Ramp Input Response of *RC* Tree Networks

Eby G. Friedman
Department of Electrical Engineering
University of Rochester
Rochester, New York 14627

and

J. H. Mulligan, Jr.*
Department of Electrical and Computer Engineering
University of California
Irvine, California 92717

Abstract - Closed form expressions are presented to accurately describe the delay characteristics of *RC* tree networks. The Penfield-Rubinstein-Horowitz approach to estimating the step function response of *RC* trees has been extended to consider ramp inputs. This improves timing accuracy by considering the shape of the input waveform driving each individual interconnect tree while maintaining computational simplicity for use in the automated timing analysis of complex VLSI circuits.

I. INTRODUCTION

The *RC* tree analysis approach originally presented by Penfield and Rubinstein [1,2] has been applied to structures of the form shown in Figure 1 in which a section of a uniform resistance-capacitance (URC) line of total series resistance R_o and total shunt capacitance C_o can be represented by an unbalanced π network of m series resistors of value R_o/m and m+1 shunt capacitors of value $C_o/m+1$ connected to ground from the input and output nodes of the section of line and the junctions between the series resistors. The Penfield-Rubinstein analysis as originally reported and subsequently applied describes results for step function input signals, although it was noted explicitly that one might apply convolution to deal with other inputs. In this paper, an equation has been derived for the approximating function used in the *RC* tree analysis on the assumption of a ramp input, and upper and lower bounds corresponding to the same input waveform are presented. The ability to deal with ramp input signals of varying slope is viewed as a more realistic approach to practical problems of interest rather than limiting consideration to a step function excitation.

In section II, the original *RC* tree analysis and its extension to deal with ramp input signals are considered. The principal results are presented in tabular form both for convenience and to save space. The calculations involved are illustrated in section III by a specific example having the form of Figure 1. Some concluding remarks are made in section IV.

II. TIME RESPONSES IN THE RC TREE NETWORK

A convenient basis for outlining the problem to be analyzed is provided in Figure 1. The input signal is assumed to be a unit ramp function, normalized to V_{DD}, which begins at $t = 0$. After modification by passage through a section of distributed *RC* interconnect, the signal appears at a node along the tree. From the Penfield-Rubinstein approach, the signal may be written as a simple exponential change from 0 to 1 volts, starting at $t = 0$. For convenience in determining the signal at its output (leaf node), the input signal is replaced by an equivalent ramp defined in terms of a ramp slope of k_i volts per second and a time at which the equivalent ramp waveform begins, assumed initially to be at t = 0. The objective of the analysis is to determine the time after t = 0 at which the waveform at a particular output node reaches the level of 0.50 normalized volts. Thus, the desired solution is obtained by computing the time response at the output of a section of interconnect represented by an *RC* network whose input is a normalized ramp function changing from 0 to 1.0 volts.

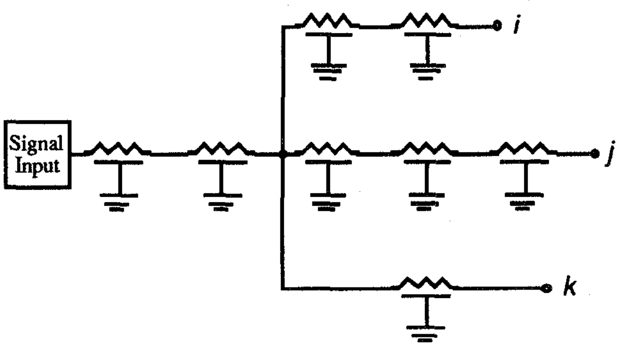

Figure 1: Distributed *RC* tree network

Calculations of the response of the sections of interconnect are made based on the work of Penfield, Rubinstein, and Horowitz (PRH) [1,2], Horowitz [3], and Wyatt and collaborators [4-9]. For a unit step function input, the work reported shows that one can approximate the time response at a node *i* in a tree of RC interconnect as [8]

$$v_i(t) = 1 - e^{\frac{-t}{T_{Di}}} . \tag{1}$$

Here T_{Di} is given by [2]

$$T_{Di} = \sum_k R_{ki} C_k \ . \tag{2}$$

R_{ki} is defined as the resistance of that portion of the unique path between the signal source and node *i* that is common with the unique path between the signal source and node *k*. C_k is the lumped capacitance between node *k* and ground. Furthermore, in the references cited, upper and lower bounds on the time

