# Nonclassical, Two-Photon Interferometry and Lithography with High-Gain Optical Parametric Amplifiers

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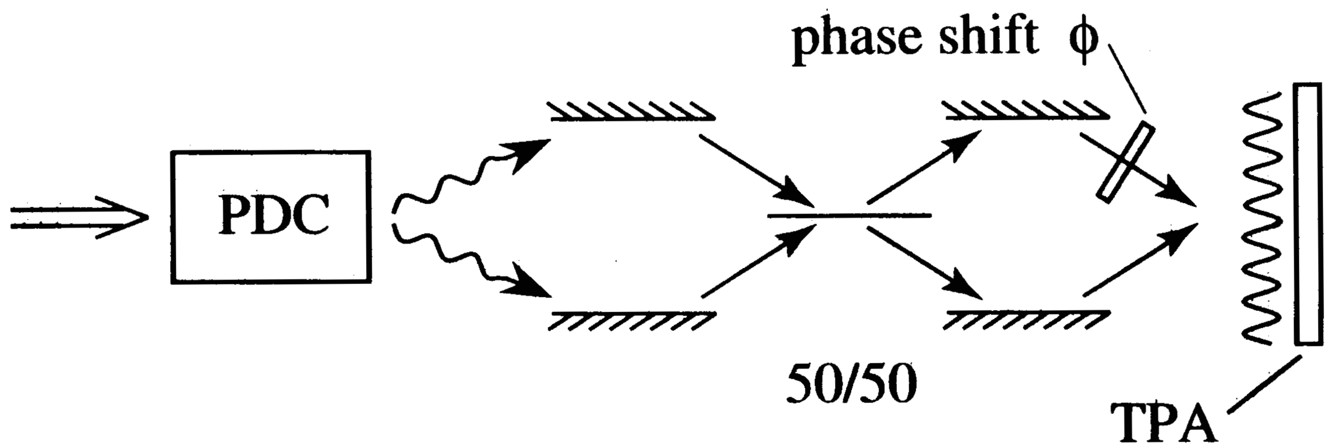
To what extent do unseeded, high-gain optical parametric amplifiers preserve the desirable quantum statistical properties of spontaneous parametric downconversion?

Presented at CQO-8, June 14, 2001.

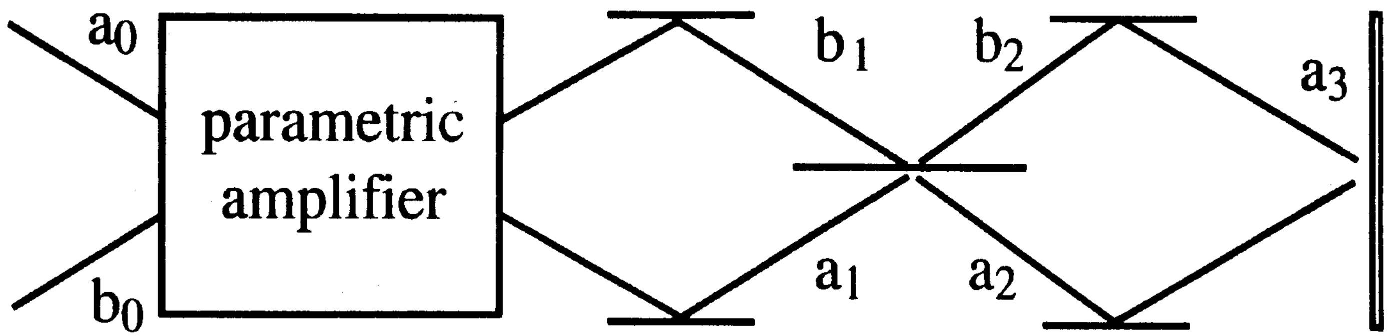
# Quantum Lithography and Microscopy

- Entangled photons can be used to form interference patterns with detail finer than the Rayleigh limit
- Process "in reverse" performs sub-Rayleigh microscopy




Boto et al, Phys. Rev. Lett. 85, 2733, 2000.

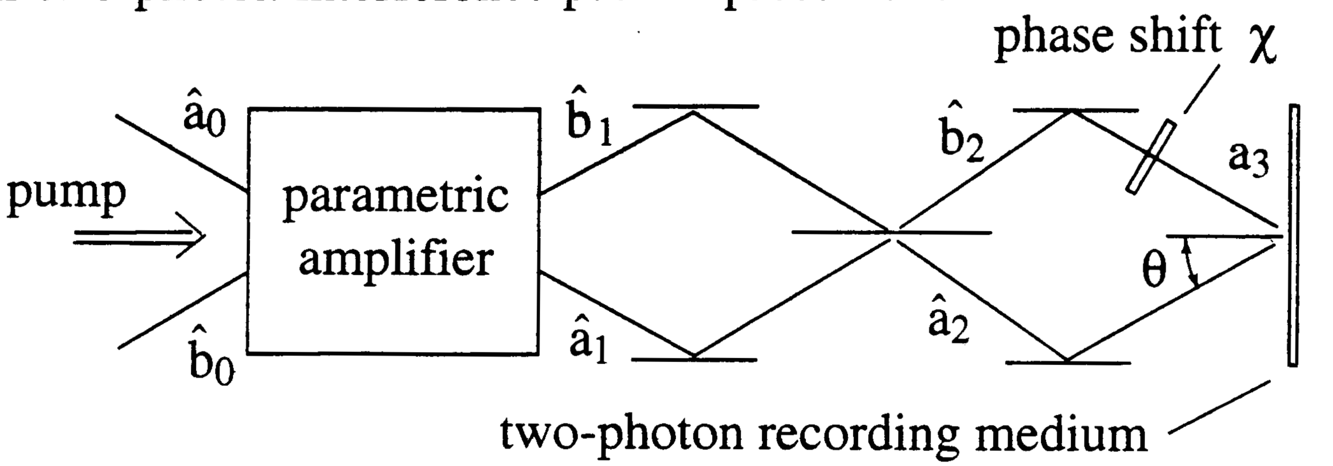
# QUANTUM LITHOGRAPHY PROPOSAL




"Replace" parametric down converter (PDC) with optical parametric amplifier (OPA)–essentially the same device, but now pumped harder to generate sufficient energy levels to be recorded by two-photon responsive lithographic plate at $a_3$.

# Use of High-Gain Parametric Amplifier

Is two-photon interference pattern preserved?




- Transfer equations of OPA

$$\hat{a}_1 = U\hat{a}_0 + V\hat{b}_0^\dagger, \quad \hat{b}_1 = U\hat{b}_0 + V\hat{a}_0^\dagger$$

where

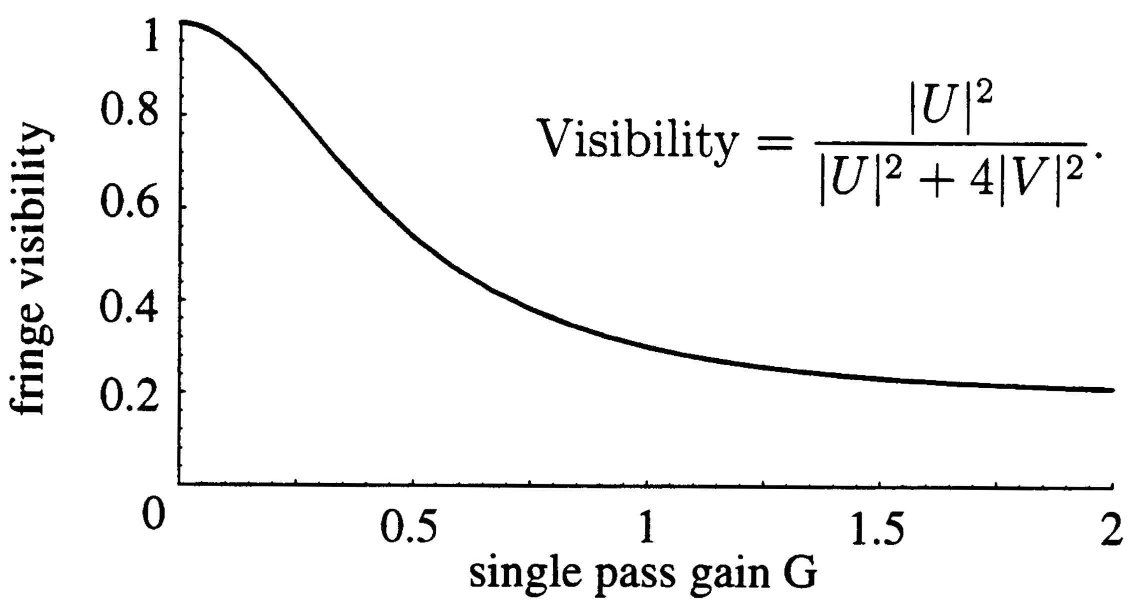
$$U = \cosh G \qquad V = -\, i\exp(i\varphi)\sinh G$$

