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Three Themes for Theory Research: Gaussian States, Coherent Laser Radars, and Multi-Photon Detectors

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Three Themes for Theory Research

- Gaussian States
  - Classical versus non-classical Gaussian states
  - Gaussian states from spontaneous parametric downconversion
  - Relevance to quantum versus thermal imaging
- Coherent Laser Radars
  - Carrier-to-noise ratio versus signal-to-noise ratio
  - Range imaging and anomalous detection
  - Relevance to quantum laser radars
- Multi-Photon Detectors
  - Multi-coincidence rates for photodetection
  - Necessity of a sensitivity function
  - Relevance to quantum lithography

Gaussian States of the Radiation Field

- Positive-frequency, photon-units field operator $\hat{E}(t)e^{-i\omega_0 t}$
- Canonical commutation relation: $\left[\hat{E}(t), \hat{E}^\dagger(u)\right] = \delta(t-u)$
- Zero-mean Gaussian quantum state

$$\left\langle \exp\left(-\int \mathrm{d}t\, \zeta^*(t)\hat{E}(t) + \int \mathrm{d}t\, \zeta(t)\hat{E}^\dagger(t)\right)\right\rangle =$$

$$\exp\left[-\frac{1}{2}\int \mathrm{d}t \int \mathrm{d}u\, \zeta^*(t)\zeta(u)[\langle \hat{E}^\dagger(u)\hat{E}(t)\rangle + \langle \hat{E}(t)\hat{E}^\dagger(u)\rangle]\right.$$

$$\left. + \operatorname{Re}\left(\int \mathrm{d}t \int \mathrm{d}u\, \zeta^*(t)\zeta^*(u)\langle \hat{E}(t)\hat{E}(u)\rangle\right)\right]$$

- Gaussian states remain Gaussian after linear filtering

Quantum Gaussian Noise: Key Properties

- Zero-mean quantum Gaussian state of $\hat{E}(t)$ is
  - completely characterized by

$$K^{(n)}_{EE}(t,u) \equiv \langle \hat{E}^\dagger(t)\hat{E}(u)\rangle$$

$$K^{(p)}_{EE}(t,u) \equiv \langle \hat{E}(t)\hat{E}(u)\rangle$$

  - when stationary, is completely characterized by the spectra

$$S^{(n)}_{EE}(\omega) \equiv \int \mathrm{d}\tau\, K^{(n)}_{EE}(\tau)e^{-i\omega\tau}$$

$$S^{(p)}_{EE}(\omega) \equiv \int \mathrm{d}\tau\, K^{(p)}_{EE}(\tau)e^{i\omega\tau}$$

  - obeys Gaussian moment factoring

$$\langle \hat{E}^\dagger(t)\hat{E}^\dagger(u)\hat{E}(t)\hat{E}(u)\rangle = \langle \hat{E}^\dagger(t)\hat{E}(t)\rangle\langle \hat{E}^\dagger(u)\hat{E}(u)\rangle + |\langle \hat{E}^\dagger(t)\hat{E}(u)\rangle|^2 + |\langle \hat{E}(t)\hat{E}(u)\rangle|^2$$

When is a Gaussian-State Field Non-Classical?

- "Classical light" + shot-noise = semiclassical photodetection

- Semiclassical photodetection is quantitatively correct for coherent states and their classically-random mixtures

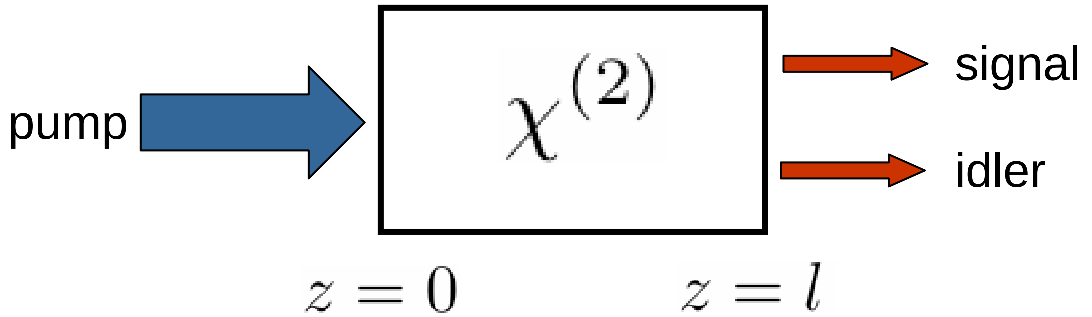
- Coherent states are displaced vacuum, hence Gaussian

