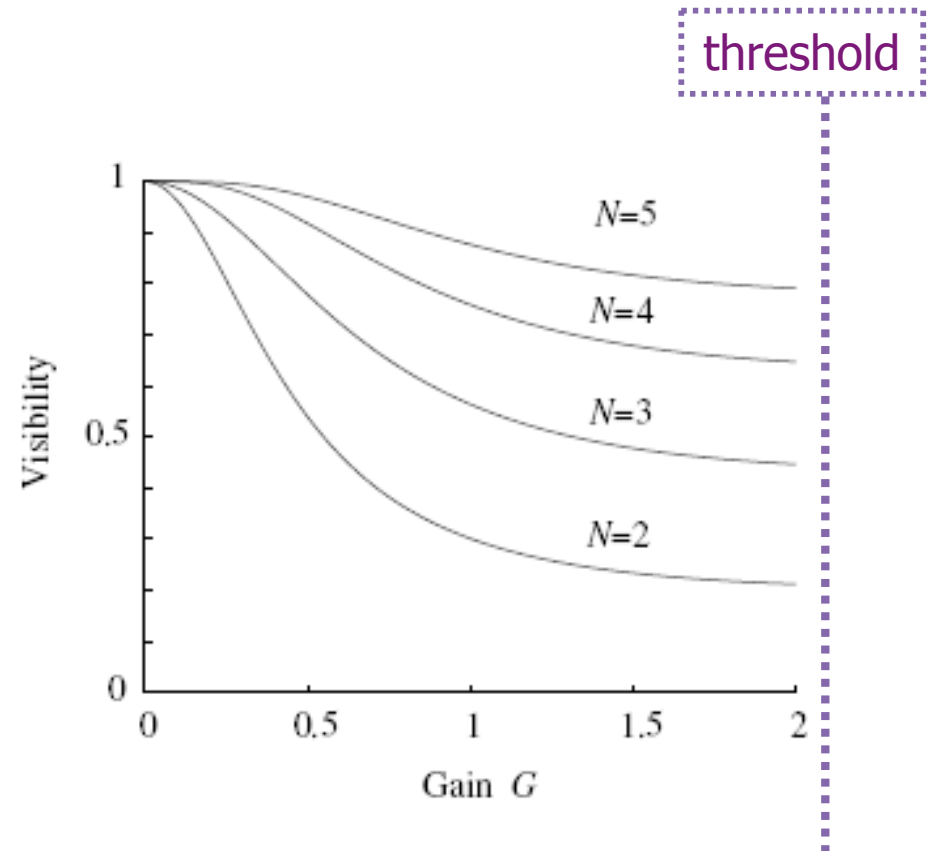
## Fringes

- For  $p = 1$  constant illumination!
- For  $p \geq 2$  a 2-photon effect



2-photon effect only. Effects of *NOON* state component countered *POOP*.

## Visibility



OPO Below Threshold Reproduces OPA Results but at Higher Intensities or Lower Gain. Above Threshold Calculation in Progress.