- Field at recording medium

$$\hat{a}_3 = \frac{1}{\sqrt{2}}\left[(-\,e^{i\chi} + i)(U\hat{a}_0 + V\hat{b}_0^\dagger) + (ie^{i\chi} - 1)(U\hat{b}_0 + V\hat{a}_0^\dagger)\right]$$

- Two-photon absorption probablility

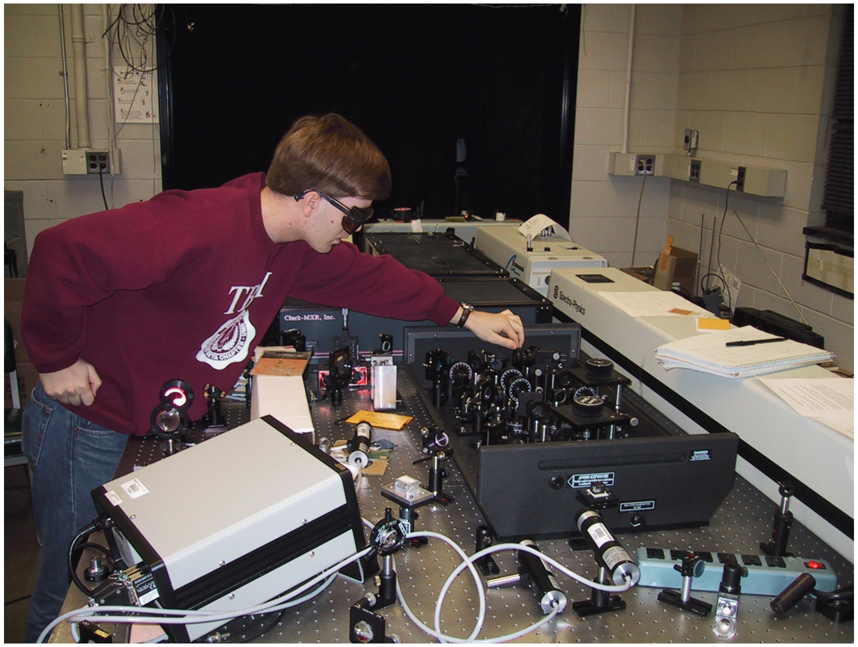
$$\langle 0,0|\hat{a}_3^\dagger\hat{a}_3^\dagger\hat{a}_3\hat{a}_3|0,0\rangle = 4|V|^2\left[|U|^2\cos^2\chi + 2|V|^2\right]$$

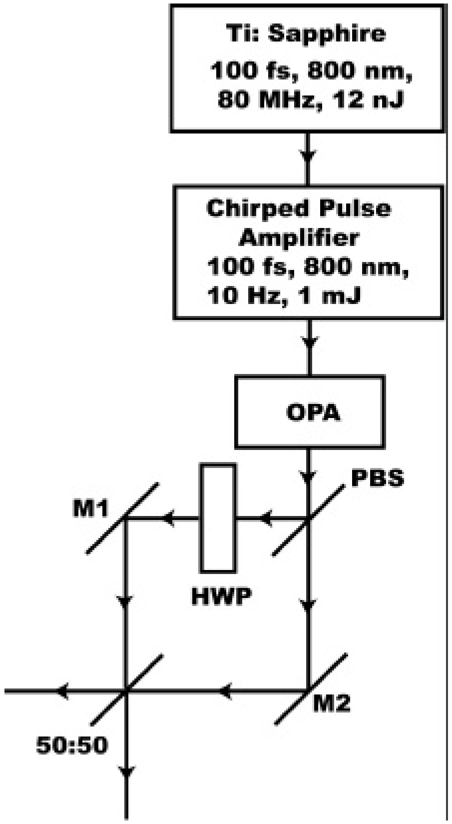



(Phys. Rev. Lett. 86, 1389, 2001)

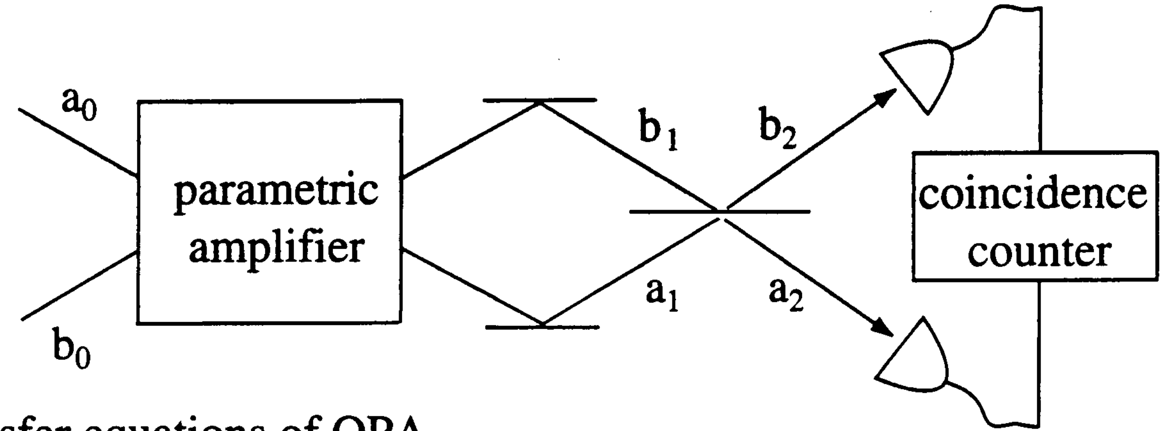
# QUANTUM LITHOGRAPHY PROPOSAL

## Experimental Layout





# Hong-Ou-Mandel Interferometer




- Transfer equations of OPA

$$\hat{a}_1 = U\hat{a}_0 + V\hat{b}_0^\dagger, \quad \hat{b}_1 = U\hat{b}_0 + V\hat{a}_0^\dagger$$

