Properties of On-Chip Inductive Current Loops

Andrey V. Mezhiba and Eby G. Friedman

Department of Electrical and Computer Engineering
University of Rochester
Rochester, New York 14627
Email: mav@ece.rochester.edu, friedman@ece.rochester.edu *

Abstract

The variation of inductance with circuit length is investigated in this paper. The nonlinear variation of inductance with length is shown to be a result of inductive coupling among circuit segments. If the distance between the forward and return current paths of a current loop is much smaller than the loop length, the inductive coupling to the forward current is similar to the coupling to the return current, resulting in negligible coupling. The inductance of these circuits therefore varies approximately linearly with length. Similarly, the effective inductive coupling between two parallel current loops is reduced through cancellation and has a negligible effect on the net inductance of a circuit. As a general rule, the inductance of circuits where the distance between the forward and return current is much smaller than the characteristic dimensions of the circuit scales linearly with circuit dimensions. This linear behavior can be used to simplify the inductance extraction and circuit analysis process.

Categories and Subject Descriptors

B.7.m [**Integrated Circuits**]: Miscellaneous—*on-chip inductance, partial inductance, inductive coupling*

General Terms

Design

Keywords

inductance, inductive coupling, loop inductance

*This research was supported in part by the Semiconductor Research Corporation under Contract No. 99–TJ–687, the DARPA/ITO under AFRL Contract F29601–00–K–0182, grants from the New York State Office of Science, Technology & Academic Research to the Center for Advanced Technology—Electronic Imaging Systems and to the Microelectronics Design Center, and by grants from Xerox Corporation, IBM Corporation, Intel Corporation, Lucent Technologies Corporation, and Eastman Kodak Company.

GLSVLSI'02 April 18-19, 2002, New York, New York, USA.

1. Introduction

IC performance has become increasingly constrained by on-chip interconnect impedances. Determining the electrical characteristics of the interconnect at early stages of the design process has therefore become necessary. The layout process is driven now by interconnect performance where the electrical characteristics of the interconnect are initially estimated and later refined. Impedance estimation is repeated throughout the circuit design and layout process and should, therefore, be computationally efficient.

The inductance of on-chip interconnect has become an important issue due to the increasing switching speeds of digital integrated circuits [1]. The inductive properties must, therefore, be effectively incorporated at all levels of the design, extraction, and simulation phases in the development of high speed ICs.

Traditionally, inductance is defined as a coefficient relating a magnetic flux through a complete current loop to the loop current. Information characterizing the current flow paths, *i.e.*, the size, shape, and mutual arrangement of the current loops, is required to determine the loop inductance. In integrated circuits, however, where the interconnects are tightly coupled, the current return path is typically not known prior to the circuit analysis. Therefore, an accurate analysis based on the loop inductance has proven difficult to achieve.

Operating an inductance extractor within a layout generator loop is computationally expensive. The efficiency of inductance estimation can be improved by extrapolating the inductive properties of a circuit from the properties of a precharacterized set of structures. Extrapolating inductance, however, is not straightforward as the inductance, in general, varies nonlinearly with the circuit dimensions. The partial inductance of a straight line, for example, is a nonlinear function of the line length. The inductive coupling of two lines decreases slowly with increasing line-to-line separation. The partial inductance representation of a circuit consists of strongly coupled elements with a nonlinear dependence on the geometric dimensions. The inductive properties of a circuit, therefore, do not, in general, vary linearly with the circuit geometric dimensions.

The objective of this work is to explore the dependence of inductance on the circuit dimensions, to provide insight from a circuit analysis perspective, and to determine the specific conditions under which the inductance properties scale linearly with the circuit dimensions with the objective of enhancing the layout extraction process. A comparison of the properties of the partial inductance with the properties of the loop inductance is shown to be effective for this purpose.

The paper is organized as follows. A brief overview of the concept of partial inductance is presented in Section 2. The dependence of inductance on the line length is discussed in Section 3. The inductive coupling of parallel conductors is described in Section 4. Application of the results in circuit analysis is discussed in Section 5. The conclusions are summarized in Section 6.

