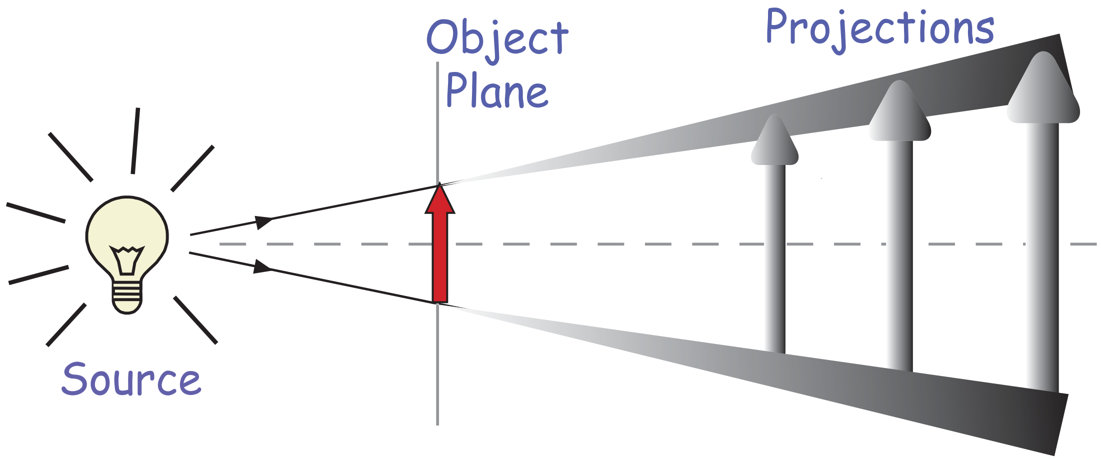
MURI Kick-Off Meeting

Rochester, June 9-10, 2005

Quantum Imaging

- Entangled state and thermal light
- Foundamental and applications

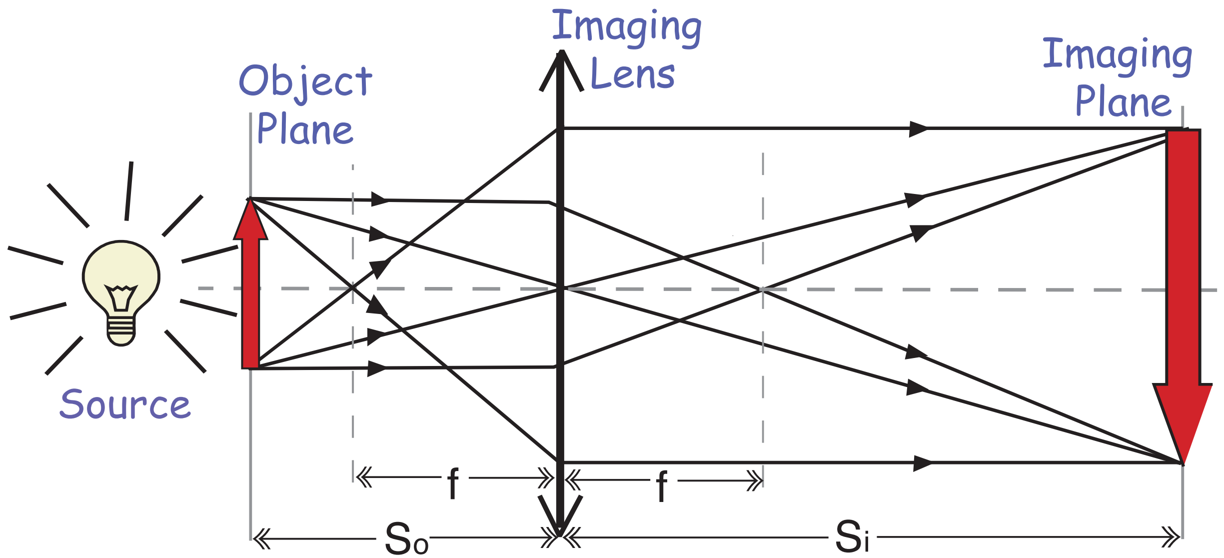
Optical Projection (Chinese shadow, x-ray, …)



Momentum (p_1) $\Longrightarrow$ Momentum (p_2)

No image plane is defined.

Optical Imaging:

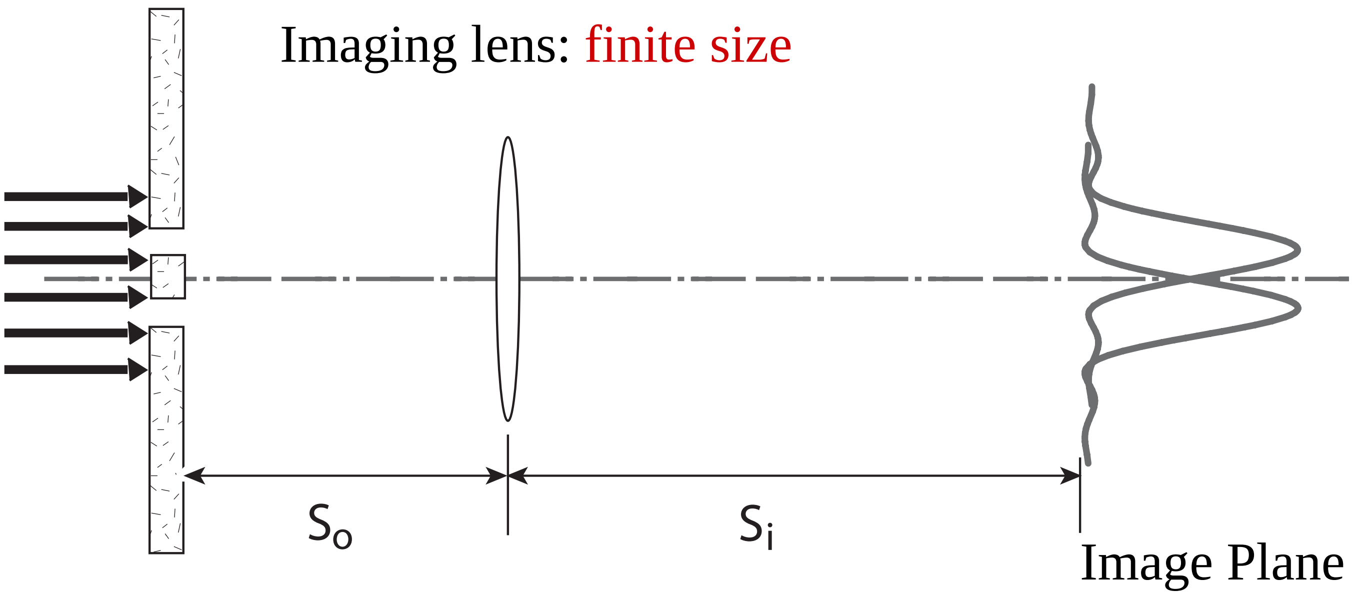


Point (object Plane) $\Longrightarrow$ Point (image plane)

Position (x_1) $\Longrightarrow$ Position (x_2)

$$\frac{1}{S_0}+\frac{1}{S_i}=\frac{1}{f} \quad \text{and} \quad \Delta(x_1-x_2)=0 \qquad \left\{ \begin{array}{l} \text{Geometric optics} \\ \text{Image lens: } \longrightarrow \infty \end{array} \right\}$$

Spatial Resolution

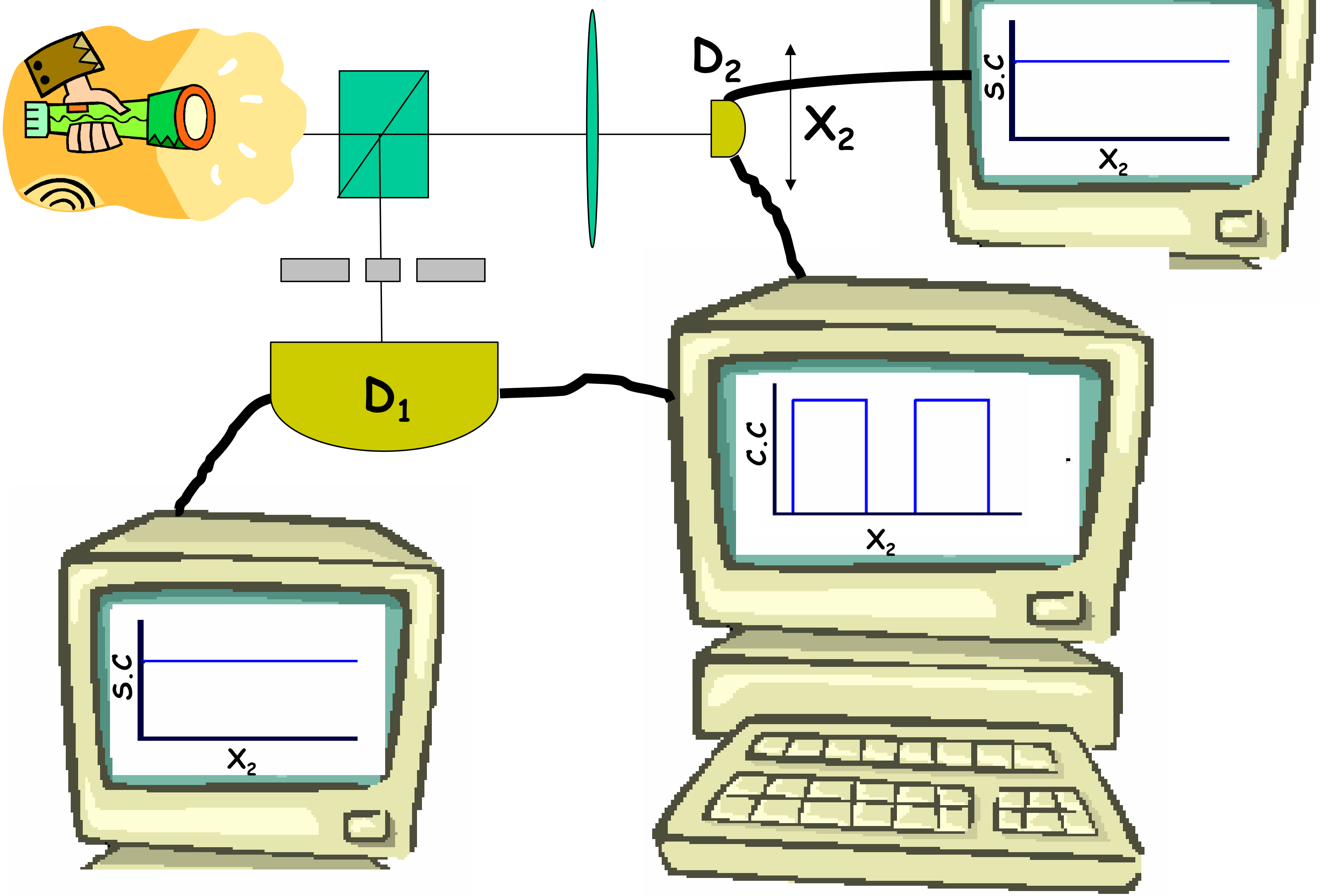


Point (object plane) $\Longrightarrow$ Spot (image plane)