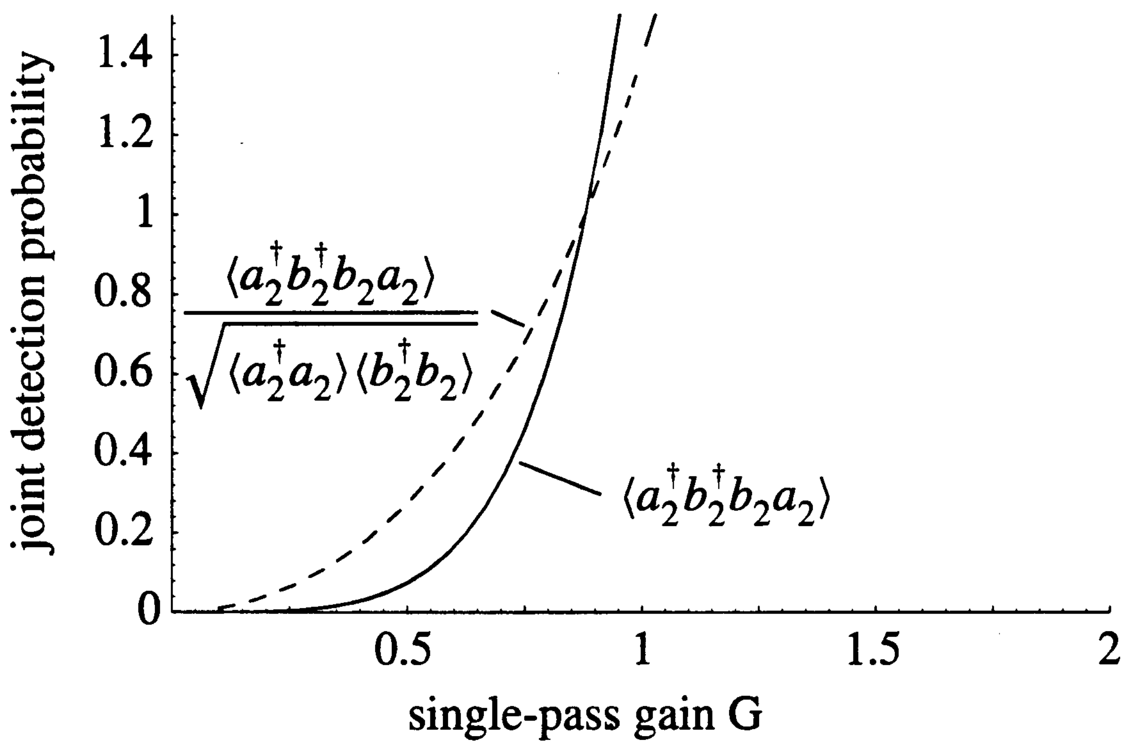
where

$$U = \cosh G \qquad V = -\, i\exp(i\varphi)\sinh G$$

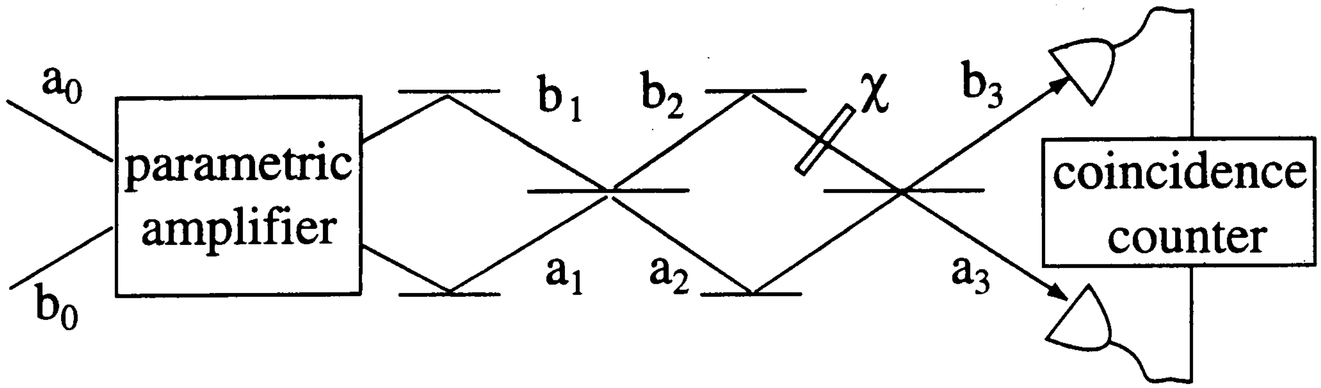
- Fields leaving the beamsplitter

$$\hat{a}_2 = \frac{1}{\sqrt{2}}[(U\hat{a}_0 + V\hat{b}_0^\dagger) - i(U\hat{b}_0 + V\hat{a}_0^\dagger)]$$

$$\hat{b}_2 = \frac{1}{\sqrt{2}}[-i(U\hat{a}_0 + V\hat{b}_0^\dagger) + (U\hat{b}_0 + V\hat{a}_0^\dagger)]$$



# Mach-Zehnder Coincidence-Count Statistics




- Transfer equations of OPA

$$\hat{a}_1 = U\hat{a}_0 + V\hat{b}_0^\dagger, \quad \hat{b}_1 = U\hat{b}_0 + V\hat{a}_0^\dagger$$

where

$$U = \cosh G \qquad V = -\,i\exp(i\varphi)\sinh G$$

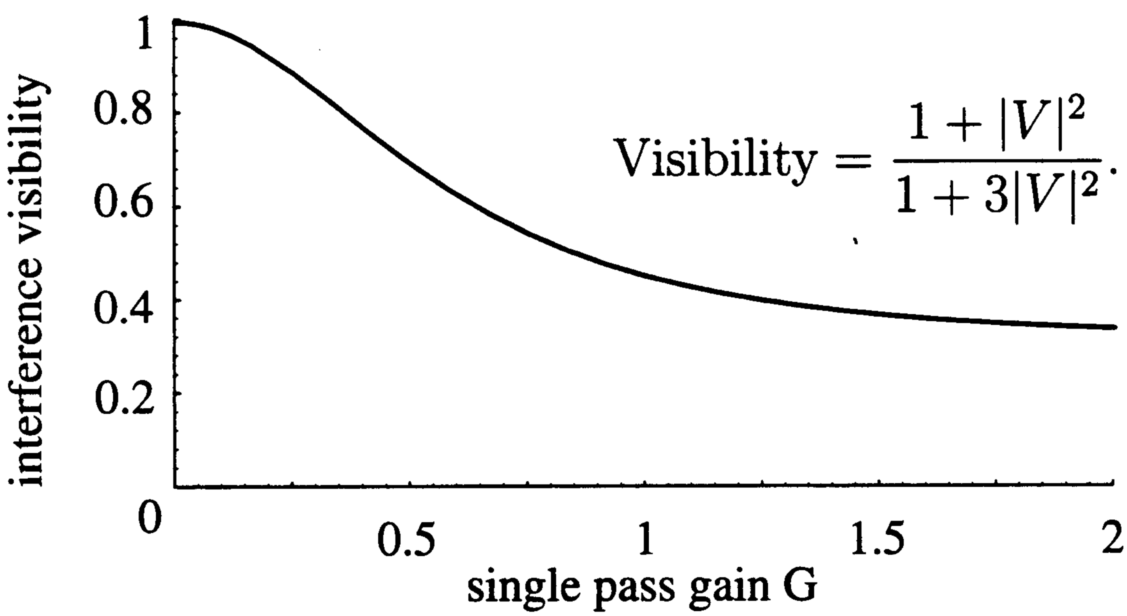
- Fields at detectors

$$\hat{a}_3 = \tfrac{1}{2}[(1-e^{i\chi})(U\hat{a}_0 + V\hat{b}_0^\dagger) - i(1+e^{i\chi})(U\hat{b}_0 + V\hat{a}_0^\dagger)]$$

$$\hat{b}_3 = \tfrac{1}{2}[-i(1+e^{i\chi})(U\hat{a}_0 + V\hat{b}_0^\dagger) - (1-e^{i\chi})(U\hat{b}_0 + V\hat{a}_0^\dagger)]$$

- Joint detection probability

$$\langle \hat{a}_3^\dagger \hat{b}_3^\dagger \hat{b}_3 \hat{a}_3 \rangle = |V|^2 \left[\tfrac{1}{2}(1+\cos 2\chi) + |V|^2(\tfrac{3}{2} + \tfrac{1}{2}\cos 2\chi)\right]$$