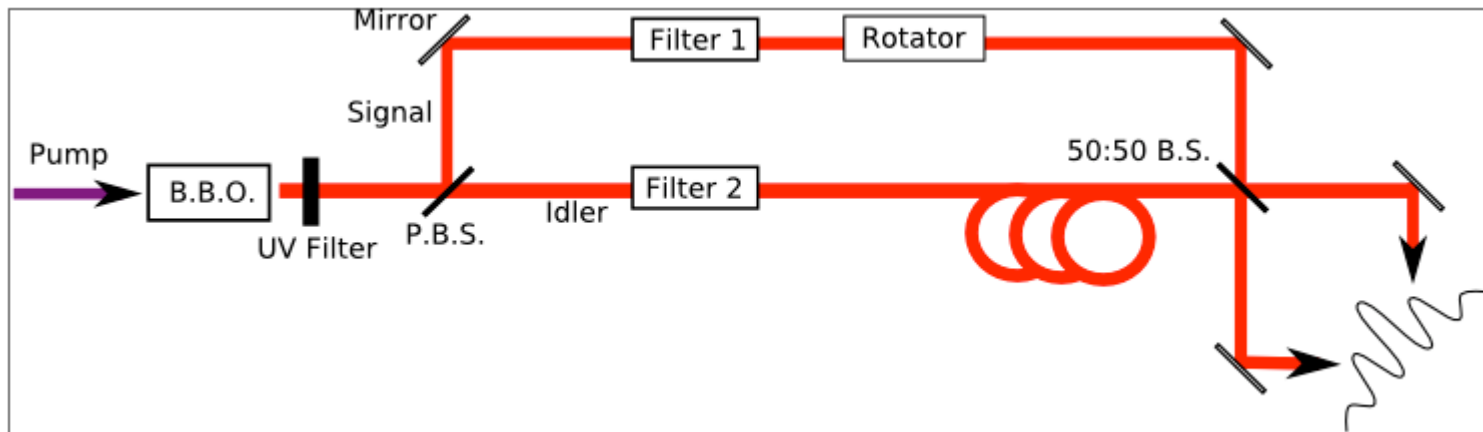
- Take again the rate of TPA

$$w_2 = 2 |g(\omega_0)|^2 \int_{-\infty}^{\infty} e^{2i\omega_f t - k_f |t|} G^2(-t, -t; t, t) dt$$

- For non-stationary states this equation becomes a probability:

$$P_2 = \iint g^*(\omega') S^{(2)}(\omega'_f - \omega', \omega'; \omega_f - \omega, \omega) g(\omega) d\omega' d\omega$$

- We take a setup like (with Type II down conversion):

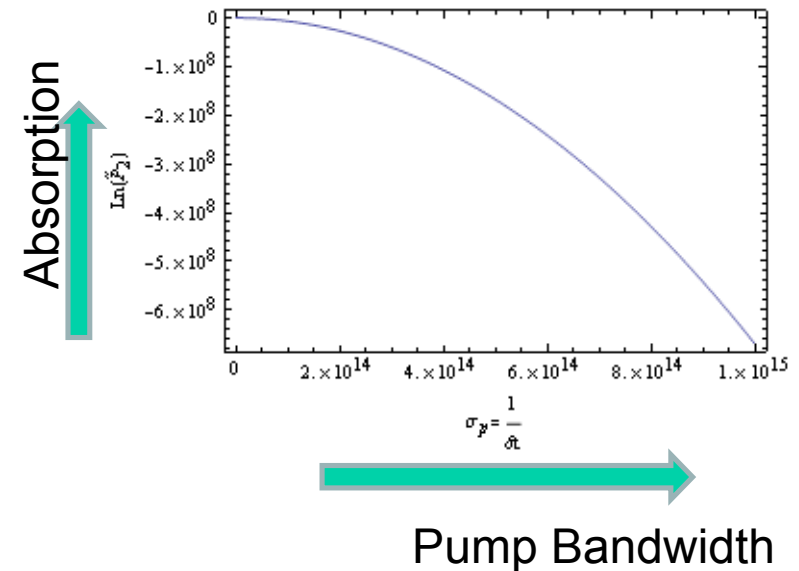
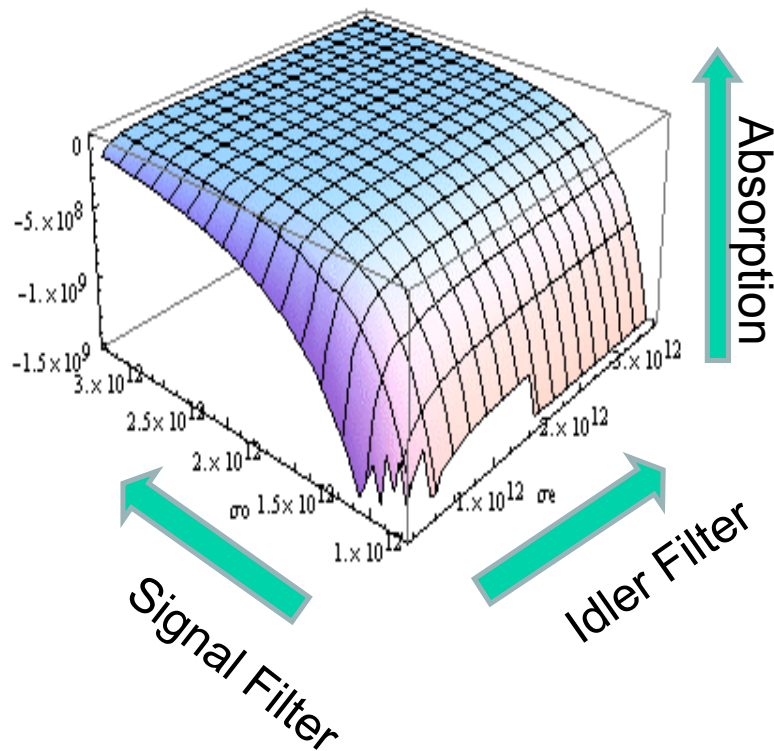