δ–function $\Longrightarrow$ *somb*-function

$$\Delta(x_1 - x_2) \Rightarrow comb(\xi) = \frac{2J_1(\pi\xi)}{\pi\xi}$$

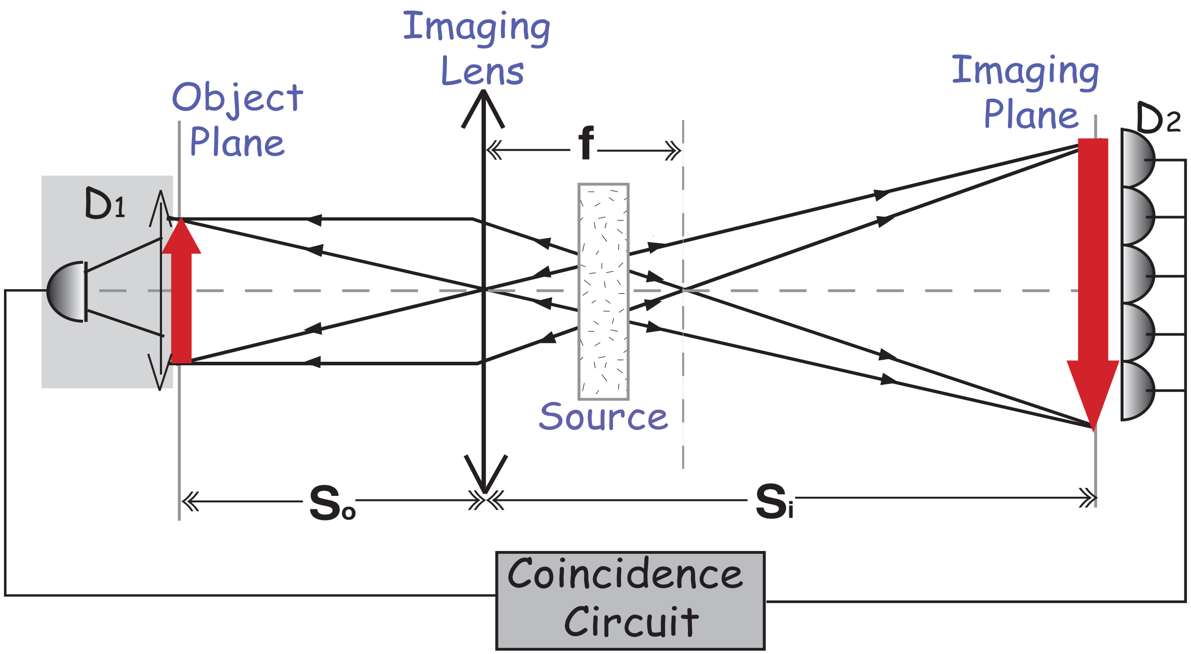
Two-Photon Imaging



“Ghost” Imaging with entangled photon pairs

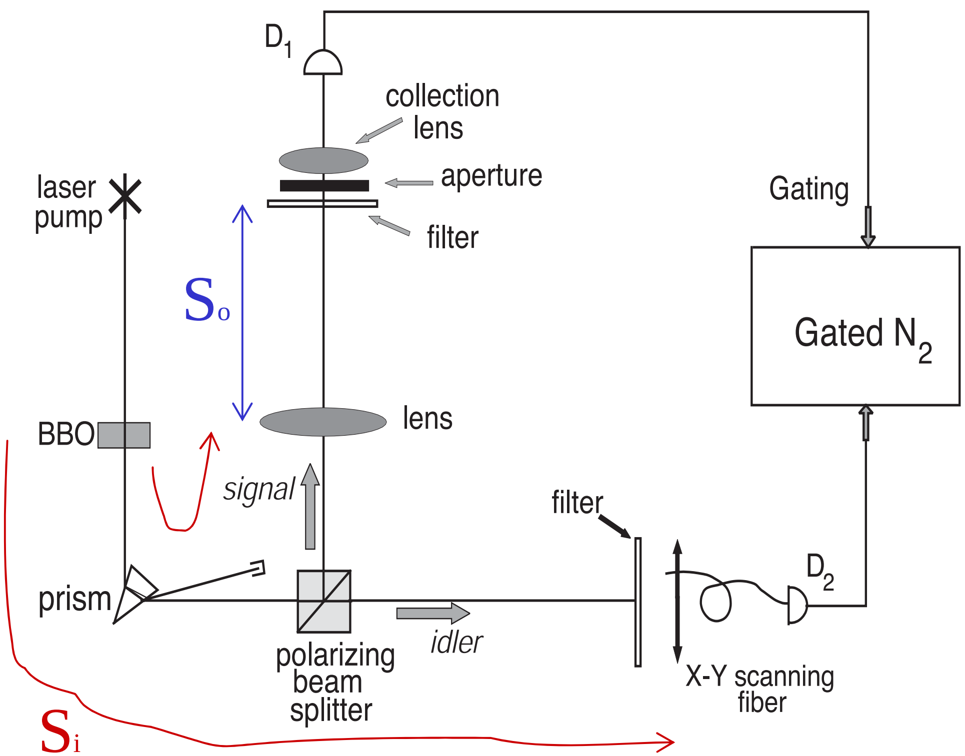
$$\Delta\vec{x}_1 = \infty \quad \Delta\vec{x}_2 = \infty \quad \Leftrightarrow \quad \vec{x}_1 - \vec{x}_2 = 0$$

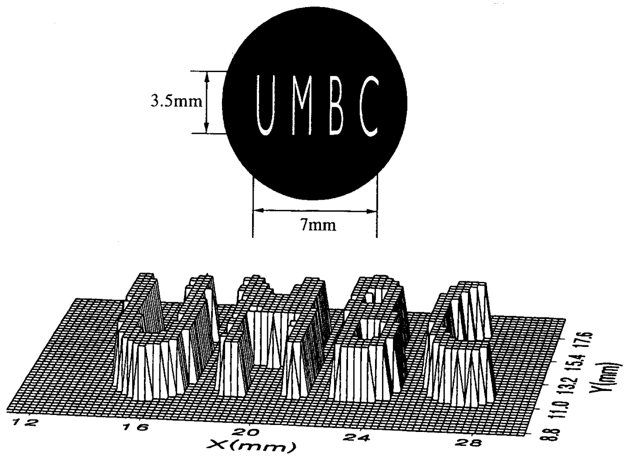
$$\Delta\vec{k}_1 = \infty \quad \Delta\vec{k}_2 = \infty \quad \Leftrightarrow \quad \vec{k}_1 + \vec{k}_2 = 0$$



Point x_1 (object plane) $\Leftrightarrow$ Point x_2 (image plane)

$$\frac{1}{S_0} + \frac{1}{S_i} = \frac{1}{f}$$





$$\frac{1}{S_0} + \frac{1}{S_i} = \frac{1}{f}$$

"Ghost" Image and "Ghost" Interference
EPR Experiment in momentum-position

PRL, 74, 3600 (1995); PRA, 52, R3429 (1995).

Classical: never!

- classical statistical measurements

$$\Delta(x_1 - x_2) = \sqrt{(\Delta x_1)^2 + (\Delta x_2)^2} > Max(\Delta x_1, \Delta x_2)$$

$$\Delta(p_1 + p_2) = \sqrt{(\Delta p_1)^2 + (\Delta p_2)^2} > Max(\Delta p_1, \Delta p_2)$$

* $H = H_1 + H_2; \quad H_{\text{interaction}} = 0$
* Space-like separated measurement events.

(1) No interaction between two distant quanta;

(2) No action-at-a-distance between individual measurements.

To EPR: the two quanta are independent as well as the measurements, so that

$$\Delta(x_1 - x_2) = \sqrt{(\Delta x_1)^2 + (\Delta x_2)^2} > Max(\Delta x_1, \Delta x_2)$$

$$\Delta(p_1 + p_2) = \sqrt{(\Delta p_1)^2 + (\Delta p_2)^2} > Max(\Delta p_1, \Delta p_2)$$

Classically correlated systems: one may consider building an ensemble of particle-pairs to force each pair with $p_1 + p_2 = p_0$ and $\Delta p_1 = 0, \ \Delta p_2 = 0,$ so that $\Delta(p_1 + p_2) = 0$. In this case, however, $\Delta(x_1 - x_2) \sim \infty$

Quantum: yes!

- EPR: if the two quanta are entangled

$$\Delta(x_1 - x_2) = 0$$

$$\Delta(p_1 + p_2) = 0$$

Although $\left\{ \begin{array}{ll} \Delta x_1 = \infty, & \Delta x_2 = \infty \\ \Delta p_1 = \infty, & \Delta p_2 = \infty \end{array} \right\}$

Can quantum mechanical physical reality be considered complete?

Einstein, Poldosky, Rosen, Phys. Rev. **47**, 777 (1935).

(1) Proposed the entangled two-particle state according to the principle of quantum superposition:

$$\Psi(x_1, x_2) = \int dp\, \psi_p(x_2) u_p(x_1) \Rightarrow \delta(x_1 - x_2 + x_0)$$

$$\overline{\Psi}(p_1, p_2) = \int dx\, \varphi_x(x_2) v_x(x_1) \Rightarrow \delta(p_1 + p_2)$$

(2) Pointed out an surprising phenomenon: the momentum (position) for neither subsystem is determinate; however, if one particle is measured to have a certain momentum (posit-ion), the momentum (position) of its "twin" is determined with certainty, *despite the distance between them!*

The apparent contradiction deeply troubled Einstein.

While one sees the measurement on (p_1+p_2) and (x_1-x_2) of two individual particles satisfy the EPR δ-function and believes the classical inequality, one might easily be trapped into considering either there is a violation of the uncertainty principle or there exists action-at-a-distance.

Violation of the uncertainty principle ?

$$\Delta\left(p_1 + p_2\right) = 0 \quad \Delta\left(x_1 - x_2\right) = 0$$

Simultaneously !

(p_1+p_2) and (x_1-x_2) are not conjugate variables !!!!

$$\Psi(x_1,x_2)=\frac{1}{2\pi\hbar}\int dp_1 dp_2\,\delta(p_1+p_2)\,e^{ip_1x_1/\hbar}e^{ip_2(x_2-x_0)/\hbar}$$

$$=\frac{1}{2\pi\hbar}\int d(p_1+p_2)\,\delta(p_1+p_2)\,e^{i(p_1+p_2)(x_1+x_2^{'})/2\hbar}$$

$$\times\int d(p_1-p_2)/2\;e^{i(p_1-p_2)(x_1-x_2^{'})/2\hbar}$$

$$=1\times\delta(x_1-x_2+x_0)$$

$$\overline{\Psi}(p_1,p_2)=\frac{1}{2\pi\hbar}\int dx_1 dx_2\,\delta(x_1-x_2+x_0)\,e^{-ip_1x_1/\hbar}e^{-ip_2(x_2-x_0)/\hbar}$$

$$=\frac{1}{2\pi\hbar}\int d(x_1+x_2^{'})\;e^{-i(p_1+p_2)(x_1+x_2^{'})/2\hbar}$$

$$\times\int d(x_1-x_2^{'})/2\;\delta(x_1-x_2^{'})\,e^{-i(p_1-p_2)(x_1-x_2^{'})/2\hbar}$$

$$=\delta(p_1+p_2)\times 1$$

Conjugate Variables:

$$(x_1 + x_2) \Leftrightarrow (p_1 + p_2)$$

$$(x_1 - x_2) \Leftrightarrow (p_1 - p_2)$$

$$\Downarrow$$

$$\Delta(x_1 + x_2) = \infty \quad \Leftrightarrow \quad \Delta(p_1 + p_2) = 0$$

$$\Delta(x_1 - x_2) = 0 \quad \Leftrightarrow \quad \Delta(p_1 - p_2) = \infty$$

EPR δ-function:

-- perfect entangled system

$$\Delta(x_1 - x_2) = 0, \quad \Delta(p_1 + p_2) = 0.$$

Although: $\Delta x_1 \approx \infty,\ \Delta x_2 \approx \infty,\ \Delta p_1 \approx \infty,\ \Delta p_2 \approx \infty.$

EPR Inequality:

-- non-perfect entangled system

$$\Delta(x_1 - x_2) < \min(\Delta x_1, \Delta x_2)$$

$$\Delta(p_1 + p_2) < \min(\Delta p_1, \Delta p_2)$$

Then, why Einstein … ?

Observation:

$$\Delta(x_1 - x_2) = 0, \quad \Delta(p_1 + p_2) = 0$$

Believing:

$$\Delta(x_1 - x_2) > Max(\Delta x_1, \Delta x_2)$$
$$\Delta(p_1 + p_2) > Max(\Delta p_1, \Delta p_2)$$

Conclusion:

$$\Delta x_1 = 0, \quad \Delta p_1 = 0$$
$$\Delta x_2 = 0, \quad \Delta p_2 = 0$$

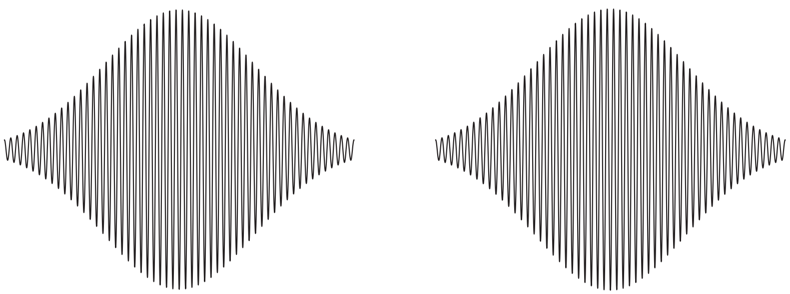
(Violation of the …)

The interpretation ?