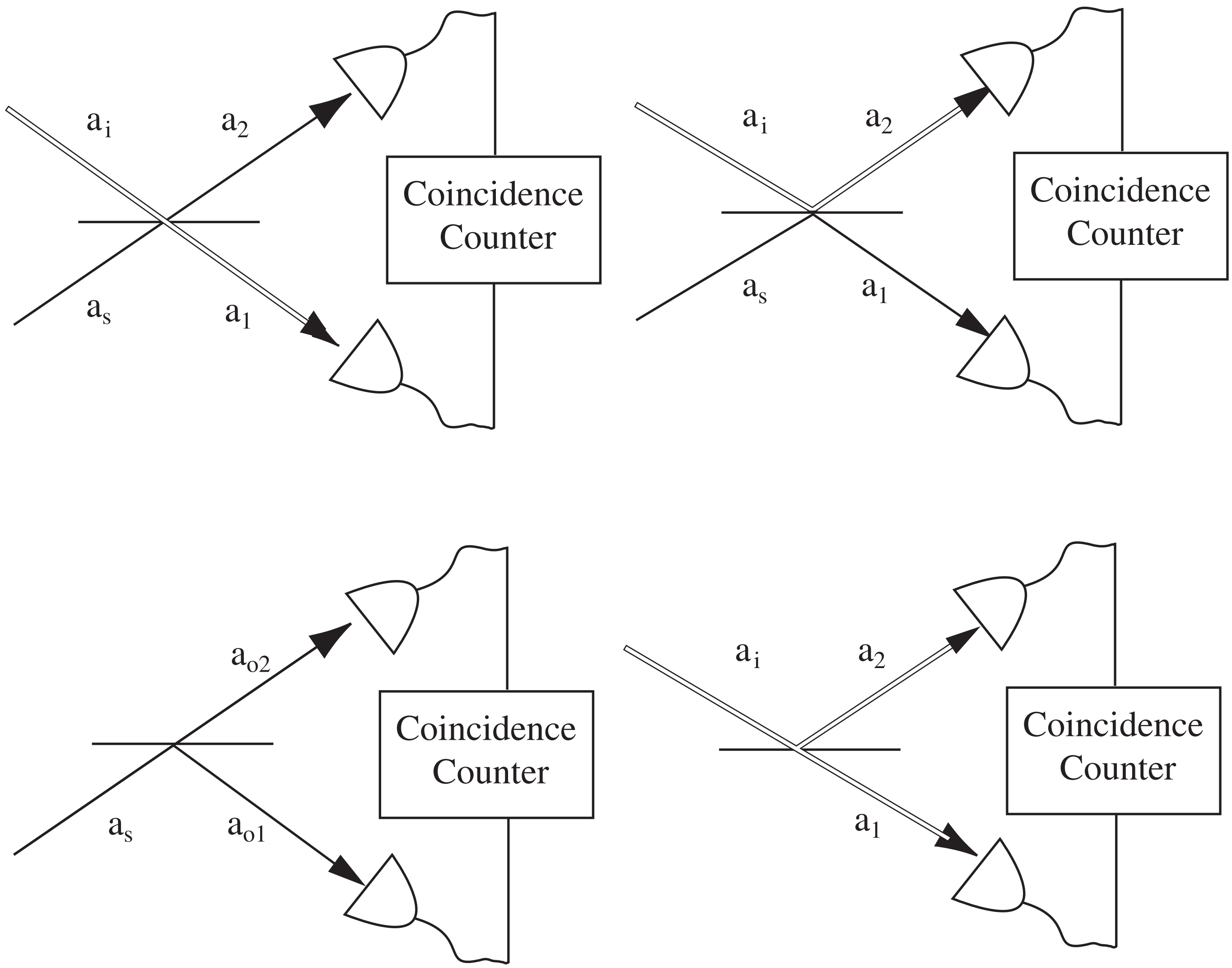

Ou, Zou, Wang, Mandel, Phys. Rev. A 42, 2957, 2000.

Conclusion: Some but not all of the quantum statistical features of the spontaneous parametric down conversion are preserved in the output of an unseeded, high-gain optical parametric amplifier.*

But why?

*Nagasako, Bentley, Boyd, and Agarwal, accepted for publication in PRA

# Processes Contributing to the Coincidence Count Rate




(And interference among these processes!)

# General Treatment of Nonclassical Interferometers

## Input/Output Relation of Interferometer

$$\begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \hat{a}_s \\ \hat{a}_i \end{pmatrix}$$

Direct Output $\quad A = D = 1 \quad B = C = 0.$

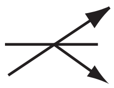
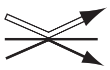
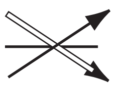
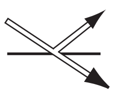
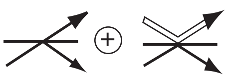
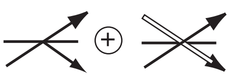
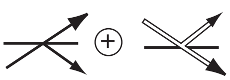
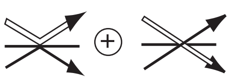
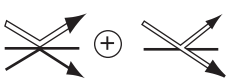
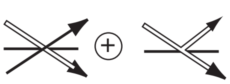
Beam Splitter $\quad A = D = \frac{1}{\sqrt{2}} \quad B = C = \frac{-i}{\sqrt{2}}$

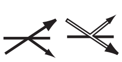
Quantum Litho. $\quad A = C = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}e^{i\chi} \quad B = D = -\frac{i}{\sqrt{2}} + \frac{1}{\sqrt{2}}e^{i\chi}$

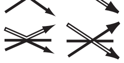
## Coincidence Detection Rate

$$\begin{aligned}
\langle \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_2 \hat{a}_1 \rangle = \; & |C|^2|A|^2 \langle \hat{a}_s^\dagger \hat{a}_s^\dagger \hat{a}_s \hat{a}_s \rangle \\
& + |D|^2|A|^2 \langle \hat{a}_s^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_s \rangle \\
& + |C|^2|B|^2 \langle \hat{a}_i^\dagger \hat{a}_s^\dagger \hat{a}_s \hat{a}_i \rangle \\
& + |D|^2|B|^2 \langle \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i \rangle \\
& + 2 \text{ Re } A^* C^* D A \langle \hat{a}_s^\dagger \hat{a}_s^\dagger \hat{a}_i \hat{a}_s \rangle \\
& + 2 \text{ Re } A^* C^* C B \langle \hat{a}_s^\dagger \hat{a}_s^\dagger \hat{a}_s \hat{a}_i \rangle \\
& + 2 \text{ Re } A^* C^* D B \langle \hat{a}_s^\dagger \hat{a}_s^\dagger \hat{a}_i \hat{a}_i \rangle \\
& + 2 \text{ Re } A^* D^* C B \langle \hat{a}_s^\dagger \hat{a}_i^\dagger \hat{a}_s \hat{a}_i \rangle \\
& + 2 \text{ Re } A^* D^* D B \langle \hat{a}_s^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i \rangle \\
& + 2 \text{ Re } B^* C^* D B \langle \hat{a}_i^\dagger \hat{a}_s^\dagger \hat{a}_i \hat{a}_i \rangle
\end{aligned}$$

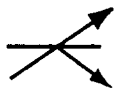
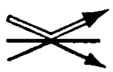
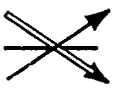
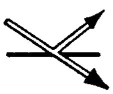
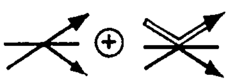
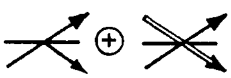
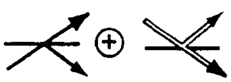
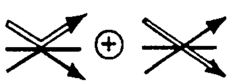
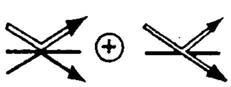
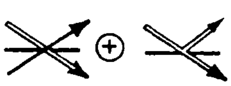
# Interferometer-Dependent Coefficients of the Individual Contributions to the Joint Detection Probability

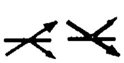
| | | OP | A | HOMI | QL |
|---|---|---|---|---|---|
|  | $\|C\|^2\|A\|^2$ | 0 | | 1/4 | (1+sin )$^2$ |
|  | $\|D\|^2\|A\|^2$ | 1 | | 1/4 | 1-sin $^2$ |
|  | $\|C\|^2\|B\|^2$ | 0 | | 1/4 | 1-sin $^2$ |
|  | $\|D\|^2\|B\|^2$ | 0 | | 1/4 | (1-sin )$^2$ |
|  | 2 Re $A^*C^*DA$ | 0 | | 0 | 2 cos (1+sin ) |
|  | 2 Re $A^*C^*CB$ | 0 | | 0 | 2 cos (1+sin ) |
|  | 2 Re $A^*C^*DB$ | 0 | | 1/2 | 2 cos $^2$ |
|  | 2 Re $A^*D^*CB$ | 0 | | -1/2 | 2 (1-sin $^2$ ) |
|  | 2 Re $A^*D^*DB$ | 0 | | 0 | 2 cos (1-sin ) |
|  | 2 Re $B^*C^*DB$ | 0 | | 0 | 2 cos (1-sin ) |