This research was supported in part by the National Science Foundation under Grant No. MIP-9208165 and Grant No. MIP-9423886, the Army Research Office under Grant No. DAAH04-93-G-0323, and by a grant from the Xerox Corporation.

* recently deceased

response (for a step function input) were developed and are given as (3) through (7) in Table 1.

The quantities T_{Ri} [2] and T_{Pi} [9] which appear in the equations in Table 1 are defined as

$$T_{Ri} = \sum_k \frac{R_{ki}^2}{R_{ii}} C_k \quad , \tag{8}$$

$$T_{Pi} = \max\left\{T_{Di}, \max_{\substack{\text{all}\\ \text{leaf}\\ \text{nodes } l}} \sum_k \frac{R_{ki}R_{kl}}{R_{il}} C_k \right\} \quad , \tag{9}$$

where R_{ii} (R_{kk}) is the resistance along the unique path between the signal source and node i (node k). A leaf node is that node which represents a final destination of the RC tree; in the resistance tree it is a node that has only one resistance connected to it.

Wyatt and Standley [9] have shown that replacing T_P in the original PRH bounds by T_{Pi} gives tighter bounds on the time response. They also noted that $v_i(t)$ perforce lies between the upper and lower bounds. Due to this last fact, it is important to calculate the upper and lower bounds to determine how well (1) approximates the desired response.

By convolving the step function response $v_i(t)$ with the derivative of a ramp input of slope k_i starting at $t = 0$ and ending at $t_1 = 1/k_i$, the expression for the approximating function $y_i(t)$ to the response produced at node i by a ramp input signal can be obtained. The result is

$$y_i(t) = \frac{t}{t_1} - \frac{T_{Di}}{t_1}\left(1 - e^{\frac{-t}{T_{Di}}}\right) \quad \text{for } t \le t_1 . \tag{10}$$

$$y_i(t) = 1 - \frac{T_{Di}}{t_1}\left(e^{\frac{t_1}{T_{Di}}} - 1\right) e^{\frac{-t}{T_{Di}}} \quad \text{for } t \ge t_1 \quad . \tag{11}$$

Before proceeding further, however, it is of interest to note that the same convolution process applied to (1) to produce (10) and (11) for the ramp input can also be used on the upper and lower bounds to the step function response. The results are corresponding upper and lower bounds for the response to a ramp starting at $t = 0$ with slope defined by $k_i t_1 = 1$. These results are given as (12) through (21) in Table 1.

The derivative of the approximating function dy_i/dt evaluated at the value of time for which $y_i(t) = 0.50$ is the slope which can be used in the representation of the waveform shape of the signal at the output of the interconnect by an equivalent input ramp of slope k_i. The slope relationships are given below:

$$\frac{d\hat{y}_i(t)}{dt} = k_o = k_i\left(1 - e^{\frac{-t}{T_{Di}}}\right) \quad for\ t \le t_1 , \tag{22}$$

$$\frac{d\hat{y}_i(t)}{dt} = k_o = k_i\left(1 - e^{\frac{-t_1}{T_{Di}}}\right) e^{\frac{-t_2}{T_{Di}}} \quad for\ t \ge t_1 , \tag{23}$$

where t_2 is the amount of time greater than t_1 (where $t_1 = 1/k_i$).