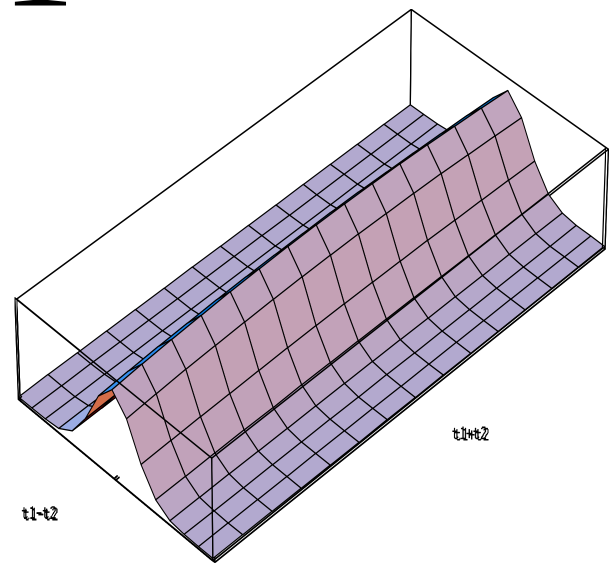
Quantum entanglement

Two-photon is not two photons !

$$2 \neq 1+1$$

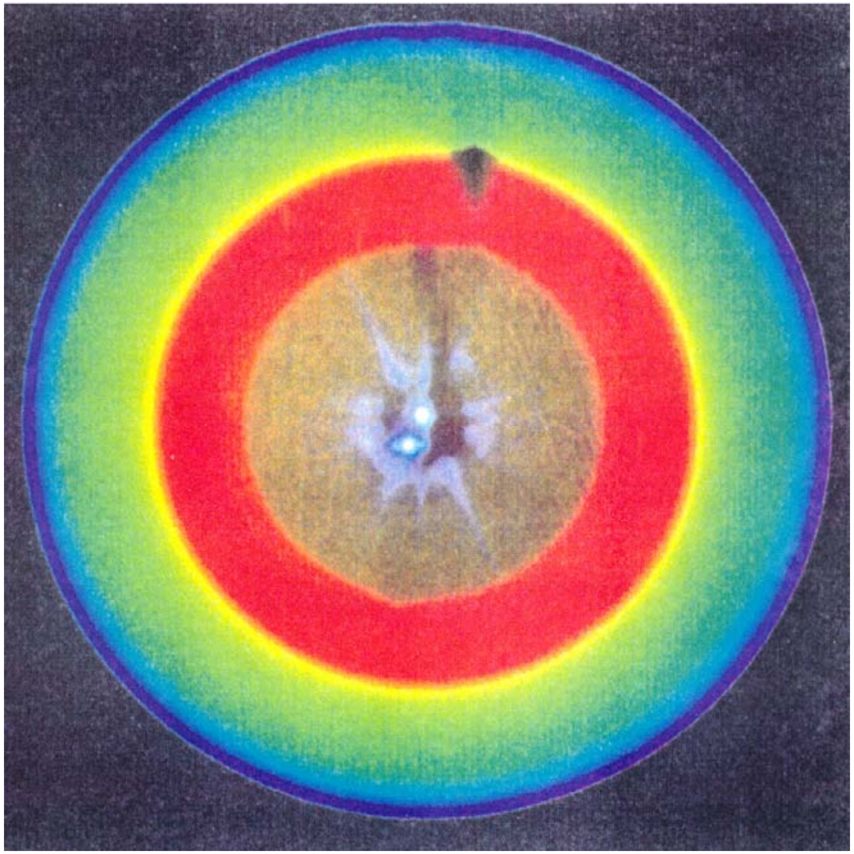


Classical:
Two Wavepackets



Entanglement:
A non-factorable
2-D Wavepacket

Biphoton State: Spontaneous Parametric Down Conversion



Two-photon Pure State

The signal (idler) photon can have any energy (momentum), however, if one of the photons is measured at certain energy (momentum) its twin must be at a certain energy (momentum).

$$\left|\Psi\right\rangle = \sum_{s,i} \delta(\omega_s + \omega_i - \omega_p)\,\delta(\mathbf{k}_s + \mathbf{k}_i - \mathbf{k}_p)\,\hat{a}_s^{+}\hat{a}_i^{+}\left|0\right\rangle$$

Operational approach:

$$G^{(2)}(x_1,x_2)=\left\langle E_1^{(-)}E_2^{(-)}E_2^{(+)}E_1^{(+)}\right\rangle$$

Pure state:

$$G^{(2)}(x_1,x_2)$$

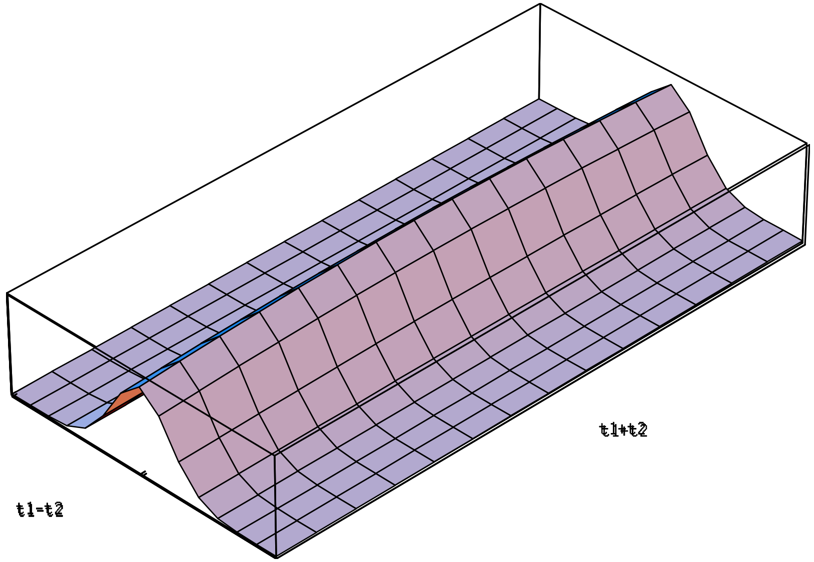
$$=\left\langle\Psi\middle|E_1^{(-)}(x_1)E_2^{(-)}(x_2)E_2^{(+)}(x_2)E_1^{(+)}(x_1)\middle|\Psi\right\rangle$$

$$=\left|\left\langle 0\middle|E_2^{(+)}(x_2)E_1^{(+)}(x_1)\middle|\Psi\right\rangle\right|^2$$

SPDC

A biphoton

$$\Psi(t_1,t_2) \equiv \langle 0 | E_2^{(+)}(t_2) E_1^{(+)}(x_1) | \Psi \rangle$$



Effective Two-photon wavefunction

$$\Psi(t_1,t_2) = F_{t_1+t_2}\{ f(\omega_p - \omega_{p0})\}\, F_{t_1-t_2}\{g(\omega_s - \omega_{s0})\}\, e^{-i\omega_{s0}t_1}\; e^{-i\omega_{i0}t_2}$$

Two-photon imaging

Field Operators:

$$E_j^{(+)}(\mathbf{r}_j, t_j) = \int d\mathbf{k}^T \int d\omega \, a(\omega, \mathbf{k}^T) \, e^{-i\omega t_j} \, g_j(\omega, \mathbf{k}^T; z_j, \vec{\rho}_j)$$

$g_j(\omega, \mathbf{k}^T; z_j, \vec{\rho}_j)$: Green's function (optical transfer function).
determined by the experimental setup.

$$\Downarrow$$

$$G^{(2)} = \left| \, \Psi(\vec{\rho}_1, \vec{\rho}_2) \, \right|^2$$

The calculation of $G^{(2)}$ is lengthy but straightforward:

$$\Delta(\vec{\rho}_1 - \vec{\rho}_2)_{EPR} \sim 0$$

It is the two-photon coherent superposition made it possible!

Although questions regarding fundamental issues of quantum theory still exist, quantum entanglement has indeed brought up a novel concept or technology in nonlocal positioning and timing measurements with high accuracy, even beyond the classical limit.

Question:

Can “ghost” image be simulated classically ?

$\Uparrow$

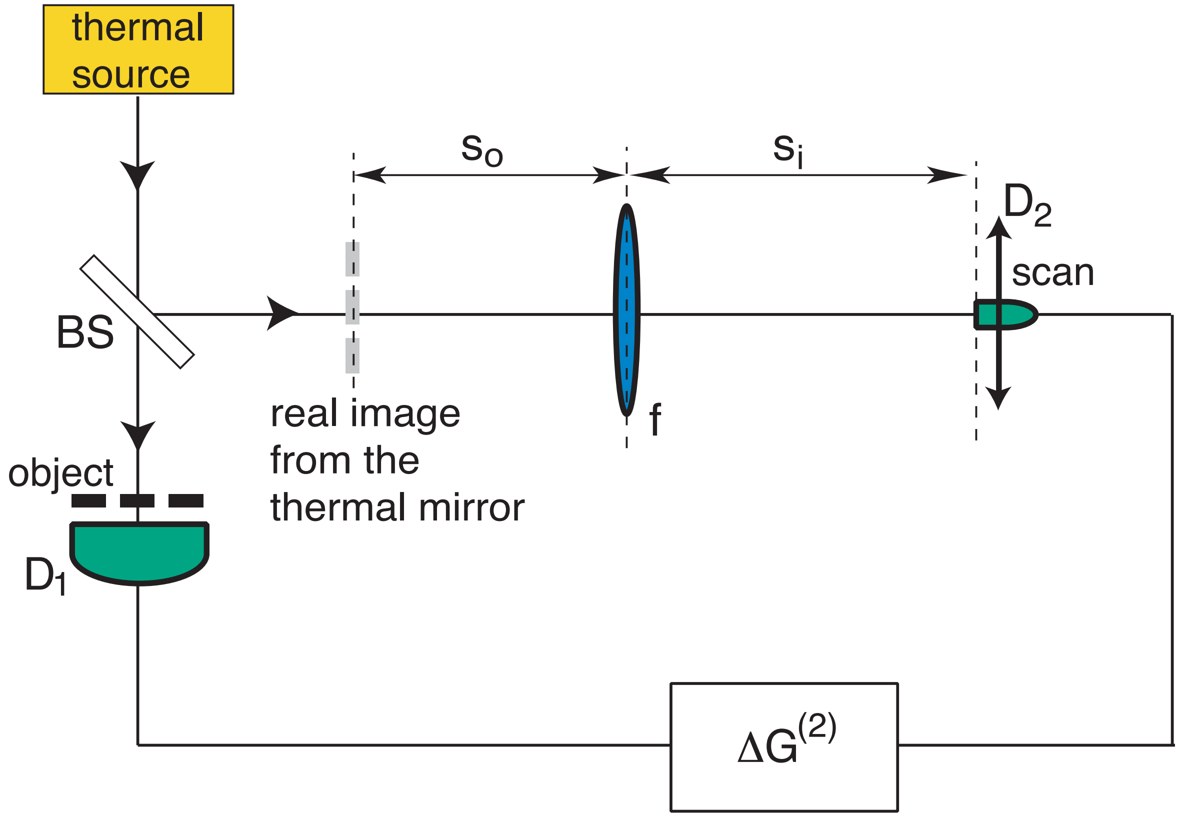
Image but not projection!!!