- Stationary, zero-mean Gaussian states are classical if

$$|S_{EE}^{(p)}(\omega)| \leq S_{EE}^{(n)}(\omega), \quad \forall\omega$$

Second-Order Nonlinear Optics

- Spontaneous parametric downconversion (SPDC)



- strong pump at frequency $\omega_P = \omega_S + \omega_I$
- no input at signal frequency ω_S
- no input at idler frequency ω_I
- nonlinear mixing in $\chi^{(2)}$ crystal produces signal and idler outputs

Quantum Coupled-Mode Equations

- Strong, monochromatic, coherent-state pump

- Positive-frequency signal and idler field operators:

$$\hat{E}_S^{(+)}(z,t) = \int \frac{\mathrm{d}\omega}{2\pi}\, \hat{A}_S(z,\omega) e^{-i[(\omega_P/2+\omega)t - k_S(\omega_P/2+\omega)z]}$$

$$\hat{E}_I^{(+)}(z,t) = \int \frac{\mathrm{d}\omega}{2\pi}\, \hat{A}_I(z,\omega) e^{-i[(\omega_P/2-\omega)t - k_I(\omega_P/2-\omega)z]}$$

- Quantum coupled-mode equations:

$$\frac{\partial \hat{A}_S(z,\omega)}{\partial z} = i\kappa \hat{A}_I^\dagger(z,\omega) e^{i\omega\Delta k' z}$$

$$\frac{\partial \hat{A}_I(z,\omega)}{\partial z} = i\kappa \hat{A}_S^\dagger(z,\omega) e^{i\omega\Delta k' z}$$

Gaussian-State Characterization

- Signal and idler at $z = 0$ are in vacuum states
- Signal and idler at $z = l$ are in zero-mean Gaussian States
- Baseband signal and idler field operators:

$$\hat{E}_m(t)e^{-i(\omega_P t/2 - k_P l)} \equiv \hat{E}_m^{(+)}(l,t), \text{ for } m = S, I$$

- Non-zero covariance functions:

$$\begin{aligned} K_{SS}^{(n)}(\tau) &= K_{II}^{(n)}(\tau) = \int \frac{\mathrm{d}\omega}{2\pi} |\nu(\omega)|^2 e^{i\omega\tau} \\ K_{SI}^{(p)}(\tau) &= \int \frac{\mathrm{d}\omega}{2\pi} \mu(\omega)\nu(\omega) e^{-i\omega(\tau - \Delta k' l)} \end{aligned}$$

Operation in the Low-Gain Regime

- Low-gain regime: $|\kappa|l \ll 1$

- Approximate Bogoliubov parameters:

$$\mu(\omega) \approx 1 \text{ and } \nu(\omega) \approx i\kappa l \frac{\sin(\omega\Delta k' l/2)}{\omega \Delta k' l/2} e^{-i\omega\Delta k' l/2}$$

- Normally-ordered and phase-sensitive spectra:

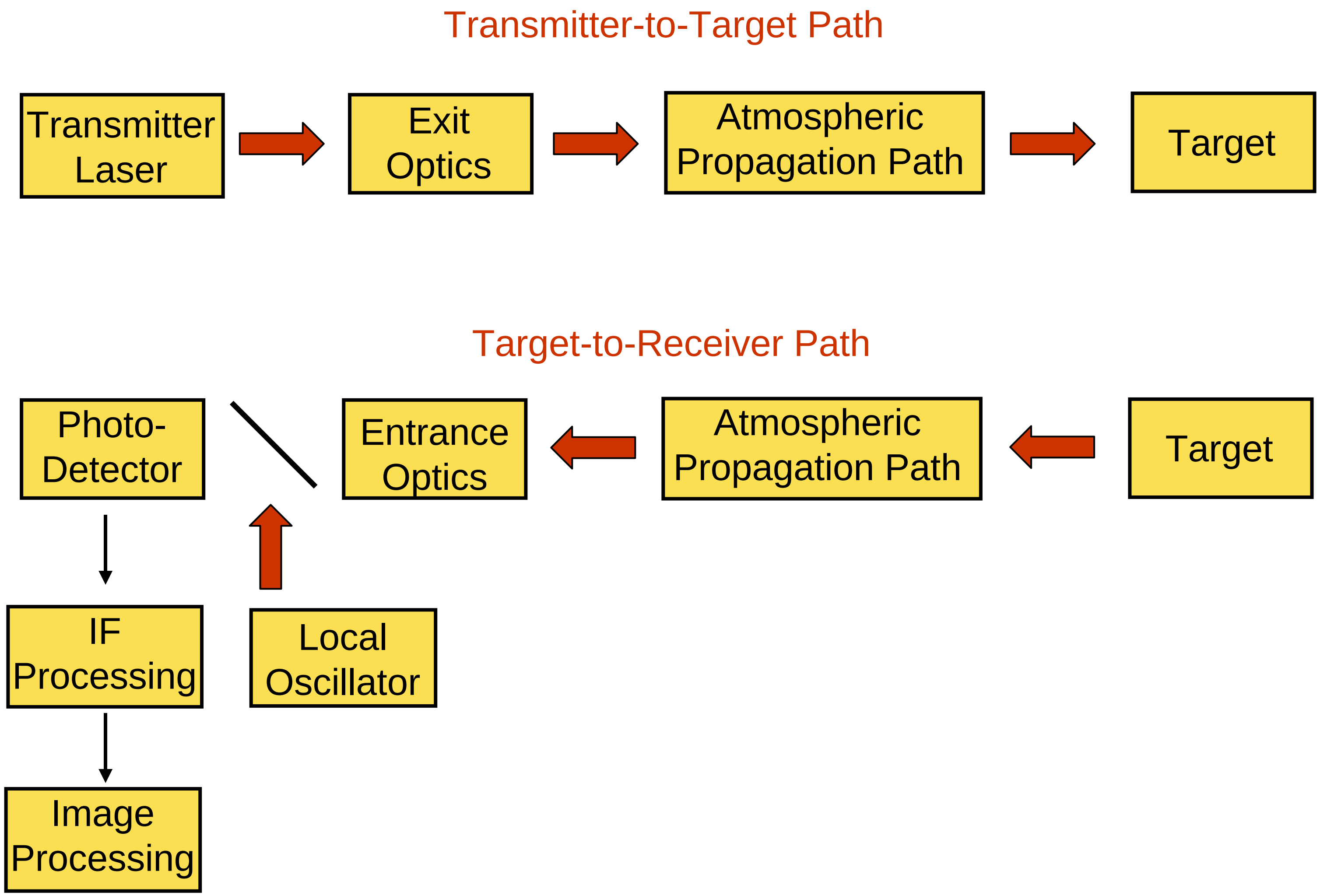
$$S_{SS}^{(n)}(\omega) = S_{II}^{(n)}(\omega) \approx (|\kappa|l)^2 \left(\frac{\sin(\omega\Delta k' l/2)}{\omega \Delta k' l/2}\right)^2$$

$$S_{SI}^{(p)}(\omega) \approx i\kappa l \frac{\sin(\omega\Delta k' l/2)}{\omega \Delta k' l/2} e^{i\omega\Delta k' l/2}$$

Looking Backwards and Forwards...

- Quantum Gaussian Noise
  - is natural extension of classical Gaussian noise
  - is completely characterized by its correlations
  - extends to vector fields with spatio-temporal dependence
  - remains Gaussian after linear filtering
  - provides convenient description for parametric downconversion
  - yields easy coincidence-counting calculations by moment factoring
- Quantum Gaussian Noise
  - will be used to study quantum imaging configurations
  - will be used to distinguish behaviors that are non-classical
  - may help to identify new classical imaging regimes

Coherent Laser Radar System



Coherent Laser Radars versus Microwave Radars

- Advantages of Coherent Laser Radars
  - finer angular resolution for same antenna aperture
  - finer range resolution for the same percentage bandwidth
  - finer velocity resolution for the same dwell time

- Disadvantages of Coherent Laser Radars
  - all-weather operation is not feasible
  - clear-weather operation affected by atmospheric turbulence
  - target roughness gives rise to speckle noise

Carrier-to-Noise Ratio and the Radar Equation

- Carrier-to-Noise Ratio Definition

$$\text{CNR} \equiv \frac{\langle \text{IF target-return power} \rangle}{\langle \text{IF LO shot-noise power} \rangle}$$