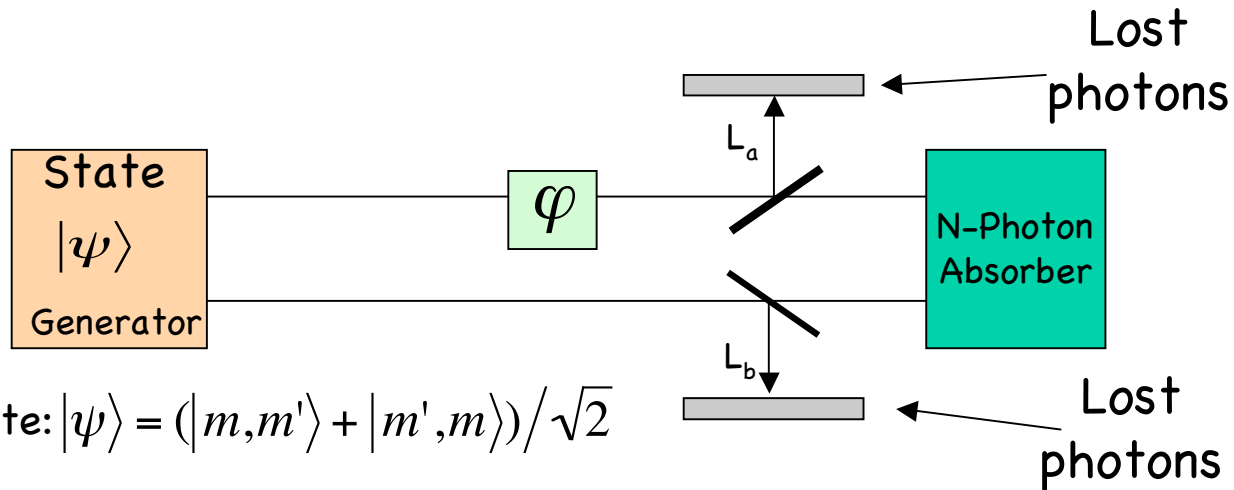
# The Absorption Properties of NOON States

William Plick, Petr Anisimov, Christoph Wildfeuer, and JPD (in preparation)

$$\tilde{P}_2 = e^{-2 \frac{4 \ln(2) U^2 \Omega_p^2}{P u^2 \sigma_o^2}} | \text{Erf}(E + L) - \text{Erf}(E - L) |^2$$

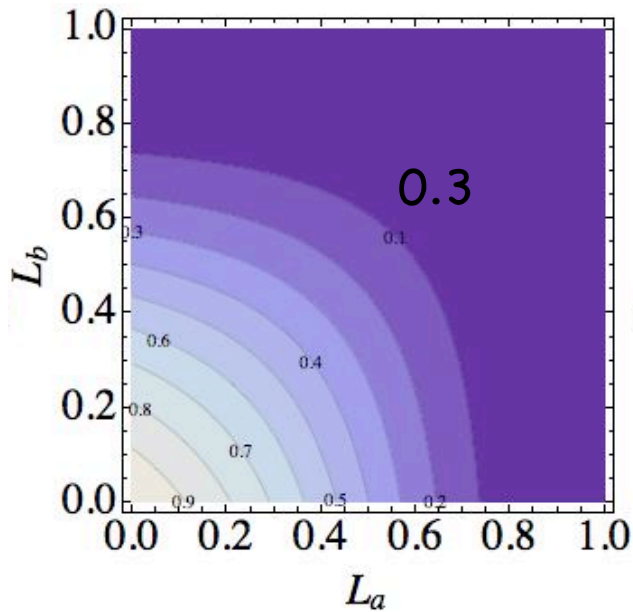


Next extend theory to N-photon absorption and consider EIT and Doppleron Absorbers.