Yes

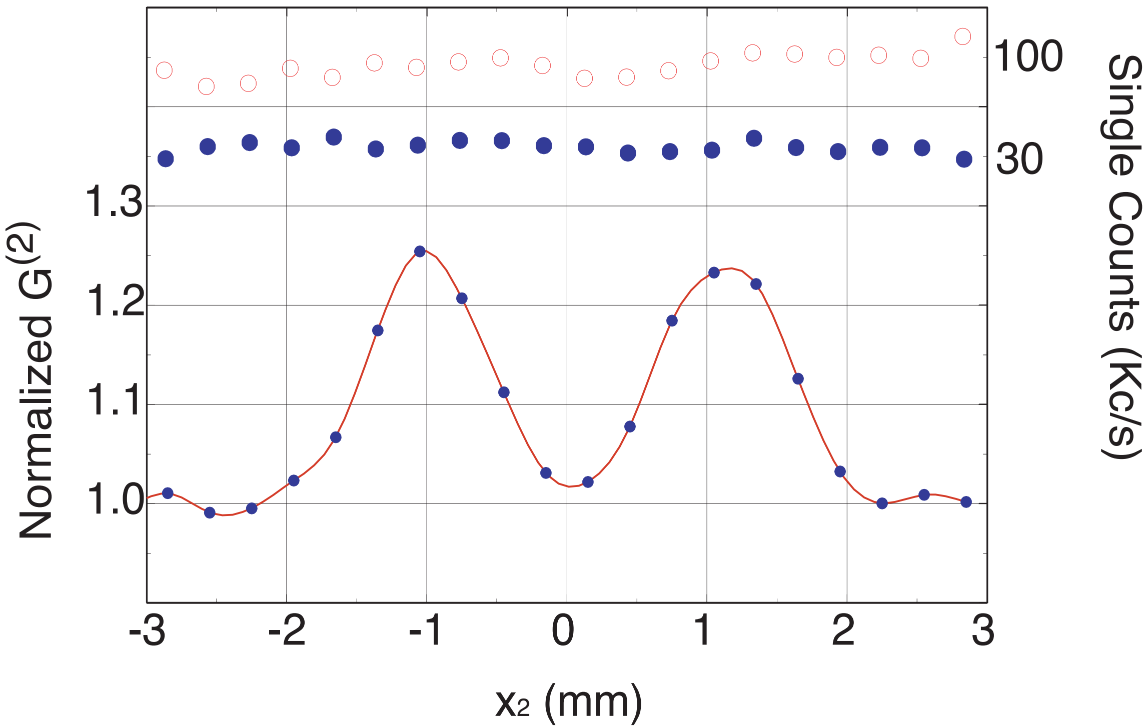
Experimentally

↕

Thermal Light Imaging



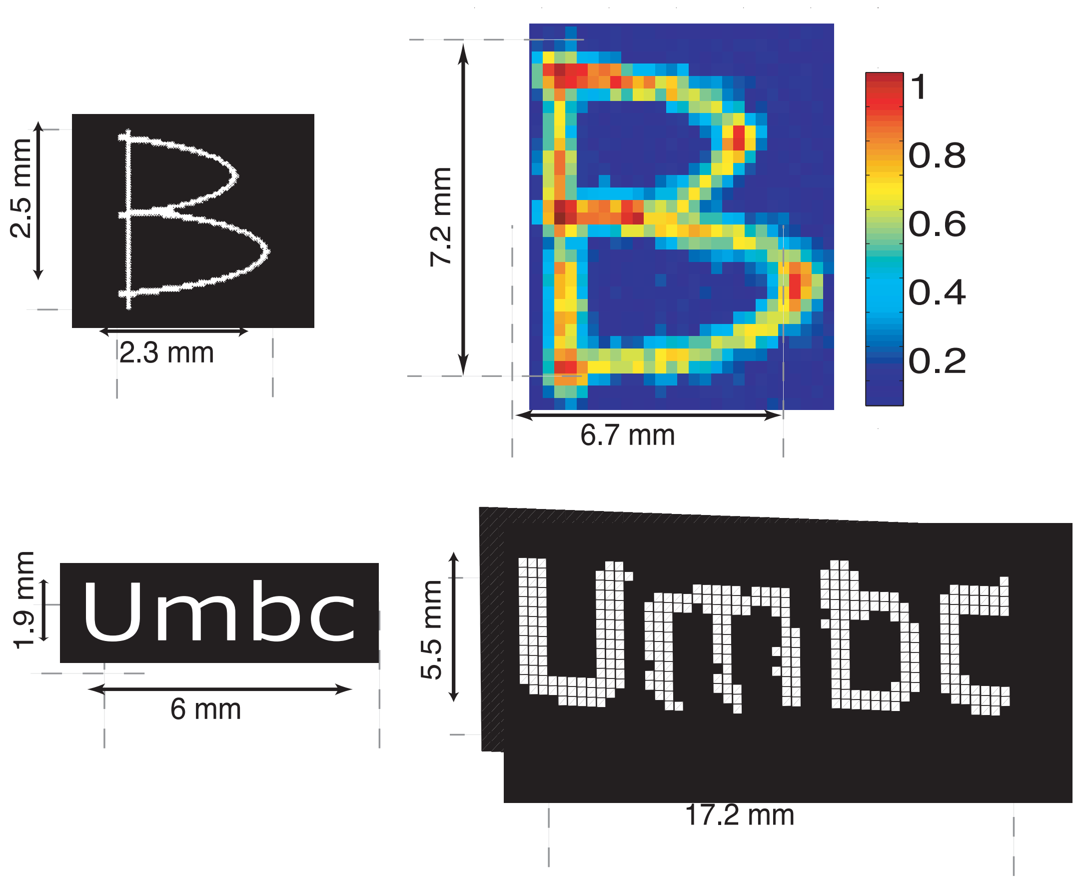
Magic Mirror and Ghost Imaging



M = 2.15 (M_{theory} = 2.16); V = 12 % (V_{theory} =16.5%)

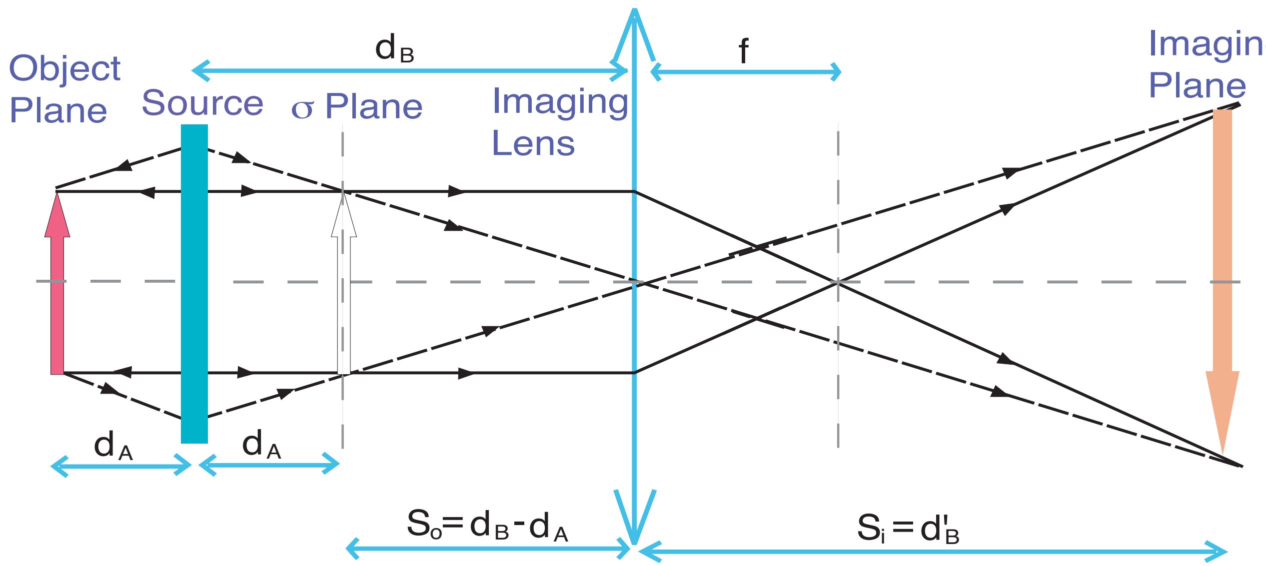
Experimental Result: Ghost image of a double-slit.

A. Valencia, G. Scarcelli, M. D'Angelo, and Y.H. Shih, Phys. Rev. Lett. **94**, 063601 (2005).



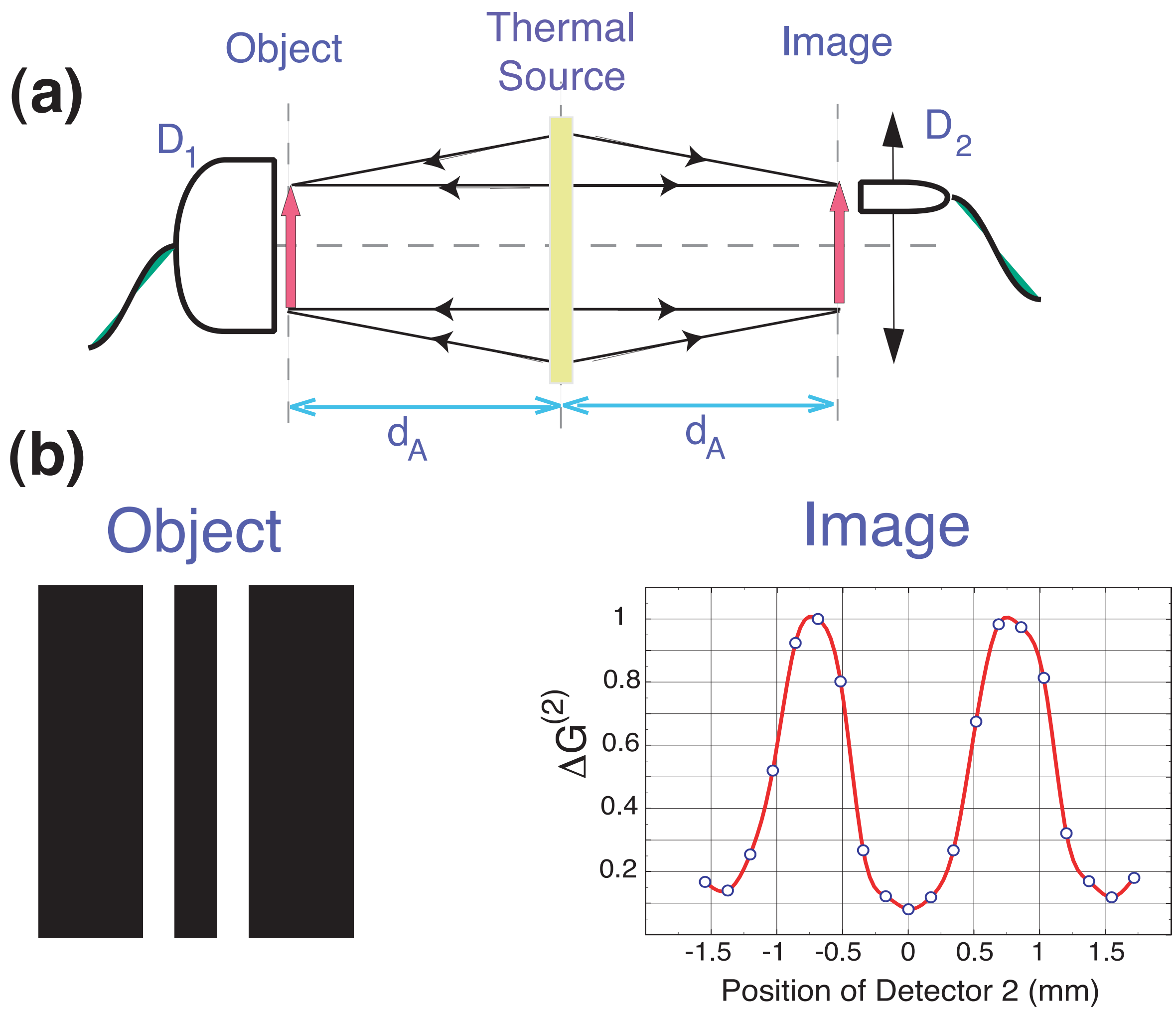
Measurement on the image plan.