 single-input terms = both detected photons arise from a single input arm

 dual-input terms = detected photons arise from both input arms

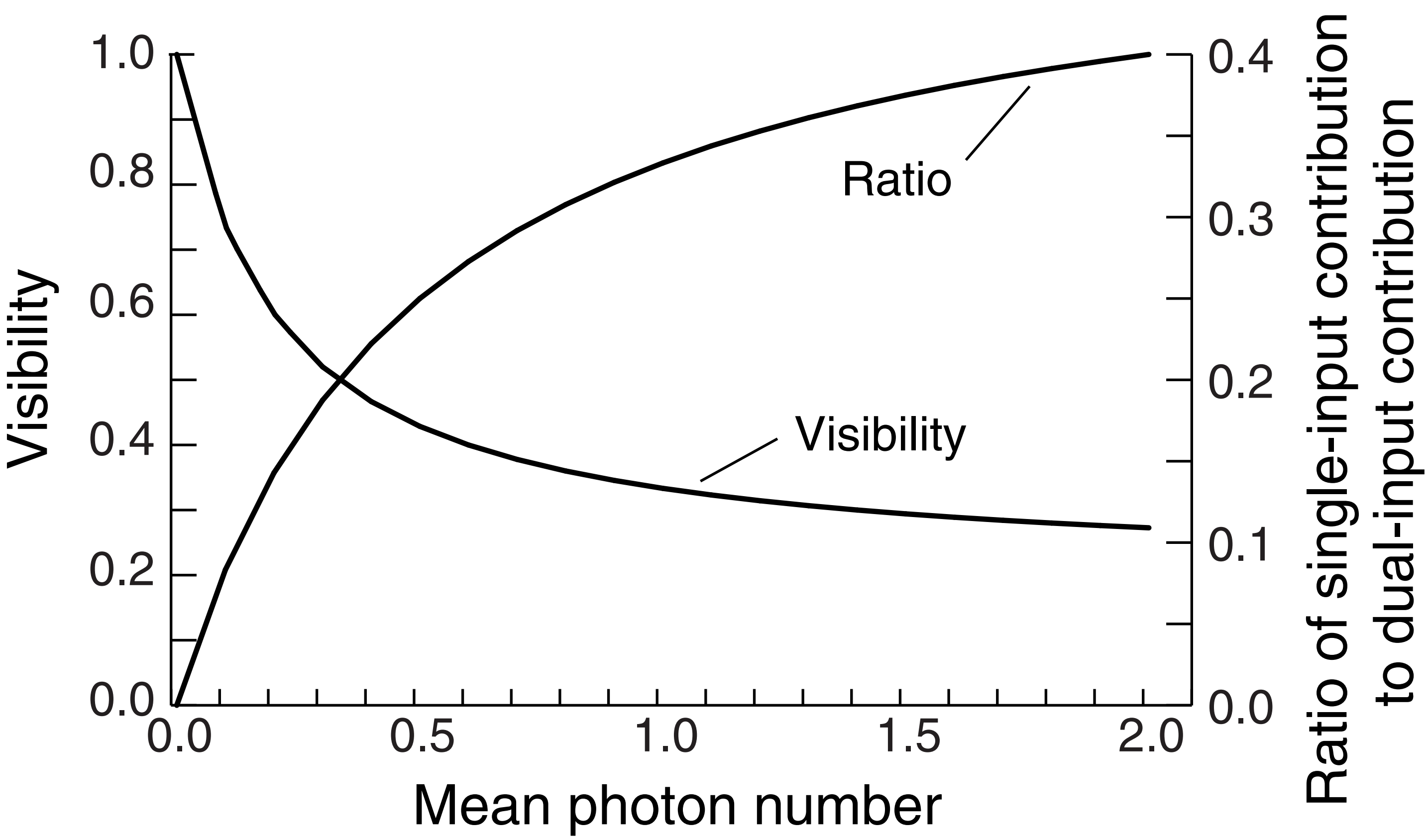
# Quantum Expectation Values of the Individual Contributions to the Joint Detection Probability

| | | $|1\,1\rangle$ | $|m\,m\rangle$ | $|\alpha_0\,\alpha_0\rangle$ | OPA |
|---|---|---|---|---|---|
|  | $\langle a_s^\dagger a_s^\dagger a_s a_s\rangle$ | 0 | m(m-1) | $|\alpha_0|^4$ | $2(\bar{m})^2$ |
|  | $\langle a_s^\dagger a_i^\dagger a_i a_s\rangle$ | 1 | $m^2$ | $|\alpha_0|^4$ | $2(\bar{m})^2 + \bar{m}$ |
|  | $\langle a_i^\dagger a_s^\dagger a_s a_i\rangle$ | 1 | $m^2$ | $|\alpha_0|^4$ | $2(\bar{m})^2 + \bar{m}$ |
|  | $\langle a_i^\dagger a_i^\dagger a_i a_i\rangle$ | 0 | m(m-1) | $|\alpha_0|^4$ | $2(\bar{m})^2$ |
|  | $\langle a_s^\dagger a_s^\dagger a_i a_s\rangle$ | 0 | 0 | $|\alpha_0|^4$ | 0 |
|  | $\langle a_s^\dagger a_s^\dagger a_s a_i\rangle$ | 0 | 0 | $|\alpha_0|^4$ | 0 |
|  | $\langle a_s^\dagger a_s^\dagger a_i a_i\rangle$ | 0 | 0 | $|\alpha_0|^4$ | 0 |
|  | $\langle a_s^\dagger a_i^\dagger a_s a_i\rangle$ | 1 | $m^2$ | $|\alpha_0|^4$ | $2(\bar{m})^2 + \bar{m}$ |
|  | $\langle a_s^\dagger a_i^\dagger a_i a_i\rangle$ | 0 | 0 | $|\alpha_0|^4$ | 0 |
|  | $\langle a_i^\dagger a_s^\dagger a_i a_i\rangle$ | 0 | 0 | $|\alpha_0|^4$ | 0 |

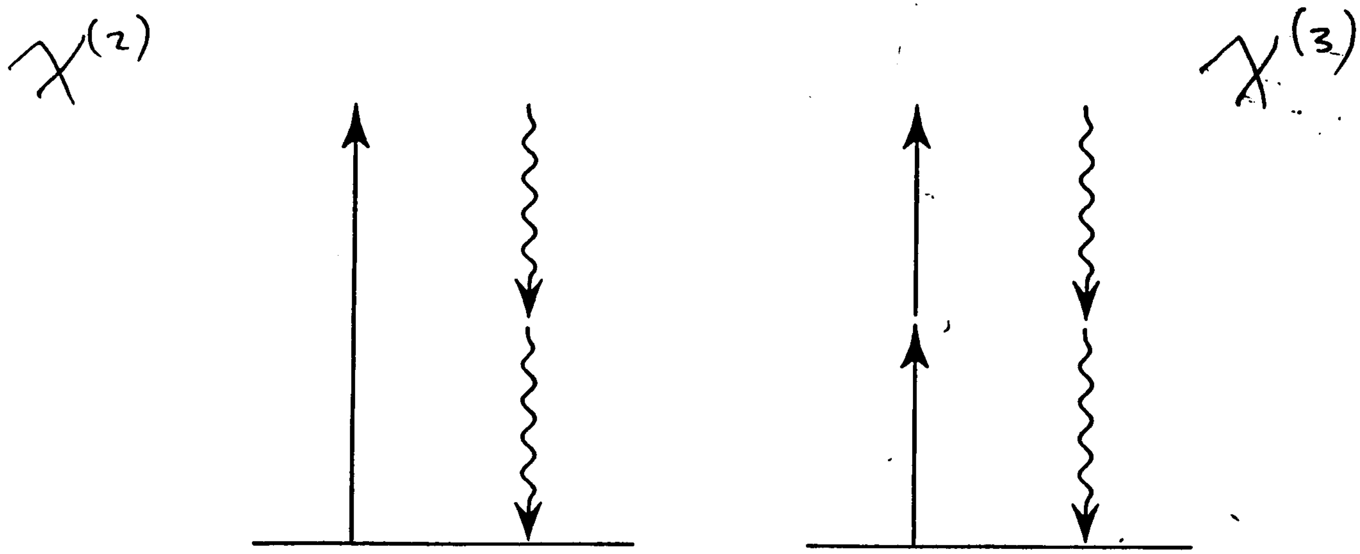
 single-input terms = both detected photons arise from a single input arm

 dual-input terms = detected photons arise from both input arms

# Nature of Decreased Fringe Visibility in a High-Gain Optical Parametric Amplifier



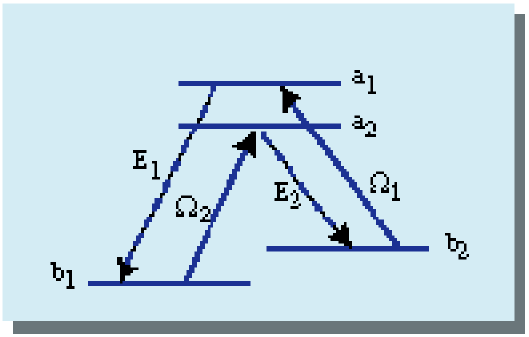
# Two Routes to Entanglement

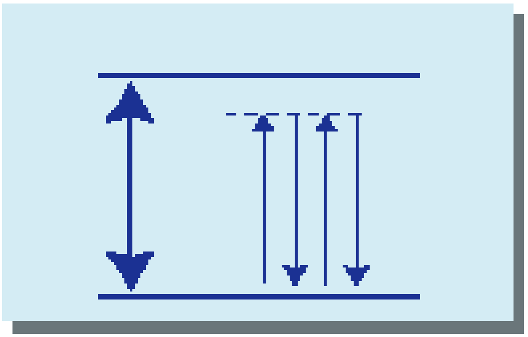


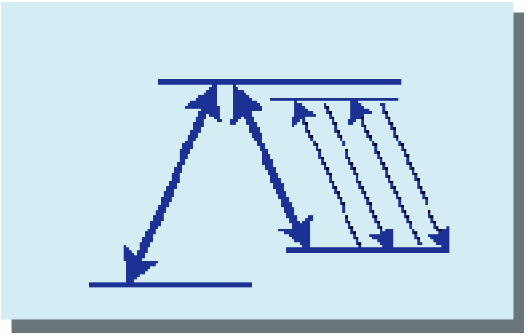
# Generation of Squeezed Light by use of EIT