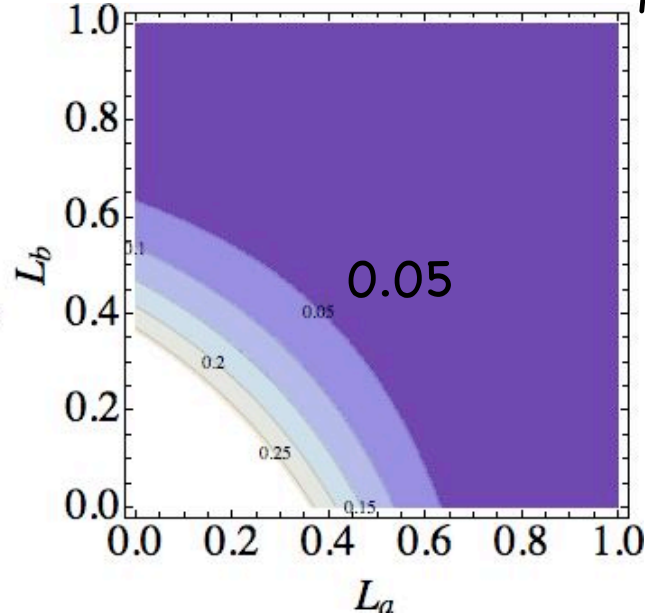
M&M Visibility

$$|\psi\rangle = (|20, 10\rangle + |10, 20\rangle) / \sqrt{2}$$

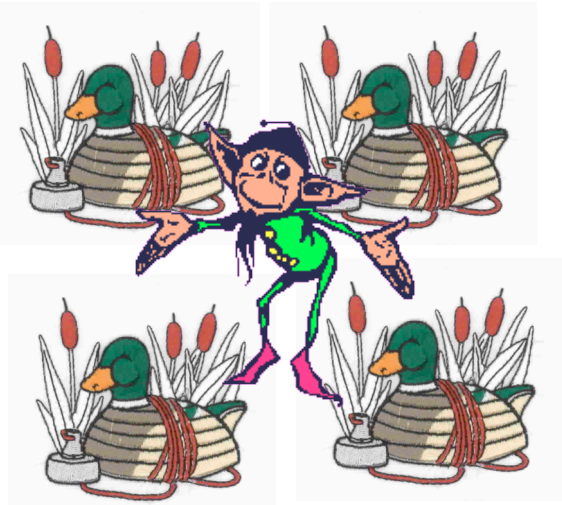


NOON Visibility

$$|\psi\rangle = (|10, 0\rangle + |0, 10\rangle) / \sqrt{2}$$



M&M' Adds Decoy Photons

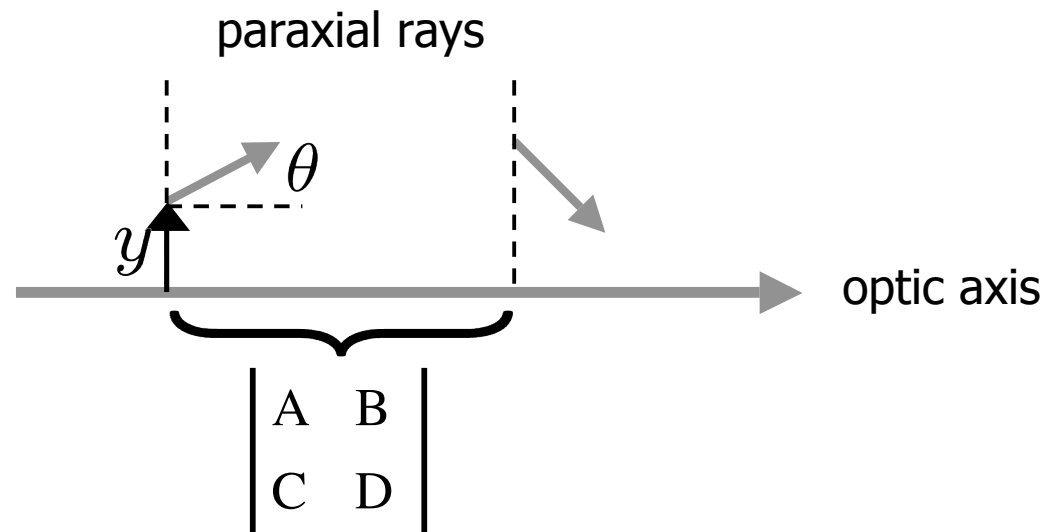
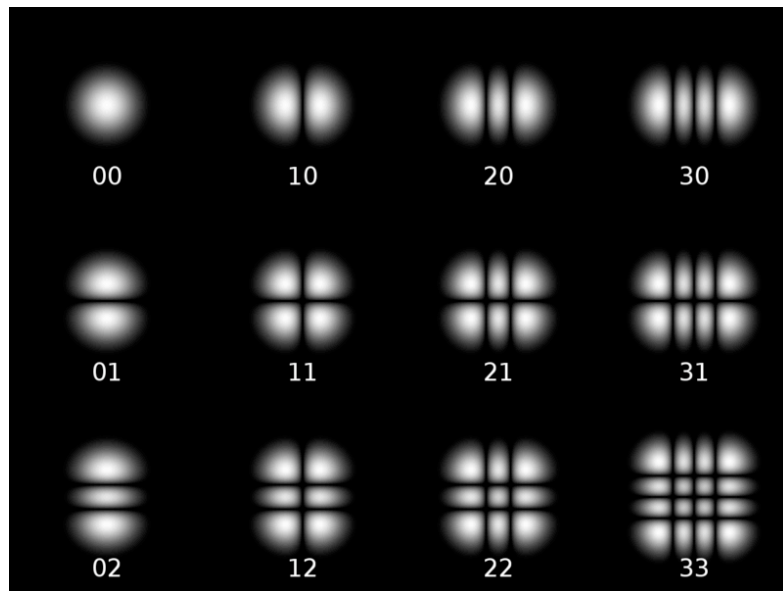


Expand electric field operator using Hermite-Gauss modes:

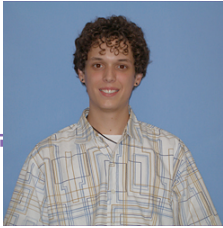