Two-photon thermal light Imaging:



Incoherent imaging: $G^{(2)} = \sum_k G_k^{(2)} = \sum_k [\, G_{11}^{(1)} G_{22}^{(1)} + G_{12}^{(1)} G_{21}^{(1)} \,]$

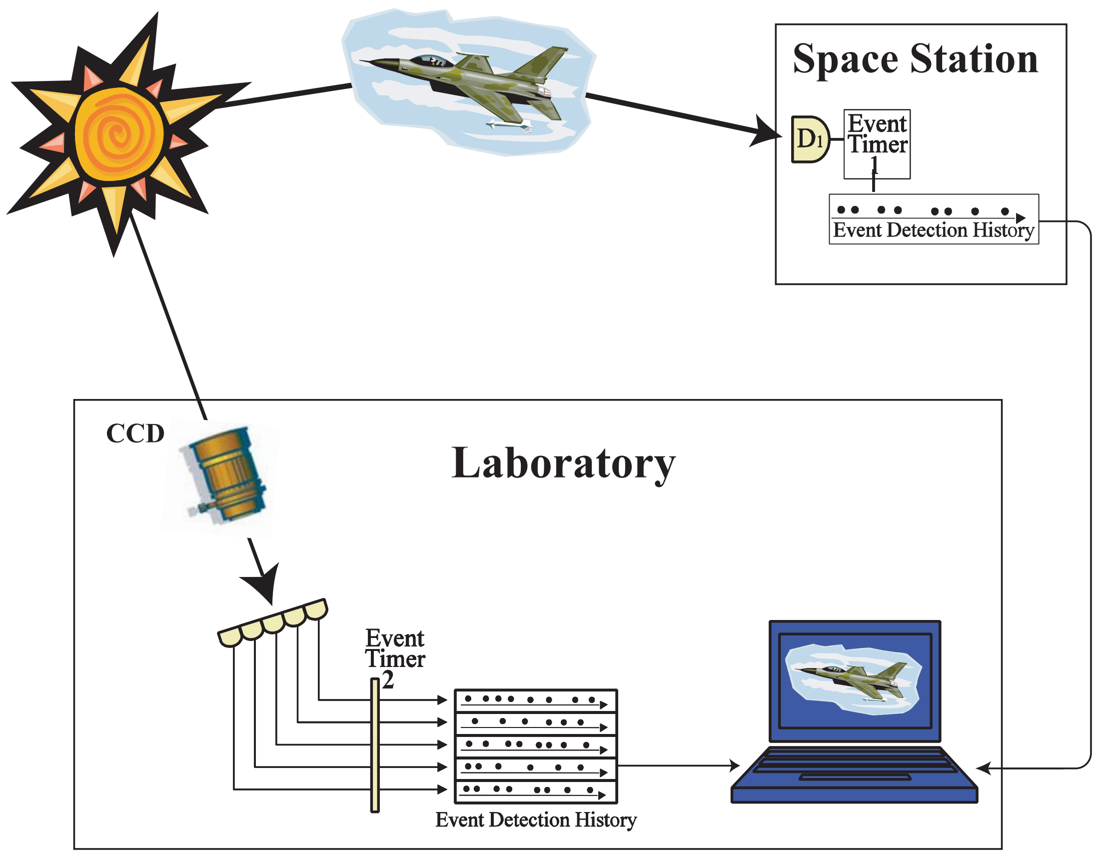
Magic Mirror ?



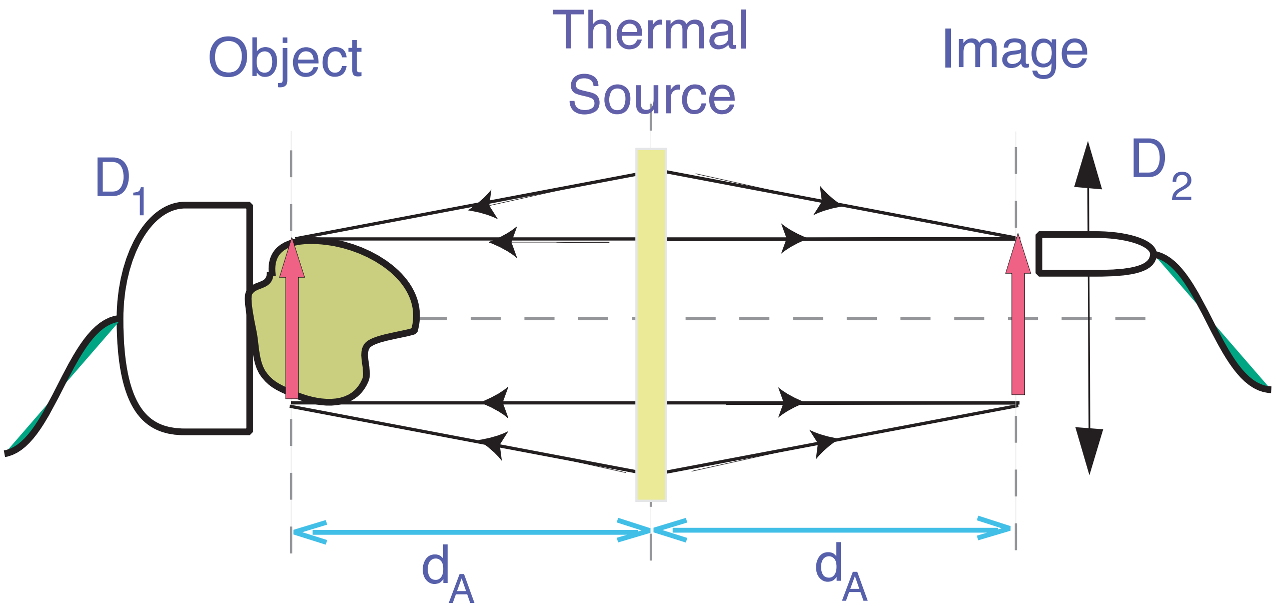
Measurement on the mirror plan.

It is useful !

A “Ghost” Camera in Space
(Nonlocal)



A “Magic Mirror” for X-ray 3-D Imaging



It is fundamentally interesting !!

50% momentum-momentum, position-position EPR correlation

Where it comes from ?

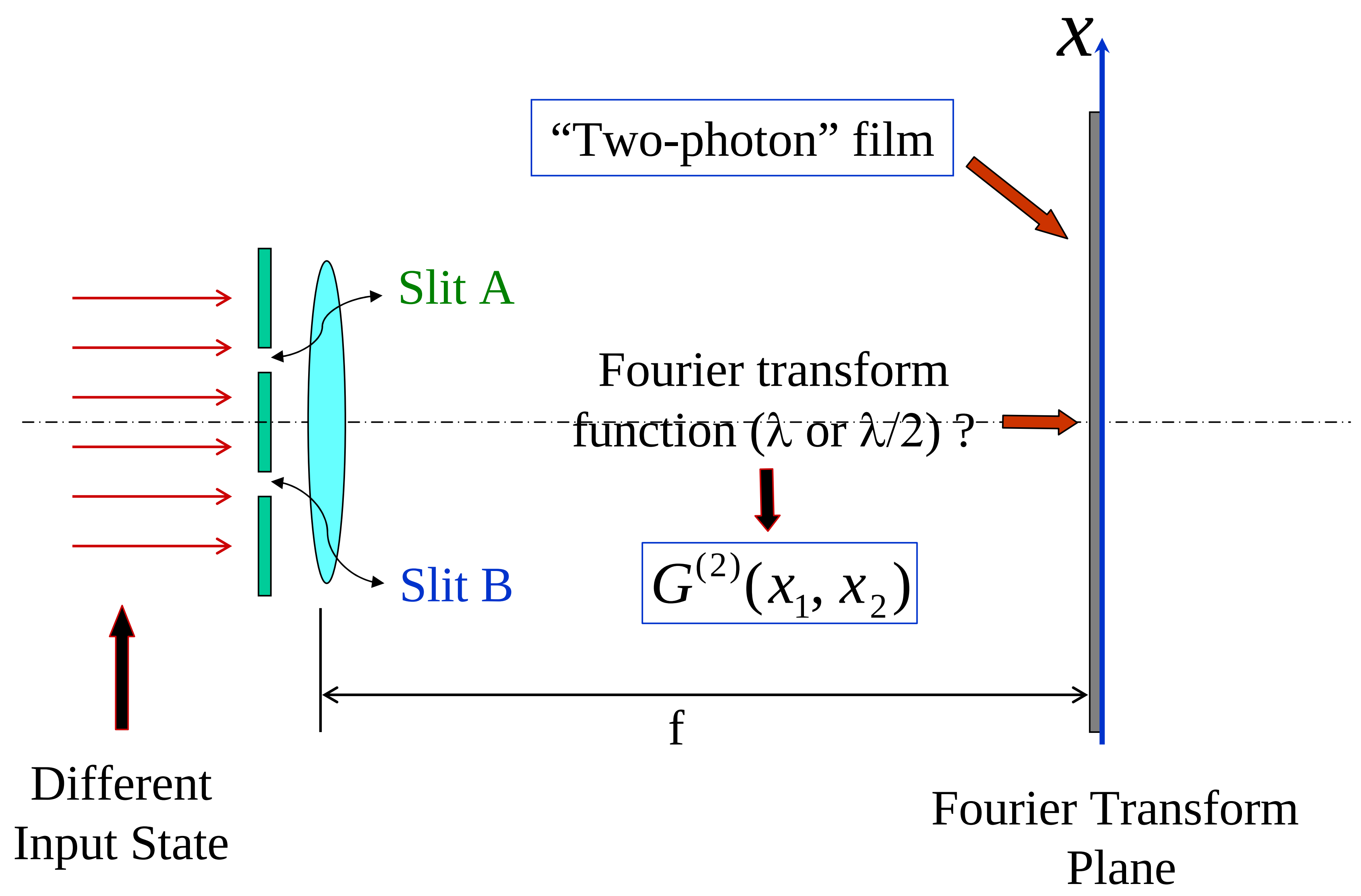
Remember: thermal light is chaotic !

It comes from Hanbeury Brown - Twiss … ???

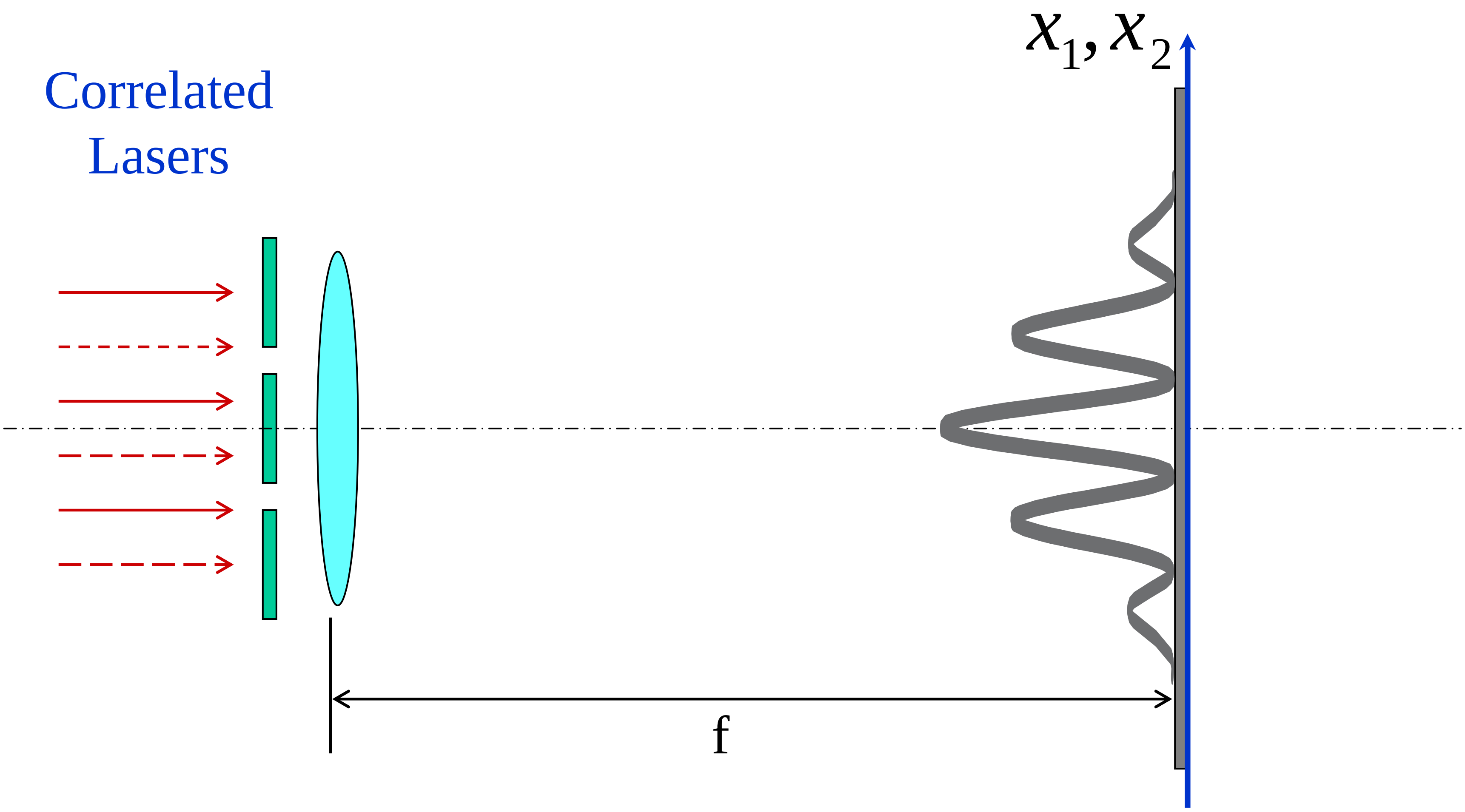
It comes from “photon bunching” … ???

We are not satisfied !

The physics behind ???