2. Background

The construct of a partial self and mutual inductance is useful in evaluating the inductive properties of integrated circuits. The concept of a partial inductance was first developed by Rosa in 1908 in application to linear conductors [2]. The need for a partial inductance arose because single linear conductors do not form closed loop circuits in which the total magnetic flux through the circuit can be clearly defined, permitting the circuit inductance to be readily determined through calculation of the magnetic flux. Thus, the partial (self and mutual) inductance is intended to represent the inductance that a circuit segment contributes *as a part of a closed loop circuit*. Rosa made an intuitive argument that for this purpose only the magnetic lines between the two planes perpendicular to the linear conductor and intersecting the conductor ends should be considered. That is, the magnetic flux of interest is the flux through the loop formed by the conductor and the two lines originating from and perpendicular to the conductor ends; the loop is closed at infinity. The remaining magnetic flux is considered when the closed loop inductance is calculated from the self and mutual partial inductance of the circuit segments according to the standard formulæ for series and parallel connections of inductors. The concept proved useful and was utilized in the inductance calculation formulæ and tables developed by Rosa and Cohen [3], Rosa and Grover [4], and Grover [5]. A rigorous theoretical treatment of the subject was first provided by Ruehli in [6], where a general definition of the partial inductance of an arbitrarily shaped conductor is given in terms of the magnetic vector potential. Ruehli also coined the term “partial inductance.”

The process of calculating the inductance based on this partial inductance concept proceeds as follows. A closed loop circuit structure is broken into smaller elements; each of these elements does not by itself form a closed circuit. Typically, the circuit is partitioned into elements of simple shape, such as straight wire elements of constant crossection, for which precise and convenient analytical partial inductance formulæ are available. Each of these elements is assigned a *partial* self inductance. Each pair of these elements is assigned a *partial* mutual inductance (although none of the elements forms a closed loop). The partial self and mutual inductances of the circuit fragments and the topology of the fragment connections are all the information required to calculate the total self and mutual inductance of the circuit. If the circuit is complete and forms a loop, the resulting total inductance is the *loop* inductance of the circuit. If the circuit is a subcircuit of a larger circuit, the resulting total inductance is the partial inductance of the open circuit. The process of calculating the total self inductance (as well as the total mutual inductance) should include both the self and mutual inductances of all of the circuit elements. Observing the process in which the partial inductance of specific circuit elements affects the characteristics of the entire circuit can lead to insight into the overall behavior of the circuit.

The relationship between the loop and partial inductances is particularly straightforward for a circuit composed of series connected elements. Given a number of series connected elements N, the partial inductances can be arranged in a matrix $L_{ij} 1 \le i, j \le N$, where L_{ii} is the partial self inductances of the element i, and L_{ij} $i \ne j$ is the partial mutual inductance between elements i and j ($L_{ij} = L_{ji}$). The value of L_{ij} is negative if the currents in the associated elements flow in opposite directions. The inductance of the entire circuit is the sum of all of the elements of the matrix: $L_0 = \sum_{i=1}^{N} \sum_{j=1}^{N} L_{ij}$ (this expression is the loop inductance if the elements complete a circuit loop or a partial self inductance of a larger subcircuit if there is no circuit loop).

A theoretical basis for this transformation and expressions for calculating the partial inductances are described in [6]. Analytical expressions characterizing the self and mutual partial inductance of straight wire segments as a function of the line dimensions are presented in [5] for various crossection shapes and mutual orientations.

The inductance extraction program FastHenry [7] is used to explore the inductive properties of grid structures. A conductivity of 58 S/μm $\simeq (1.72\ \mu\Omega \cdot \text{cm})^{-1}$ is assumed for the interconnect material. The inductive portion of the impedances is shown to be relatively insensitive to the interconnect resistivity in the range of 1.7 $\mu\Omega \cdot$cm to 2.5 $\mu\Omega \cdot$cm (typical for advanced processes with copper interconnect [8, 9, 10]).

3. Dependence of inductance on line length

A non-intuitive property of the partial inductance is the characteristic that the partial inductance of a line is a nonlinear function of the line length. The partial inductance of a straight line is a superlinear function of length. The partial self inductance of a rectangular line at low frequency can be described by [5]

$$L_{part} = 0.2l \left(\ln \frac{2l}{T+W} + \frac{1}{2} - \ln \gamma \right) \mu\text{H}, \tag{1}$$

where T and W are the thickness and width of the line, and l is the length of the line in meters. The $\ln \gamma$ term is a function of only the T/W ratio, is small as compared to the other terms (varying from 0 to 0.0025), and has a negligible effect on the dependence of the inductance with length. This expression is an approximation, valid for $l \gg T+W$; a precise formula for round conductors can be found in [2].

From a circuit analysis point of view, this nonlinearity is caused by a significant inductive coupling among the different segments of the same line. Consider a straight line; the corresponding circuit representation of the self inductance of the line is a single inductor, as shown in Fig. 1a. Consider also the same line as two shorter lines connected in series. The corresponding circuit representation of the inductance of these two lines is two coupled inductors connected in series, as shown in Fig. 1b.