To determine the value of $t = t_o$ at which $y_i(t) = 0.50$, it is convenient to introduce the normalized time $u = t/T_{Di}$. Then the value of u at which $y_i(t) = 0.50$ is given by the solution of

$$f(u) = u - 1 + e^{-u} \tag{24}$$

with

$$f(u) = \frac{1}{2k_iT_{Di}} \quad . \tag{25}$$

For values of $f(u) > 3.5$, a good approximation to $u = u_o$ which satisfies the equation is

$$u_o \doteq 1 + \frac{1}{2k_iT_{Di}} \tag{26}$$

and

$$t_o \doteq T_{Di}u_o = T_{Di} + \frac{1}{2k_i} = T_{Di} + \frac{t_1}{2} \tag{27}$$

is the value at which $y_i(t) = 0.50$.

When the numerical values do not permit this approximation, an alternative approach is suggested by Figure 2. The abscissa shows values of the normalized time variable $u = t/T_{Di}$. The left ordinate is the normalized variable $f(u) = y_i(t)/k_iT_{Di}$, while the right ordinate is the normalized variable $f'(u) = (1/k_i)(dy_i/dt)$. Thus, for an arbitrary value of normalized value $y_i(t)$ and known values of the slope of the input ramp (k) and interconnect characteristic T_{Di}, the transcendental equation (24) can be solved directly by iteration or by the use of Figure 2.

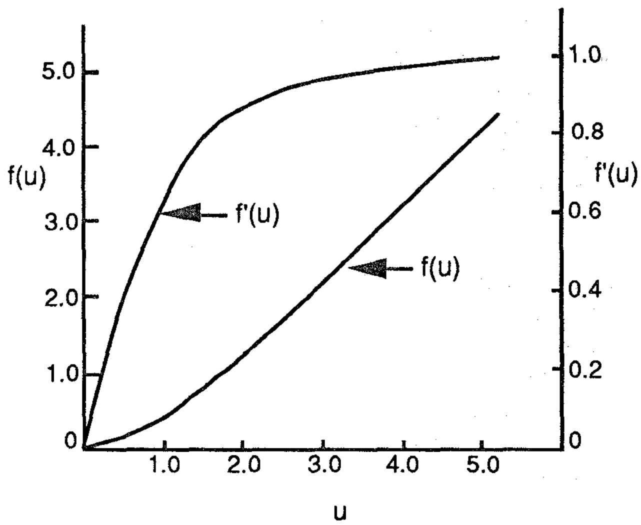

Figure 2: Normalized response of ramp driven interconnect

The slope of the equivalent ramp signal at the output of the section of interconnect k_0 (which might be at a leaf node) is given by $dy_i(t)/dt$ evaluated at $y_i(t) = 0.50$. Thus,

$$k_0 = k_i\left(1 - e^{-u_o}\right) \quad for\ t < t_1, \tag{28}$$

where u_o satisfies (24). This equivalent ramp starts at a time delay from the start of the input ramp to the interconnect of