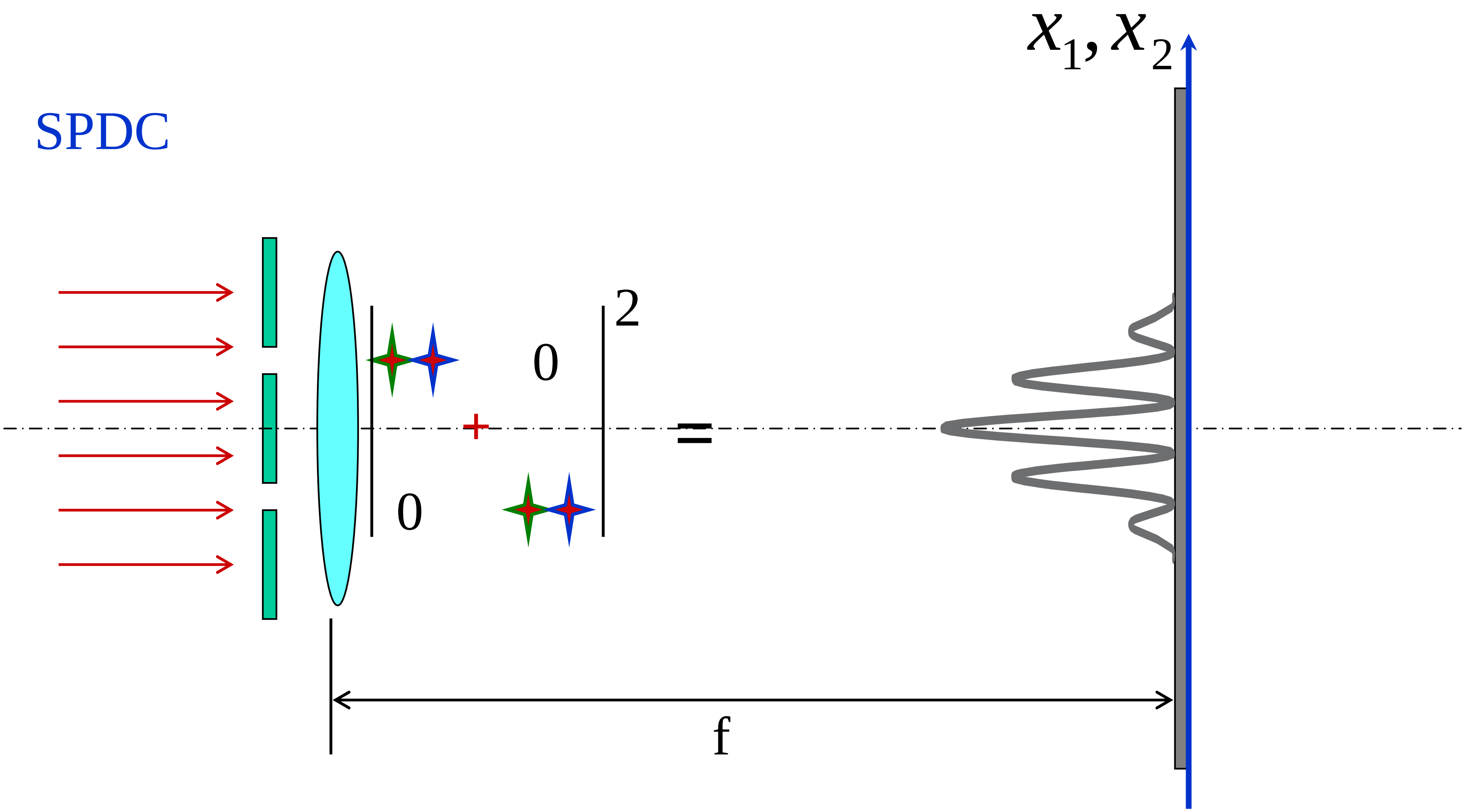


x
"Two-photon" film
Slit A
Fourier transform
function (λ or λ/2) ?
$G^{(2)}(x_1, x_2)$
Slit B
f
Different
Input State
Fourier Transform
Plane



$$G^{(2)} = G^{(2)}(x_1) \times G^{(2)}(x_2)$$

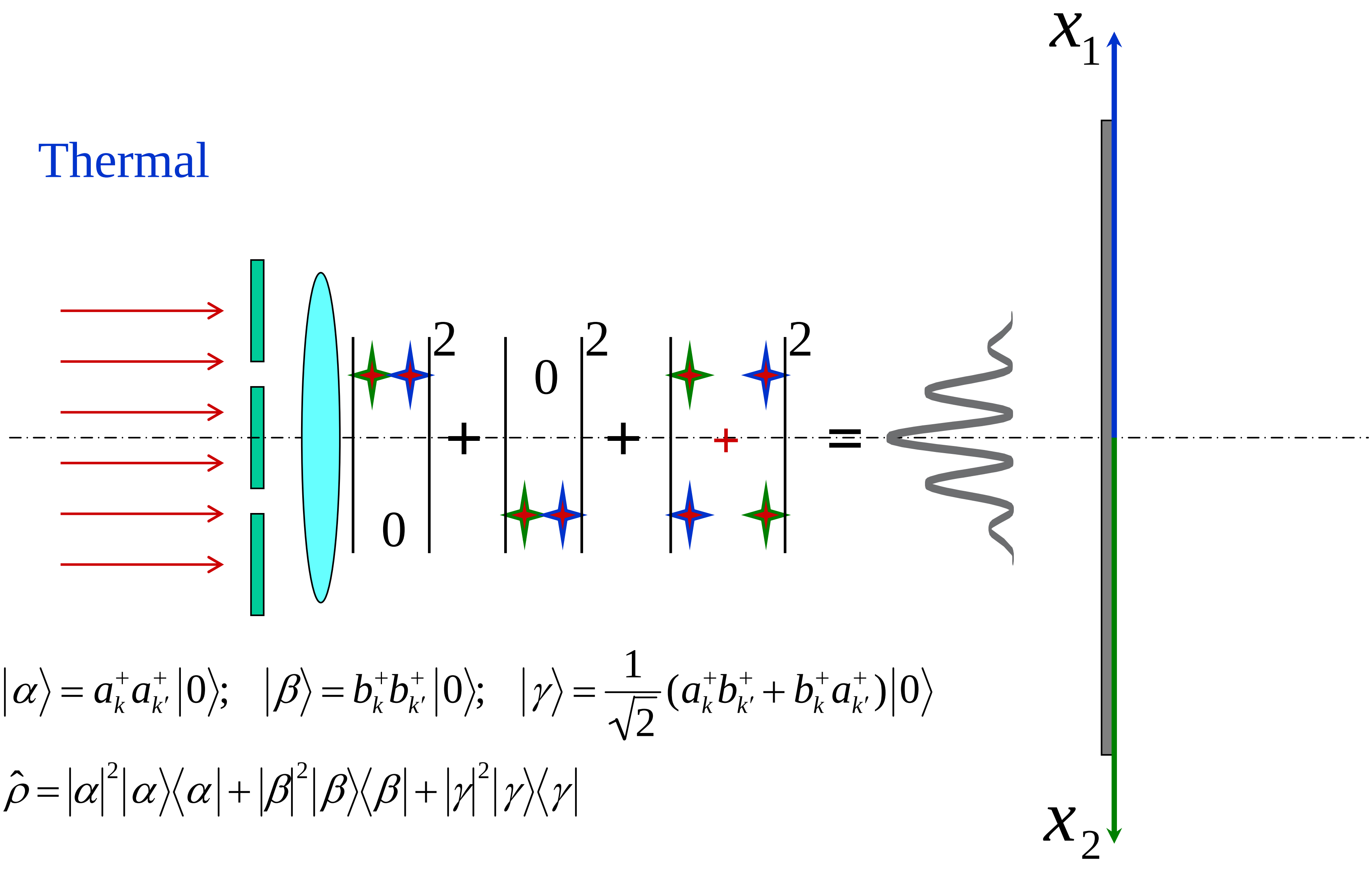
A product of two independent first-order-pattern.



$$\left|\Psi\right\rangle = [a_s^+ a_i^+ + b_s^+ b_i^+ e^{i\phi}]\left|0\right\rangle; \quad \phi = const.$$

$$G^{(2)} \propto \operatorname{sinc}^2[\frac{\pi a f}{\lambda}(x_1 + x_2)] \cos^2[\frac{\pi b f}{\lambda}(x_1 + x_2)] = \operatorname{sinc}^2(\frac{\pi a f}{\lambda/2} x) \cos^2 \frac{\pi b f}{\lambda/2} x$$

Thermal



$$|\alpha\rangle = a_k^+ a_{k'}^+ |0\rangle; \quad |\beta\rangle = b_k^+ b_{k'}^+ |0\rangle; \quad |\gamma\rangle = \frac{1}{\sqrt{2}}(a_k^+ b_{k'}^+ + b_k^+ a_{k'}^+)|0\rangle$$

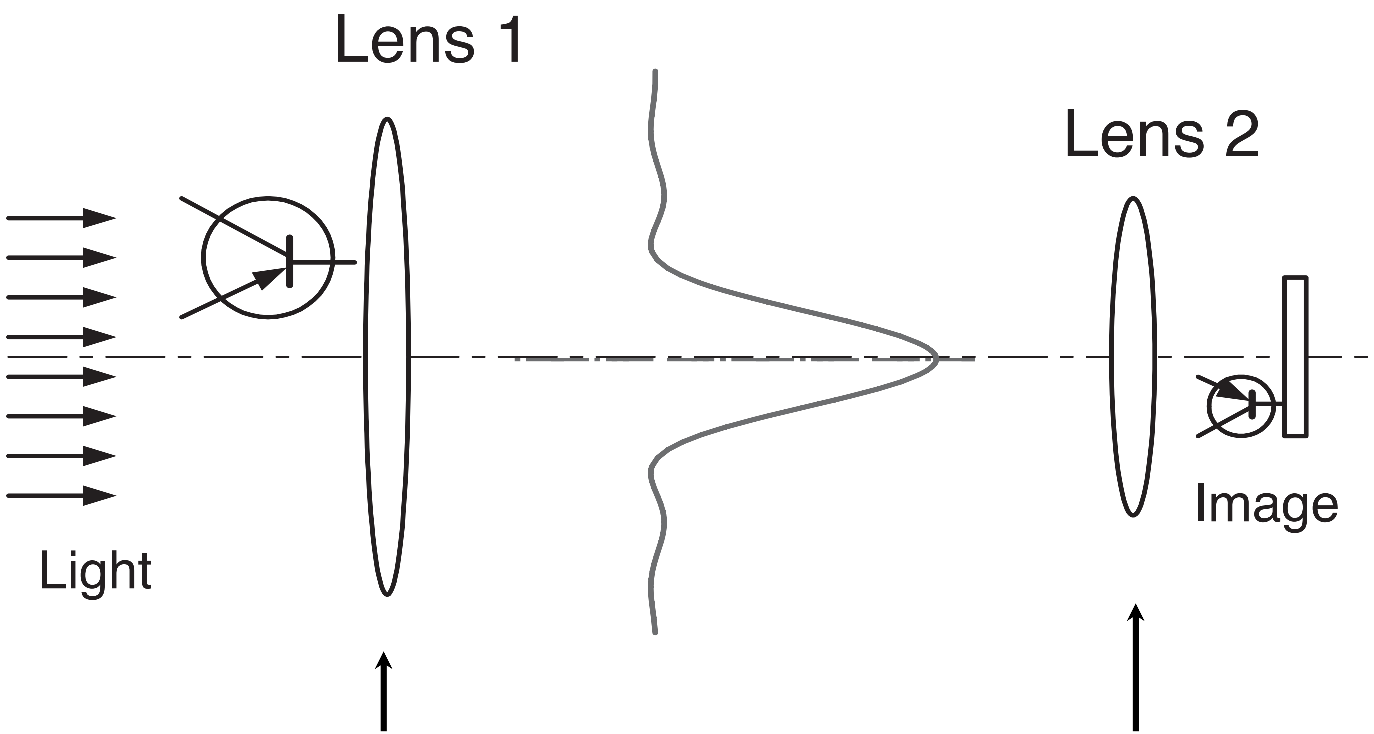
$$\hat{\rho} = |\alpha|^2 |\alpha\rangle\langle\alpha| + |\beta|^2 |\beta\rangle\langle\beta| + |\gamma|^2 |\gamma\rangle\langle\gamma|$$

$$G^{(2)} \propto 1 + \operatorname{sinc}^2[\frac{\pi a f}{\lambda}(x_1 - x_2)] \cos^2[\frac{\pi b f}{\lambda}(x_1 - x_2)]$$