- Monostatic Radar Equations for CNR

$$\text{CNR} = \begin{cases} \dfrac{\eta P_T}{\hbar\omega B} \dfrac{G_T}{4\pi R^2} \dfrac{\epsilon_g \sigma A_R}{4\pi R^2} e^{-2\alpha R}, & \text{glint target} \\ \dfrac{\eta P_T}{\hbar\omega B} \dfrac{\epsilon_s \rho A_R}{\pi R^2} e^{-2\alpha R}, & \text{speckle target} \end{cases}$$

Image Signal-to-Noise Ratio

- Pixel output from square-law IF processor: $I(i,j)$

- Signal-to-Noise Ratio Definition

$$\text{SNR} \equiv \frac{\langle \text{signal in } I(i,j) \rangle^2}{\text{variance of } I(i,j)}$$

- SNR Behavior

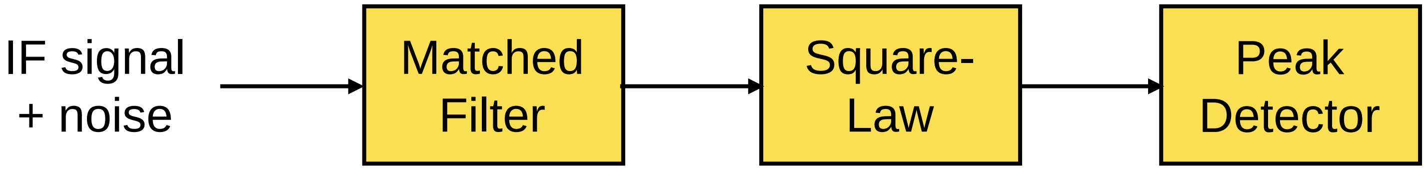
$$\text{SNR} = \frac{\text{CNR}/2}{1 + \text{CNR}/2\text{SNR}_{\text{sat}} + 1/2\text{CNR}}$$

- Saturation SNR

$$\begin{aligned} \text{SNR}_{\text{sat}} &= \lim_{\text{CNR}\to\infty} \text{SNR} \\ &= 1 \text{ for a speckle target} \end{aligned}$$

Range Imaging and Anomalous Detection

- Range Processor



- Range Resolution $R_{\mathrm{res}} = cT/2$

- Range Accuracy without Anomaly $\delta R \sim R_{\mathrm{res}}/\sqrt{\mathrm{CNR}}$

- Probability of Anomalous Range for a Speckle Target

$$\Pr(A) \sim \ln(\Delta R/R_{\mathrm{res}})/\mathrm{CNR}$$

Quantum Laser Radars

- Non-classical Light for the Transmit Beam
  - phase-sensitive power amplifier will produce squeezed states
  - homodyne detection of squeezed states improves noise performance
  - *BUT* system impractical because non-classicality degraded by loss

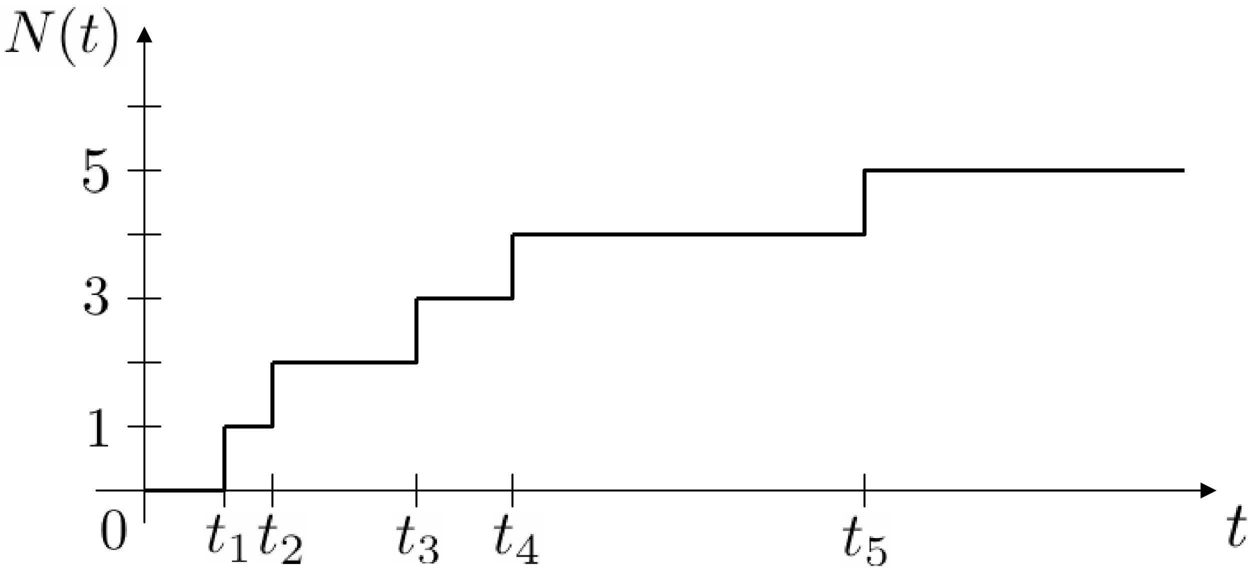
- Phase-Sensitive Preamplifier on the Receive Beam
  - phase-sensitive preamplifier squeezes the target-return beam
  - noiseless image amplification of one quadrature is obtained
  - homodyne detection achieves improved noise performance