Applicable Time Range	Step Response Bounds	Ramp Response Bounds	
		$t \le t_1$	$t > t_1$
<u>Lower Bounds</u>			
$t \le T_{Di} - T_{Ri}$	$v_i(t) \ge 0$ (3)	$y_i(t) \ge 0$ (12)	$y_i(t) \ge 0$ (15)
$T_{Di} - T_{Ri} \le t \le T_{Pi} - T_{Ri}$	$v_i(t) \ge 1 - \frac{T_{Di}}{t + T_{Ri}}$ (4)	$y_i(t) \ge \frac{t}{t_1} - \frac{T_{Di}}{t_1} ln \left(\frac{t + T_{Ri}}{T_{Ri}}\right)$ (13)	$y_i(t) \ge 1 - \frac{T_{Di}}{t_1} ln\left(\frac{t + T_{Ri}}{t - t_1 + T_{Ri}}\right)$ (16)
$T_{Pi} - T_{Ri} \le t$	$v_i(t) \ge 1 - \frac{T_{Di}}{T_{Pi}} e^{\left(\frac{T_{Pi} - T_{Ri}}{T_{Pi}}\right)} e^{\frac{-t}{T_{Pi}}}$ (5)	$y_i(t) \ge k_i \left[t - T_{Di} e^{\left(\frac{T_{Pi} - T_{Ri}}{T_{Pi}}\right)} \left(1 - e^{\frac{-t}{T_{Pi}}}\right) \right]$ (14)	$y_i(t) \ge 1 - \frac{T_{Di}}{t_1} e^{\left(\frac{T_{Pi} - T_{Ri}}{T_{Pi}}\right)} \left[e^{\frac{t_1}{T_{Pi}}} - 1 \right] e^{\frac{-t}{T_{Pi}}}$ (17)
<u>Upper Bounds</u>			
$t \le T_{Di} - T_{Ri}$	$v_i(t) \le 1 - \frac{T_{Di} - t}{T_{Pi}}$ (6)	$y_i(t) \le \frac{t_1}{t} \left[1 - \frac{T_{Di} - \frac{t}{2}}{T_{Pi}} \right]$ (18)	$y_i(t) \le 1 - \frac{T_{Di} + \frac{t_1}{2} - t}{T_{Pi}}$ (20)
$t \ge T_{Di} - T_{Ri}$	$v_i(t) \le 1 - \frac{T_{Ri}}{T_{Pi}} e^{\left(\frac{T_{Di} - T_{Ri}}{T_{Ri}}\right)} e^{\frac{-t}{T_{Ri}}}$ (7)	$y_i(t) \le k_i \left[t - \frac{T_{Ri}^2}{T_{Pi}} e^{\left(\frac{T_{Di} - T_{Ri}}{T_{Ri}}\right)} \left[1 - e^{\frac{-t}{T_{Ri}}} \right] \right]$ (19)	$y_i(t) \le 1 - \frac{T_{Ri}^2}{t_1 T_{Pi}} e^{\left(\frac{T_{Di} - T_{Ri}}{T_{Ri}}\right)} \left[e^{\frac{t_1}{T_{Ri}}} - 1 \right] e^{\frac{-t}{T_{Ri}}}$ (21)

Table 1: Time Response of RC Interconnect Tree

amount

$$T_{IC} = u_o T_{Di}\left[1 - \frac{1}{2k_i T_{Di}(1 - e^{-u_o})}\right]. \qquad (29)$$

For values of $f(u)$ given by (24) > 3.5, the last two results become

$$k_o = k_i \qquad (30)$$

$$T_{IC} \doteq T_{Di} \qquad (31)$$

III. ILLUSTRATIVE EXAMPLE

To illustrate the application of these results, delay calculations are presented for the *RC* tree network shown in Figure 3. Nodes *i*, *j*, and *k* represent leaf nodes of the *RC* tree. Other assumed element and parameter values are indicated in Figure 3. The upper and lower bounds and the approximate delay from the signal source to node *i* are plotted in Figure 4. Further, these curves are compared to the exact solution derived from circuit simulation defined at the 50% point.

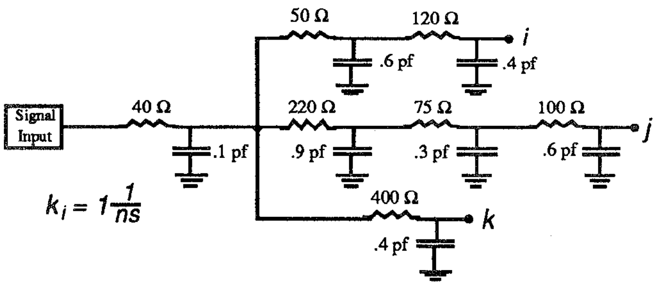

Figure 3: *RC* tree network example

IV. CONCLUSIONS

In this paper, the Penfield, Rubinstein, and Horowitz algorithm (PRH) has been extended to handle ramp as well as step function inputs and utilizes enhancements by Standley and Wyatt which tighten the bounds on the time response. The approximating function $y_i(t)$, the upper and lower bounds, and the output ramp $dy_i(t)/dt$ are developed for an *RC* tree with a ramp input. Thus, this paper provides a more accurate, systematic approach for developing and analyzing *RC* tree networks in high performance applications.