Quantum lithography

(ultra-resolution: beyond classical limit)

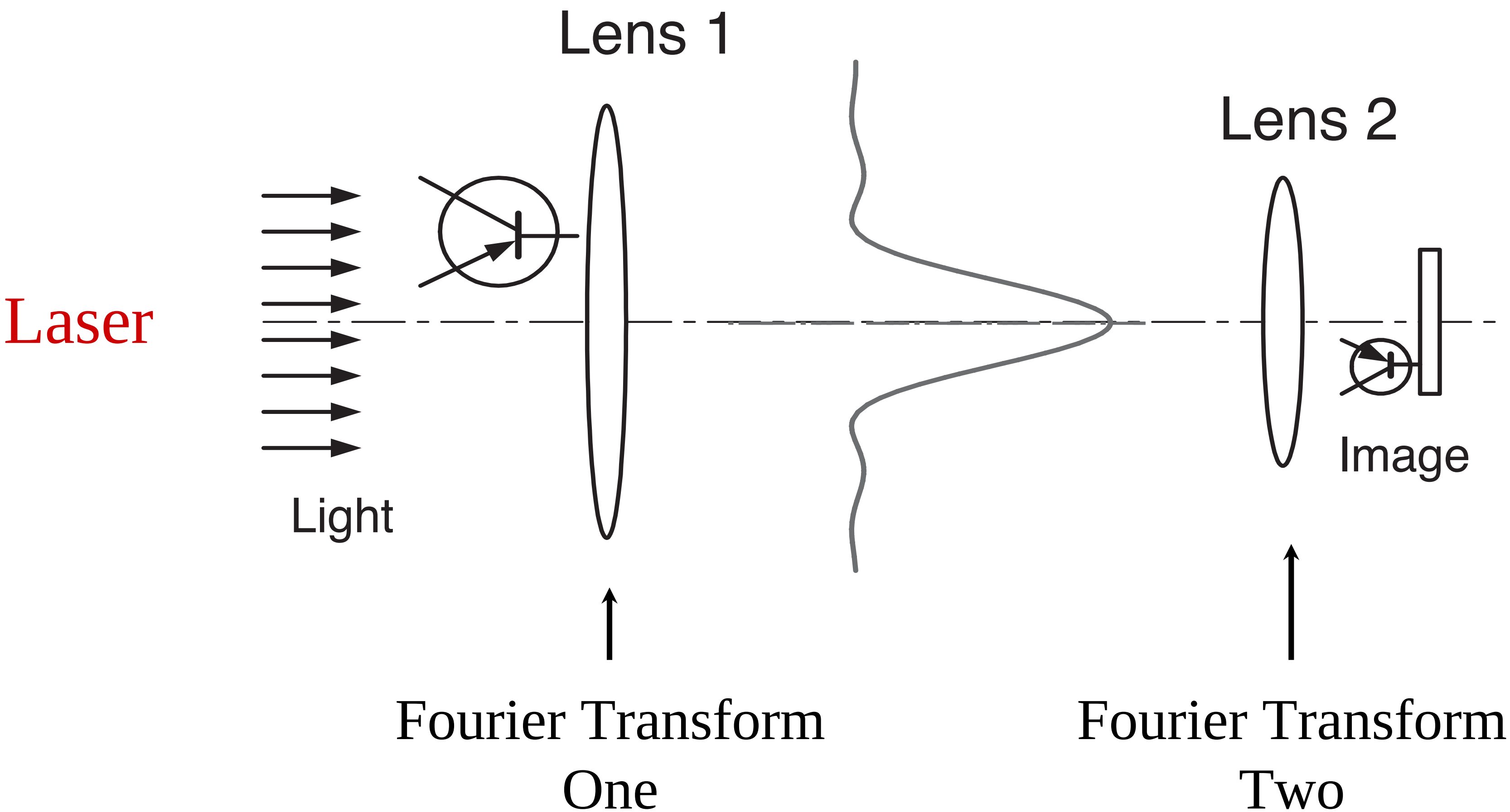
Optical Lithography



Fourier Transform One

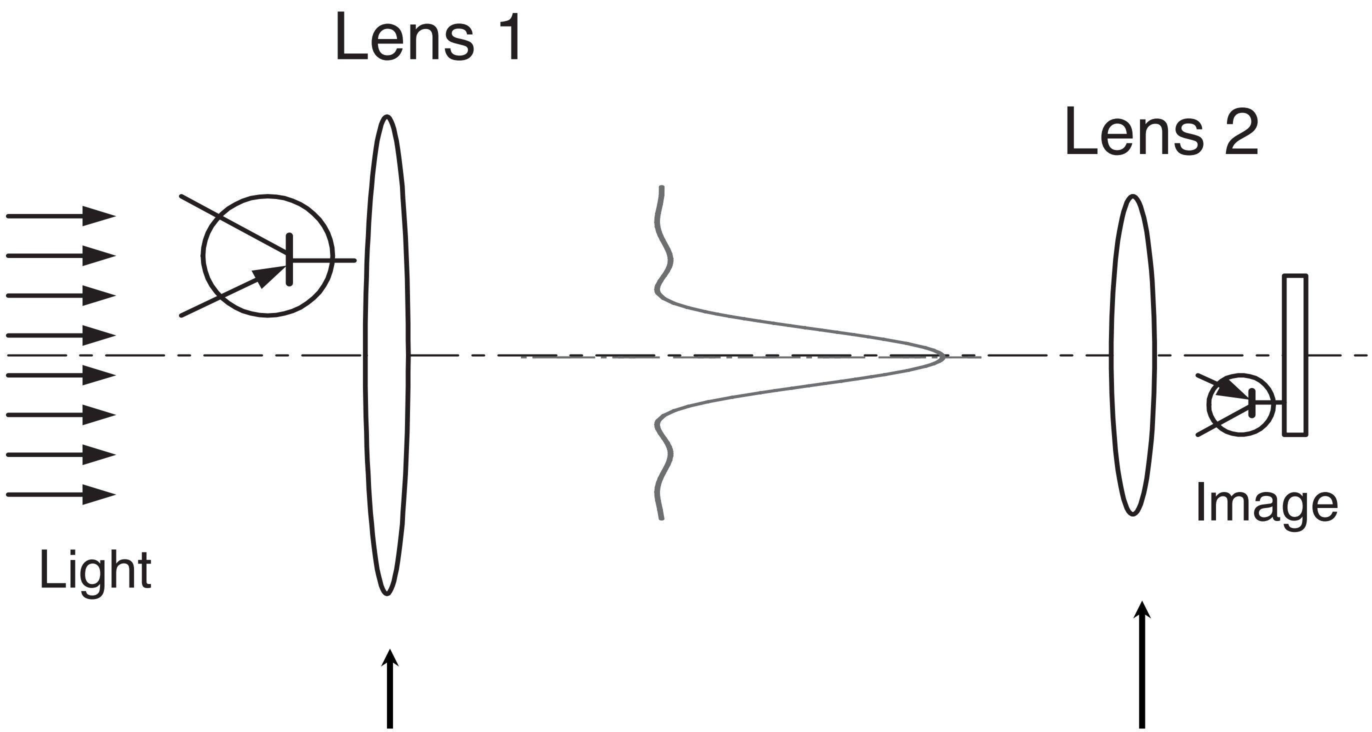
Fourier Transform Two

Optical Lithography



Optical Lithography

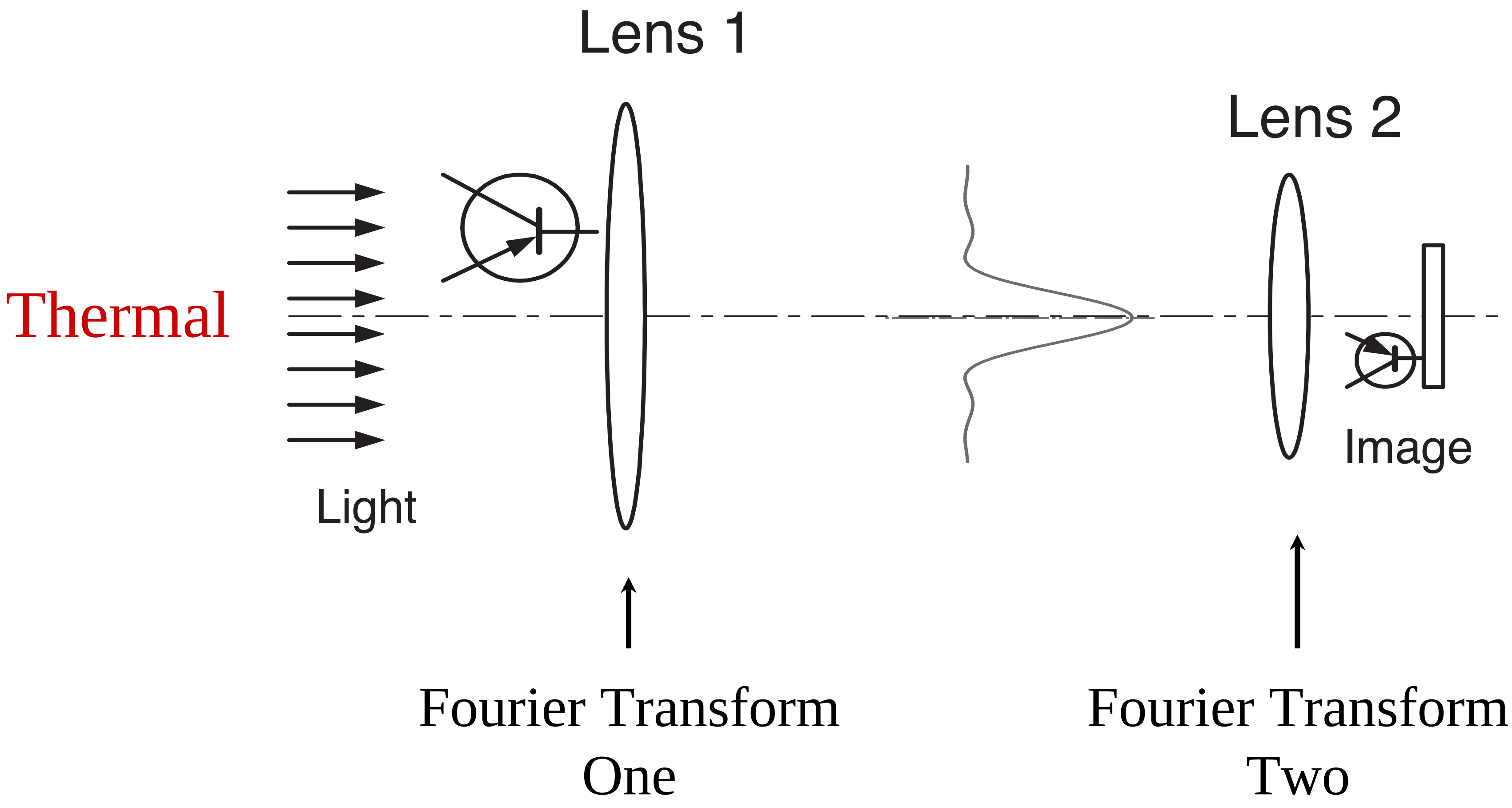
SPDC



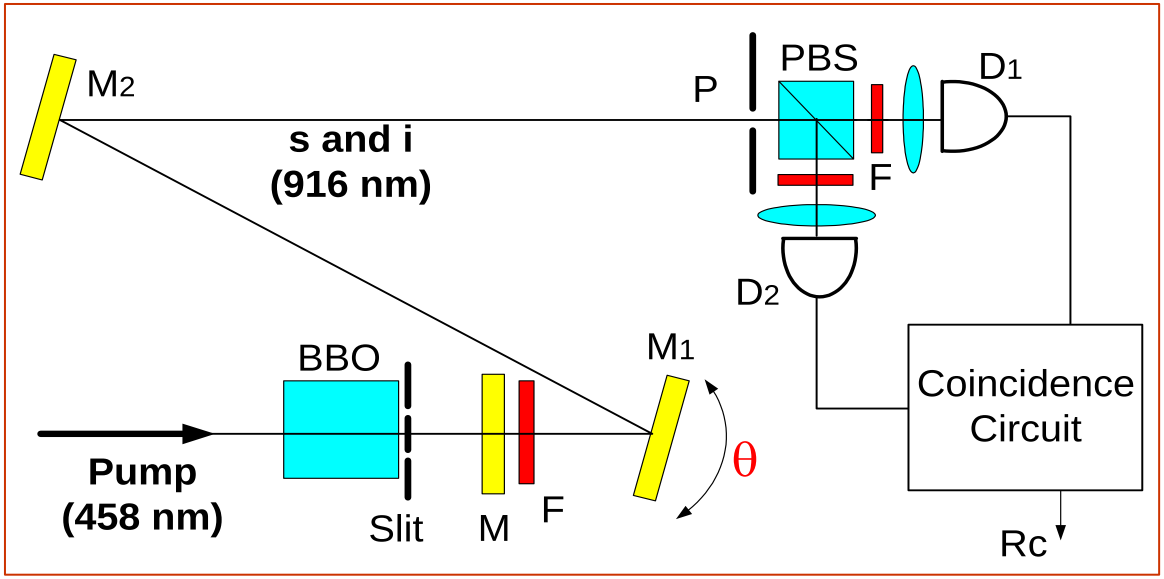
Fourier Transform
One

Fourier Transform
Two

Optical Lithography



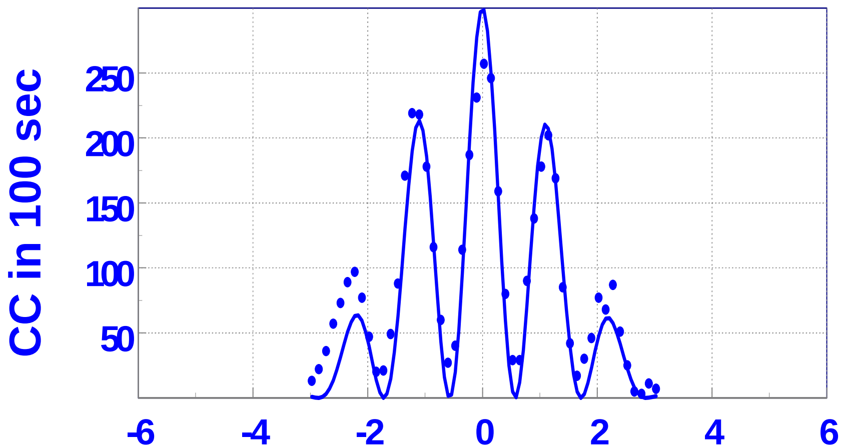
Two-photon diffraction and quantum lithography



Experiment: M. D'Angelo, et al, PRL, **87**, 013602 (2001).

Theory: A.N. Boto, et al. PRL **85**, 2733 (2000).

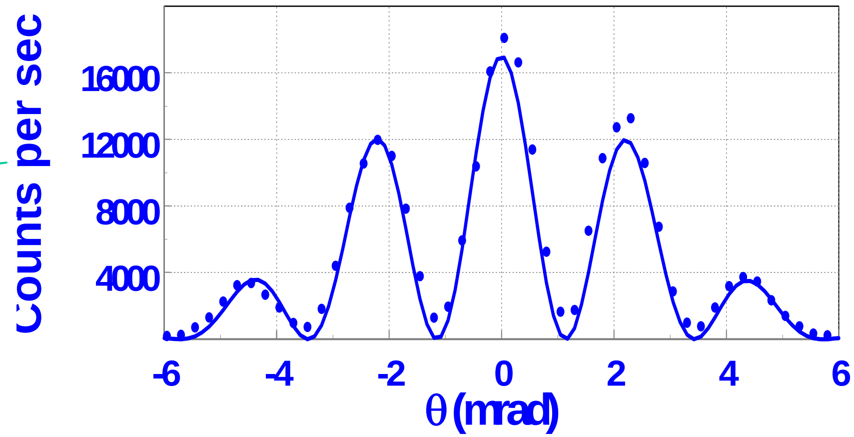
Experimental Data



$$R_c(\theta) \;=\; \mathrm{sinc}^2[(2\pi a/\lambda)\;\theta] \;\times\; \cos^2[(2\pi b/\lambda)\;\theta]$$

SPDC two-photon at

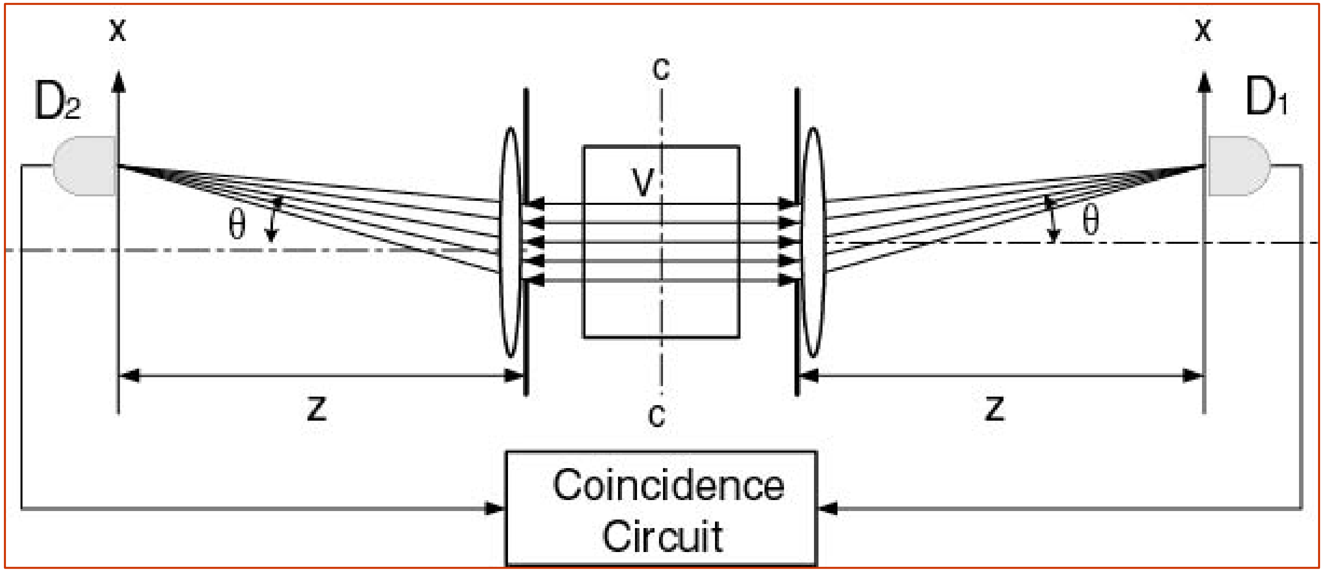
$$\lambda_s = \lambda_i = 916nm$$



$$I(\theta) \;=\; \mathrm{sinc}^2[(\pi a/\lambda)\;\theta] \;\times\; \cos^2[(\pi b/\lambda)\;\theta]$$