Robert W. Boyd and C. R. Stroud, Jr., University of Rochester

## Three Approaches



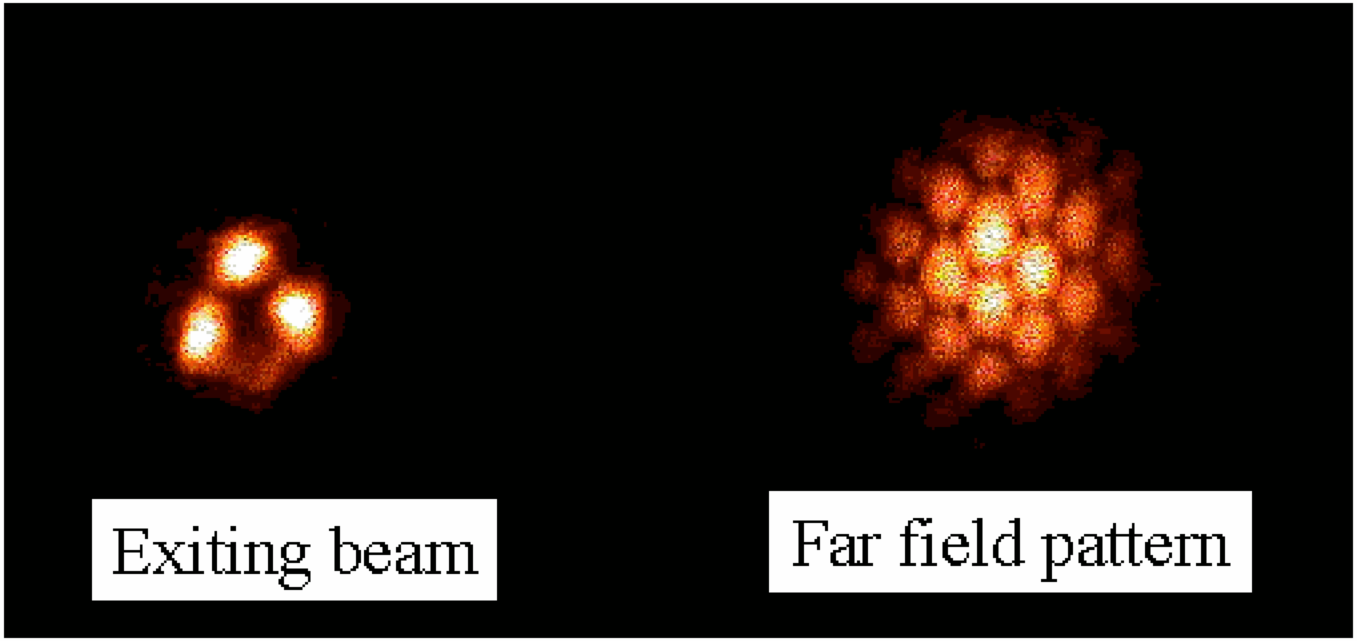


**Fundamental idea:** EIT eliminates linear absorption so that there is no spontaneous emission background noise.

# Honey Comb Pattern Formation

Robert W. Boyd and C. R. Stroud, Jr., University of Rochester

Output from cell with single gaussian beam input




Quantum image?

Input power 150 mW
Input beam diameter 0.22 mm
$\lambda = 588.995$ nm

Sodium vapor cell
T = 220° C

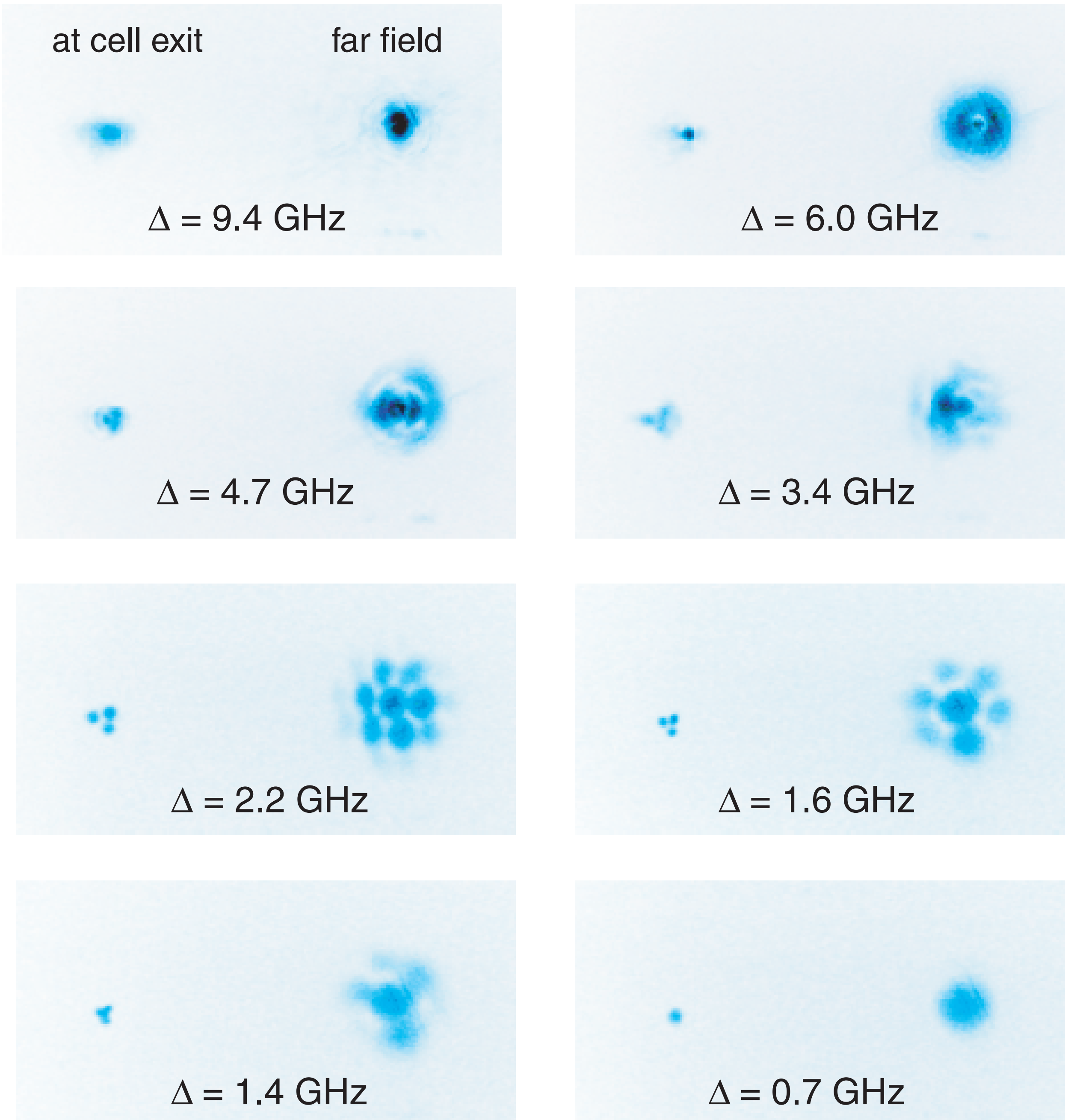
# Honeycomb Pattern Formation by Laser-Beam Filamentation in Atomic Sodium Vapor

Ryan S. Bennink
Vincent Wong
David L. Aronstein
Robert W. Boyd
Svetlana G. Lukishova
Alberto M. Marino
Carlos R. Stroud, Jr.