$$E(x, y, z) = \sum_{nm} A_{nm} U_{nm}^{\text{HG}}(x, y, z)$$

Special solutions to paraxial Helmholtz equation:

$$\left( \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} \right) - 2ik \frac{\partial A}{\partial z} = 0$$



Design Quantum Imaging System  
Via Matrix Multiplication!

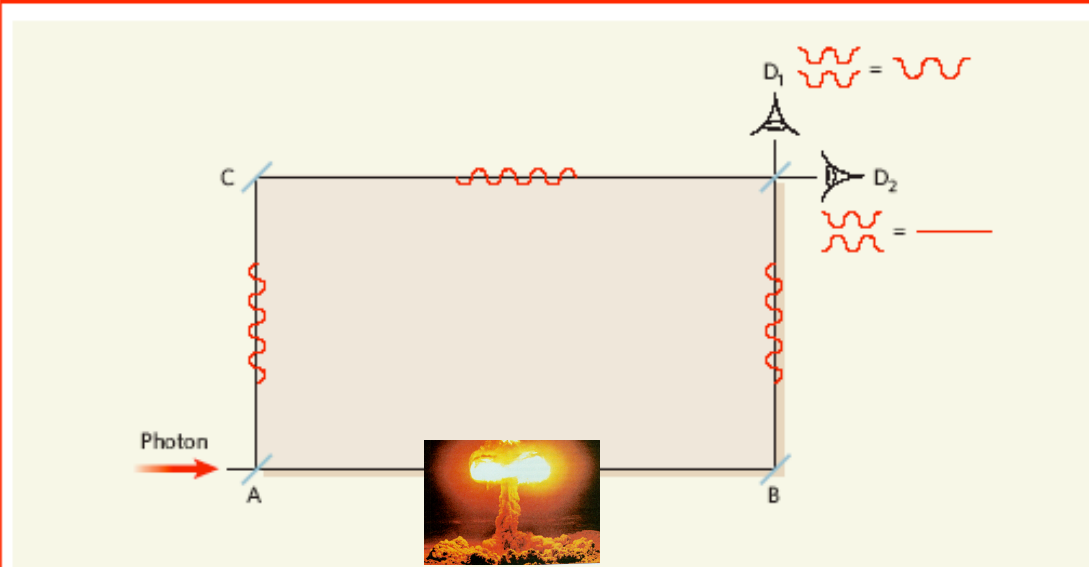


# Loss & Noise in Interaction-Free Imaging

Daniel Lum, Blane McCracken, & JPD (in progress)