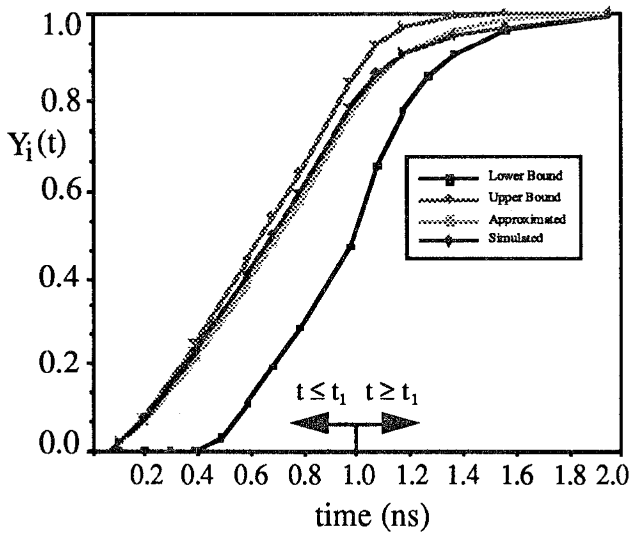

Figure 4: Upper and lower bounds and approximate solution for node *i* as compared to the exact solution derived from circuit simulation

REFERENCES

[1] P. Penfield, Jr. and J. Rubinstein, "Signal Delay in RC Tree Networks," *Proceedings of Caltech Conference on VLSI*, pp. 269-283, January 1981.

[2] J. Rubinstein, P. Penfield, Jr., and M. A. Horowitz, "Signal Delay in RC Tree Networks," *IEEE Transactions on Computer-Aided Design*, Vol. CAD-2, No. 3, pp. 202-211, July 1983.

[3] M. A. Horowitz, "Timing Models of MOS Circuits," Technical Report No. SEL 83-003, Integrated Circuits Laboratory, Stanford University, Stanford, California, December 1983.

[4] J. L. Wyatt, Jr., "Waveform Bounding for Fast Timing Analysis of MOS VLSI Circuits," *Proceedings of IEEE International Symposium on Circuits and Systems*, pp. 760-761, May 1983.

[5] J. L. Wyatt, Jr., and Q. Yu, "Signal Delay in RC Meshes, Trees and Lines," *Proceedings of IEEE International Conference on Computer-Aided Design*, pp. 15-17, November 1984.

[6] Q. Yu, J. L. Wyatt, Jr., C. Zukowski, H-N. Tan, and P. O'Brien, "Improved Bounds on Signal Delay in Linear RC Models for MOS Interconnect," *Proceedings of IEEE International Symposium on Circuits and Systems*, pp. 903-906, June 1985.

[7] J. L. Wyatt, Jr., "Signal Delay in RC Mesh Networks," *IEEE Transactions on Circuits and Systems*, Vol. CAS-32, No. 5, pp. 507-510, May 1985.

[8] J. L. Wyatt, Jr., "Signal Propagation Delay in RC Models for Interconnect," *Circuit Analysis, Simulation and Design, Part II: VLSI Circuit Analysis and Simulation*, A. Ruehli, ed., Vol. 3 in the series Advances in CAD for VLSI, North-Holland, 1987.

[9] D. Standley and J. L. Wyatt, Jr., "Improved Signal Delay Bounds for RC Tree Networks," VLSI Memo No. 86-317, Massachusetts Institute of Technology, Cambridge, Massachusetts, May 1986.