*Institute of Optics, University of Rochester*

# Experimental Results

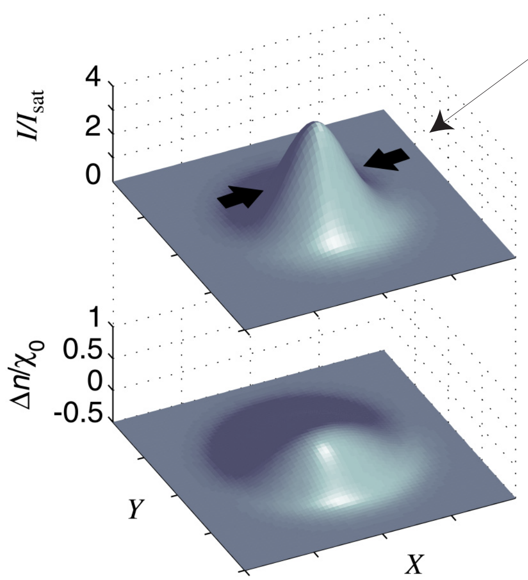
## Frequency dependence




$N = 3 \times 10^{12}\ \mathrm{cm}^{-3}$, $P = 110$ mW, $2w = 180\ \mu\mathrm{m}$

# *Spontaneous Pattern Formation in Sodium Vapor*

A sodium vapor may be thought of as a medium composed of two-level atoms. Light whose frequency is near the atomic transition frequency experiences a refractive index $n$ which depends strongly on the intensity $I$:



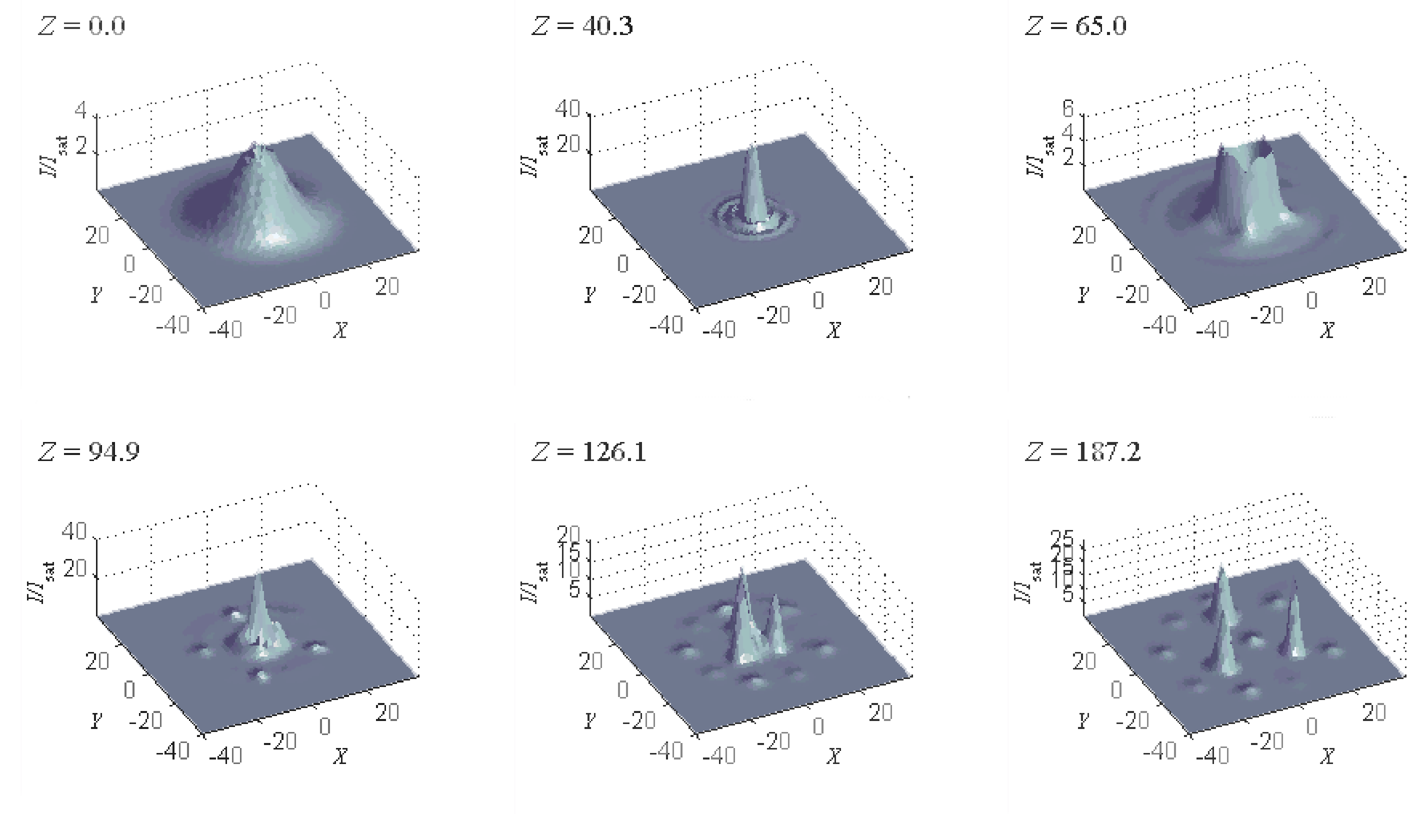

$$n = 1 + \quad n$$

$$1 + \frac{1}{2}\,\frac{0}{1 + I/I_{sat}}$$

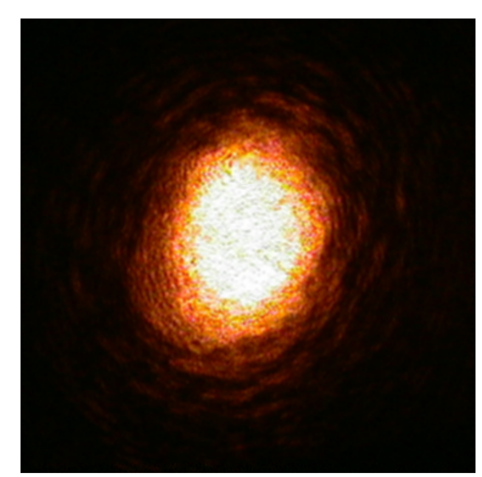
Since light refracts in the direction of increasing index, in a medium with negative saturable nonlinearity it refracts toward regions of higher intensity. This causes smooth beams to narrow or self-focus. But it also tends to destabilize a beam as small amplitude fluctuations grow due to local self-focusing. Thus beams with even small amplitude noise can spontaneously split into two or more separate beams.

**For sodium at 200°C, $_0$ -0.05 and $I_{sat}$ 6 mW/cm$^2$*

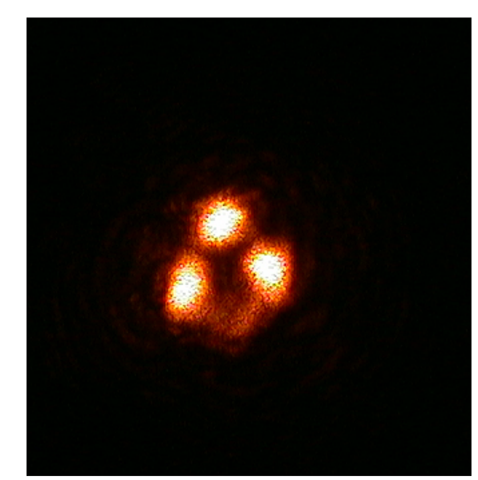
A simulation of spontaneous break-up into 3 stable beams:



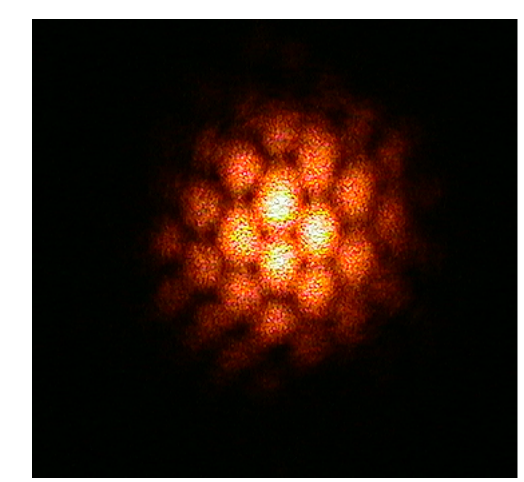

Experimental observation of spontaneous break-up resulting in a striking far-field pattern:



beam entering sodium



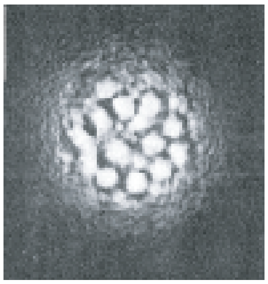
beam leaving sodium



far-field pattern

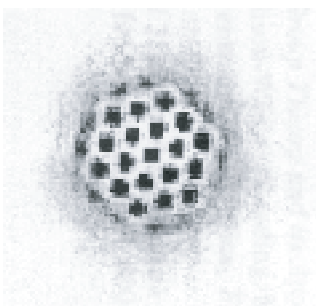
*Pictures taken by R. Bennink, S. Lukishova, and V. Wong.*