## Box 1 How to stop worrying and learn to love the bomb



Avshalom Elitzur and Lev Vaidman's bomb thought-experiment<sup>2</sup> uses a simple wave-splitting device known as an optical interferometer and the quantum phenomenon of wave-particle duality. If just one photon enters their interferometer at A, its associated wave splits, going with a certain probability by way of either B or C. The geometry of the interferometer is set up such that, at one of two detectors at its far end ( $D_1$ ), the waves from the two paths interfere constructively, so light is detected. At the other detector ( $D_2$ ), on the other hand, the waves interfere destructively, so it is never triggered — unless one of the pathways, say B, is blocked off, preventing the passage of a photon. In this case, there can be no interference,

so the photon will be detected with equal probability by  $D_1$  or  $D_2$ .

Now suppose that B is — possibly — blocked by a photon-triggered bomb. A single photon enters the interferometer. Three outcomes are possible. First, the bomb goes off: so there is a bomb. Second,  $D_1$  triggers. This outcome does not help us, as it could mean one of two things: there is a bomb, but the photon passed by C, or there is n't a bomb, and waves from the two paths are interfering constructively. Third,  $D_2$  triggers. In this case, there must be a bomb blocking B, but our photon passed by C. In other words, we have 'seen' the bomb without a photon ever touching it.

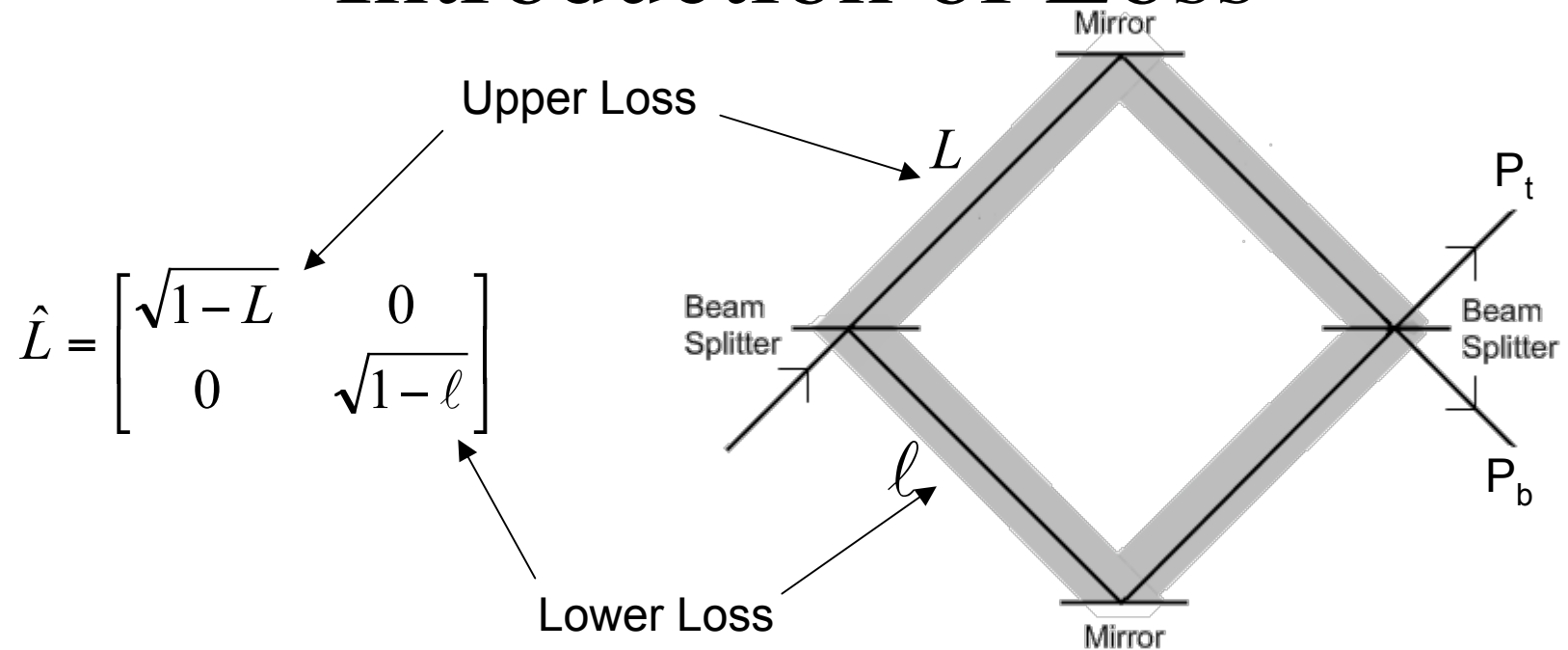
J.P.D.