Classical laser light at

$$\lambda = 916nm$$

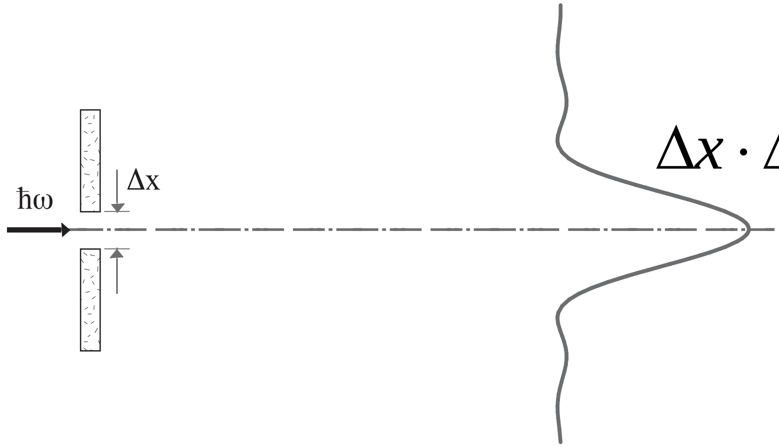


It is the result of two-photon coherent superposition. It measures the second-order correlation between the object plane and the image plane, defined by the Gaussian thin lens equation.

The published measurement was on the Fourier transform plane (far-field).
PRL, **87**, 013602 (2001).

Super-resolution:

Classical diffraction

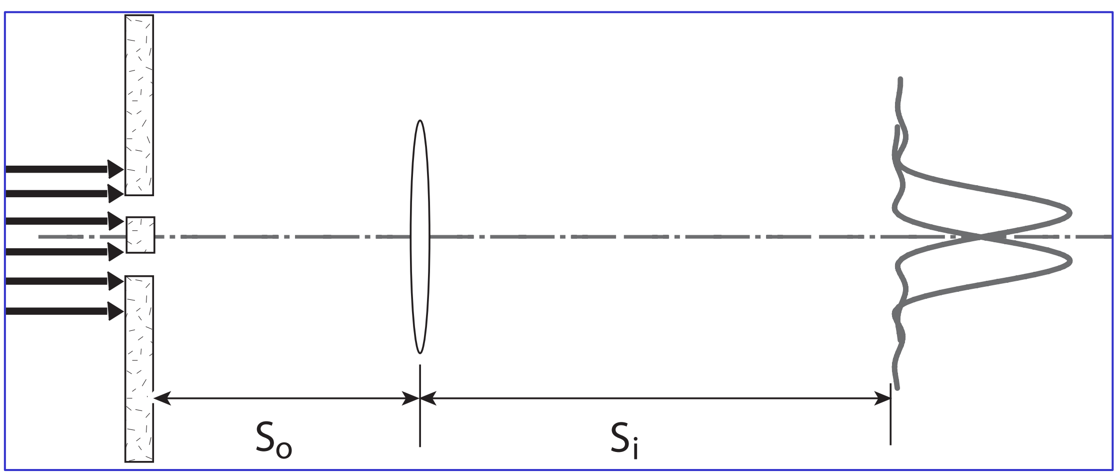


$$\Delta x \cdot \Delta p_x = h$$

Diffraction of a pair

$$\Delta x \cdot \Delta p_x < h$$

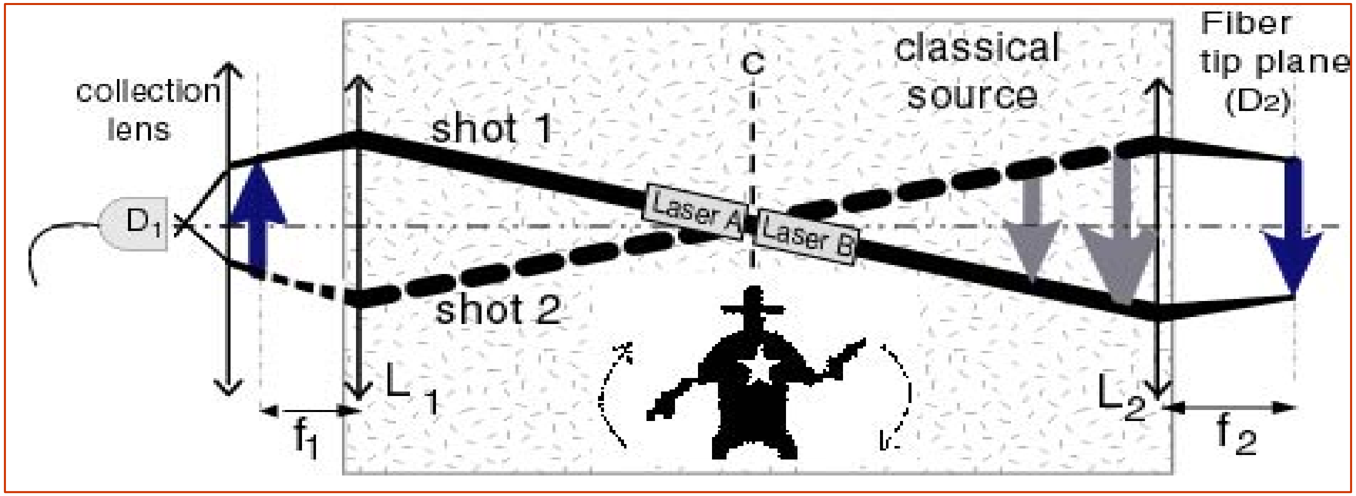
⇔



Double (super) Spatial Resolution on the Image Plane

“Ghost” Shadow (Projection)

$$\hat{\rho}_{cl} = \int d\mathbf{k}_1 \int d\mathbf{k}_2\, P(\mathbf{k}_1)\,\delta(\mathbf{k}_1 + \mathbf{k}_2)\,\rho_1^{(\mathbf{k}_1)} \otimes \rho_2^{(\mathbf{k}_2)}$$



Bennink ***et al*****. PRL 89, 113601 (2002)**