# *Some Related Findings*



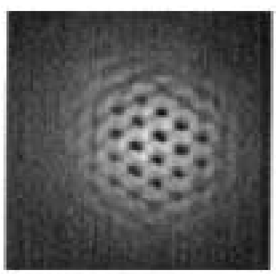
- spontaneous pattern formation in nematic LC with mirror feedback

R. MacDonald and H.J. Eichler, Opt. Comm. **89** (1992) 289-295.



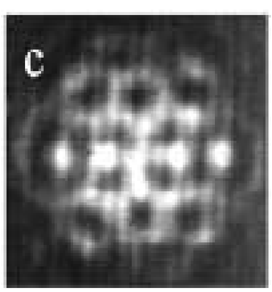
- simulation of pattern formation in a Kerr slice with mirror feedback

F. Papoff, G. D'Alessandro, G.-L. Oppo, and W.J. Firth, Phys. Rev. A **48** (1993) 634.



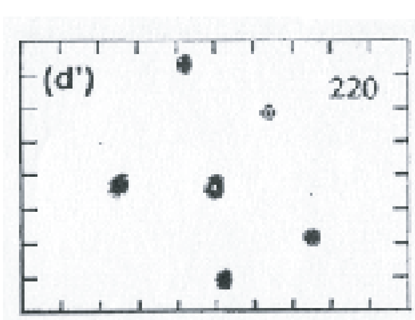
- spontaneous pattern formation in sodium vapor with a feedback mirror

R. Herrero, E. Grosse Westhoff, A. Aumann, T. Ackemann, Y. A. Logvin, and W. Lange, Phys. Rev. Lett. **82** (1999) 4627.




- spontaneous pattern formation in a near-degenerate OPO

M. Vaupel, A. Maitre, and C. Fabre, Phys. Rev. Lett. **83** (1999) 5278.




- filementation of an aberrated beam in sodium vapor

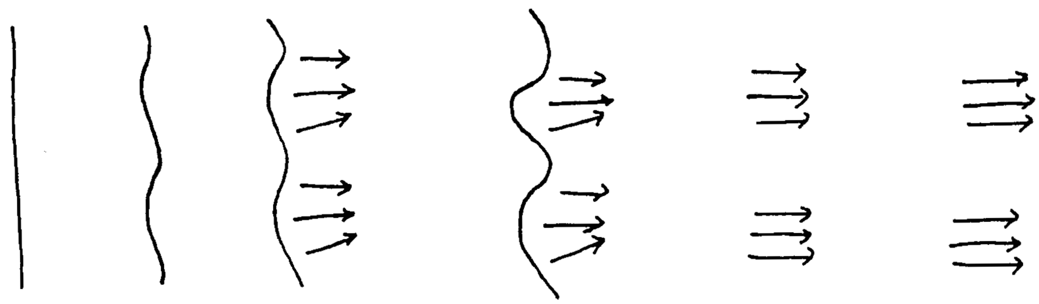
J.W. Grantham, H.M. Gibbs, G. Khitrova, J.F. Valley, and Xu Jiajin, Phys Rev. Lett. **66** (1991) 1422.

# Some Underlying Issues in Nonlinear Optics

- Self-Assembly/Self-Organization in Nonlinear Systems
- Stability vs. Instability (and Chaos) in Nonlinear Systems

# Laser Beam Filamentation

Spatial growth of wavefront perturbations



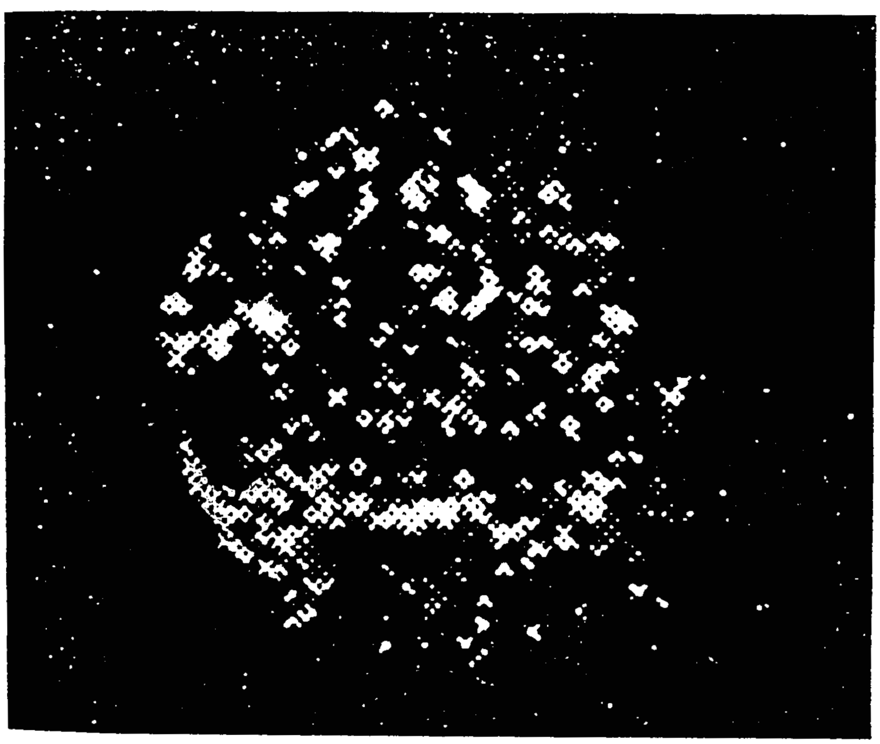
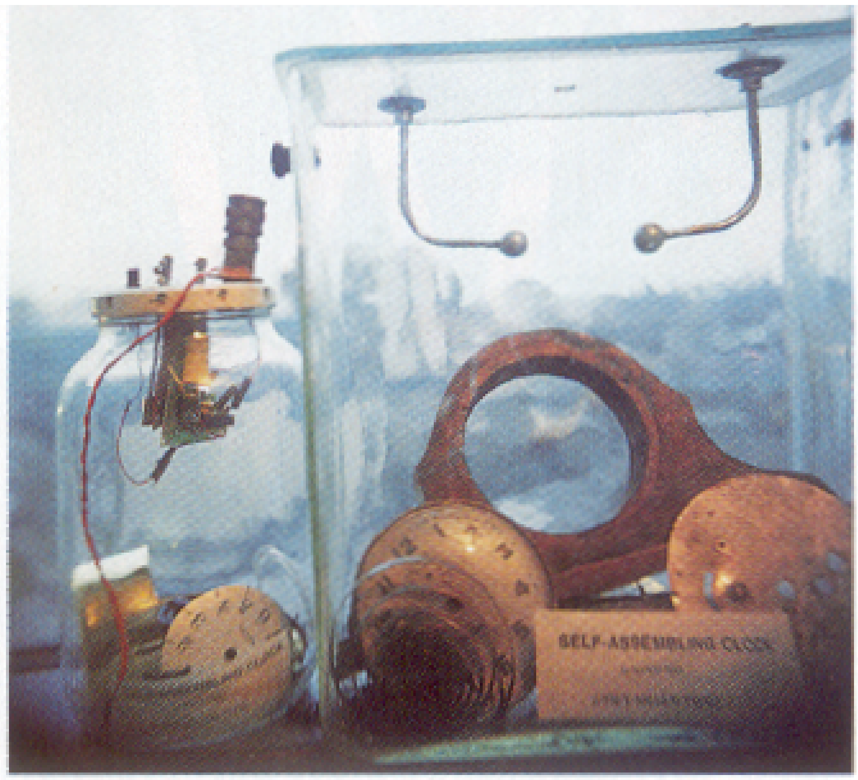


Fig. 17.2 Image of small-scale filaments at the exit windows of a $CS_2$ cell created by self-focusing of a multimode laser beam. [After S. C. Abbi and H. Mahr, *Phys. Rev. Lett.* 26, 604 (1971).]

# Experiment in Self Assembly




Joe Davis, MIT