## Modeling:

- Loss
- Scattering
- Turbulence
- Clutter

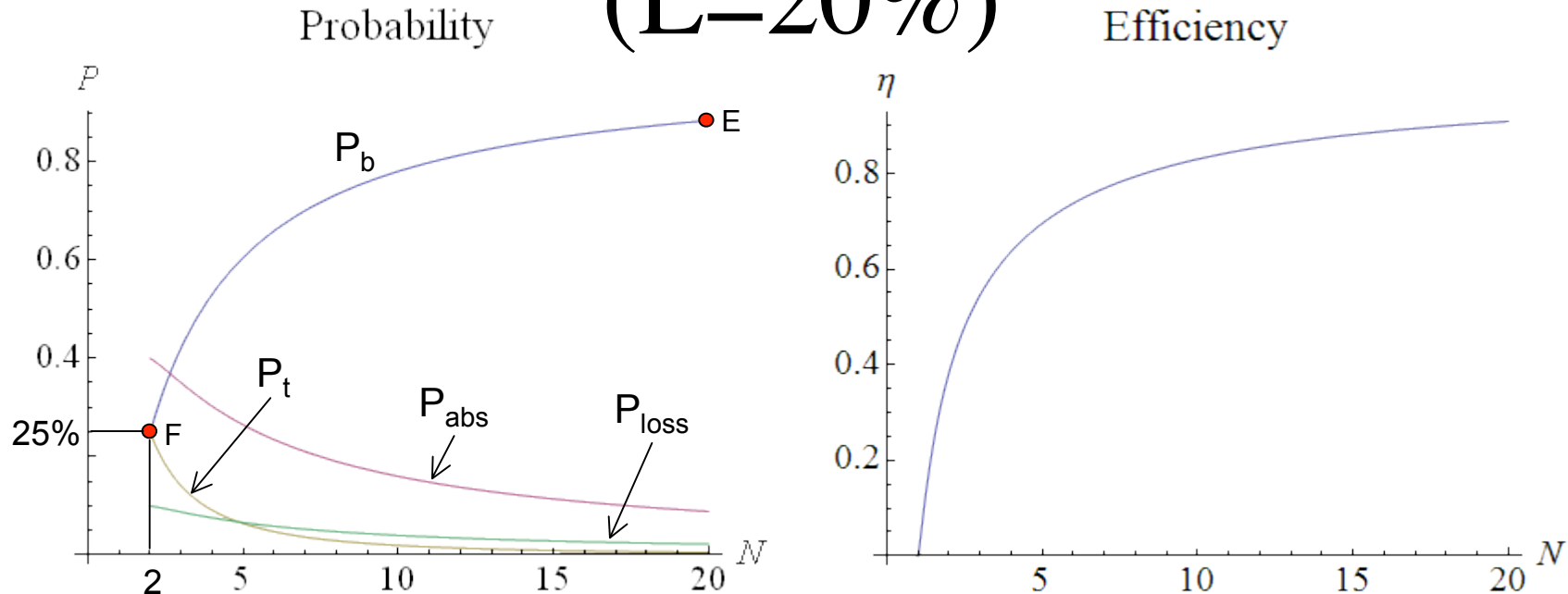


# Introduction of Loss



Independent loss parameters are introduced in the two independent modes of the interferometer by a nonunitary loss operator.

# With Object / Upper Loss ( $L=20\%$ )



When an object is present, the detection probabilities ( $P_b$  and  $P_t$ ) are independent of loss. The probability of photon absorption by the object, however, is decreased if the loss is encountered before the object. This results in increased efficiency with more beam splitters.