Looking Backwards and Forwards...

- Laser Radars
  - offer much finer resolutions in angle, range, and velocity
  - are not all-weather systems
  - have been studied and developed for coherent and direct detection
- Quantum Laser Radars
  - will use phase-sensitive preamplifiers to achieve noise advantages
  - will be the subject of system analyses of CNR, SNR, range imaging
  - their performance will be compared to that of conventional systems

Multi-Coincidence Rates for Photodetection

- Photodetection Counting Process



- Multi-Coincidence Rates (MCRs)

$$w_m(t_1, t_2, \ldots, t_m) = \lim_{\Delta t \to 0} \frac{\Pr[(\prod_{i=1}^{m}[N(t_i + \Delta t) - N(t_i)]) = 1]}{(\Delta t)^m}$$

Quantum MCRs for a Single-Photon Detector

- Photodetector is illuminated by field operator $\hat{E}(t)$

- MCRs of the form

$$w_m(t_1, t_2, \ldots, t_m) = \eta^m \left\langle \left(\prod_{i=1}^{m} \hat{E}^\dagger(t_i) \right) \left(\prod_{i=1}^{m} \hat{E}(t_i) \right) \right\rangle$$

imply that $N(t) \longleftrightarrow \hat{N}(t) \equiv \int_0^t \mathrm{d}\tau \, \hat{E}'^\dagger(\tau) \hat{E}'(\tau)$

where $\hat{E}'(t) \equiv \sqrt{\eta}\, \hat{E}(t) + \sqrt{1-\eta}\, \hat{E}_v(t)$

with $\hat{E}_v(t)$ in its vacuum state

Quantum MCRs for a Two-Photon Detector

- Photodetector is illuminated by field operator $\hat{E}(t)$

- MCRs of the form

$$w_m(t_1, t_2, \ldots, t_m) = \tilde{\eta}^m \left\langle \left(\prod_{i=1}^{m} \hat{E}^{\dagger 2}(t_i) \right) \left(\prod_{i=1}^{m} \hat{E}^{2}(t_i) \right) \right\rangle$$

  and a coherent-state field yield Poisson process $N(t)$

- *Same* MCRs and a number-state field can yield

$$\langle [\Delta N(t)]^2 \rangle < 0$$

Ad Hoc Model for Removing the Contradiction

- Two-Photon Detection with a Sensitivity Function

$$N(t) \longleftrightarrow \hat{N}(t) \equiv$$

$$\tilde{\eta} \int_0^t \mathrm{d}\tau \int_0^\infty \mathrm{d}\mu \, s(\mu) \hat{E}^\dagger(\tau) \hat{E}(\tau) \hat{E}^\dagger(\tau - \mu) \hat{E}(\tau - \mu)$$

- Bondurant (1980) has examined this model
  - coherent-state illumination yields an average photocount rate with terms both linear and quadratic in the average photon flux, as has been seen in two-photon detection experiments
  - the contradiction found in the previous MCR approach is eliminated

Looking Backwards and Forwards…

- Quantum Theory of Photodetection
  - is obtainable from MCRs for single-photon detectors
  - does *not* follow from the usual MCRs for multi-photon detectors
  - sensitivity-function theory yields consistent multi-photon results
- Quantum Lithography
  - relies on multi-photon detection of entangled photons
  - will be studied using a spatio-temporal sensitivity-function theory