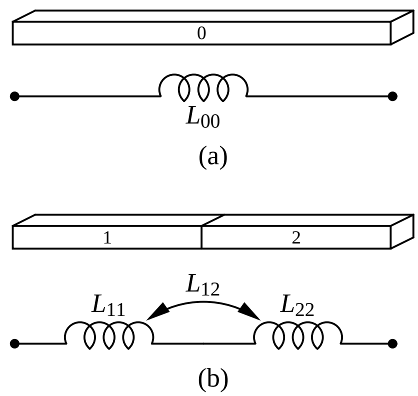

Figure 1: Two representations of a straight line inductance. (a) The line can be considered as a single element, with a corresponding circuit representation as a single inductor. (b) The same line can also be considered as two lines connected in series with a corresponding circuit representation as two coupled inductors connected in series.

The inductance of this circuit is

$$L_{1+2} = L_{11} + L_{22} + 2L_{12}. \quad (2)$$

If the partial inductance is a linear function of length, the inductance of the circuit is the sum of the inductance of its elements, *i.e.*,

$$L_{linear} = L_{11} + L_{22}. \quad (3)$$

The difference between the nonlinear dependence [see (2)] and the linear dependence [see (3)] is the presence of the cross coupling term $2L_{12}$. This term increases the inductance beyond the linear value, *i.e.*, the sum $L_{11}+L_{22}$. The cross coupling term increases with line length and does not become small as compared to the self inductance of the lines L_{11} as the line length increases.

However, the loop inductance of a complete current loop formed by two parallel straight rectangular conductors, shown in Fig. 2, is given by [5]

$$L_{loop} = 0.4l \left(\ln \frac{P}{H+W} + \frac{3}{2} - \ln \gamma + \ln k \right) \mu\text{H}, \quad (4)$$

where P is the distance between the center of the lines (the pitch) and $\ln k$ is a tabulated function of the H/W ratio. This expression is accurate for long lines (*i.e.*, for $l \gg P$). The expression is a linear function of the line length l.

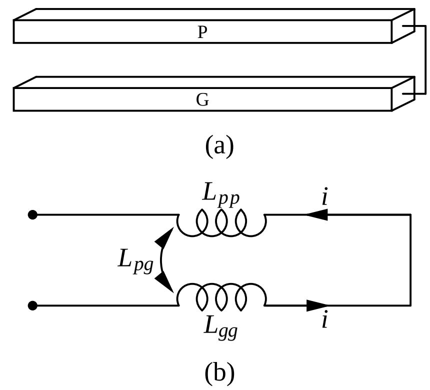

Figure 2: A complete current loop formed by two straight parallel lines; (a) a physical structure and (b) a circuit diagram of the partial inductance.

To compare the dependence of inductance on length for both a single line and a complete loop and to assess the accuracy of the long line approximation assumed in (4), the inductance extractor FastHenry is used to evaluate the partial inductance of a single line and the loop inductance of two identical parallel lines forming forward and return current paths. The crossection of the lines is 1 μm $\times$ 1 μm. The spacing between the lines in the complete loop is 4 μm. The length is varied from 10 μm to 10 mm.

The linearity of a function is difficult to visualize when the function argument spans three orders of magnitude (particularly when plotted in a semi-logarithmic coordinate system). The inductance per length, alternatively, is a convenient measure of the linearity of the inductance. The inductance per length (the inductance of the structure divided by the length of the structure) is independent of the length if the inductance is linear with length, and varies with length otherwise.

The inductance per length is calculated for a single line and a two line loop of various length. As the linearity of the data rather than the absolute magnitude of the data is of primary interest here, the data are plotted as a per cent deviation from the reference value. The inductance per length at a length of 1 mm is chosen as a reference. Thus, the inductance per length versus line length is plotted in Fig. 3 as a per cent deviation from the magnitude of inductance per length at a line length of 1 mm. As shown in Fig. 3, the inductance per length of a single line changes significantly with length. The inductance per length of a complete loop is practically constant for a wide range of lengths (varying approximately 4% over the range from 50 μm to 10,000 μm). The inductance of a complete loop can, therefore, be considered linear when the length of the loop exceeds the loop width by approximately a factor of ten. Note that in the case of a simple structure, such as the two line loop shown in Fig. 2, it is not necessary to use FastHenry to produce the data shown in Fig. 3. The formulæ for inductance in [5] can be applied to derive the data shown in Fig. 3 with sufficient accuracy.

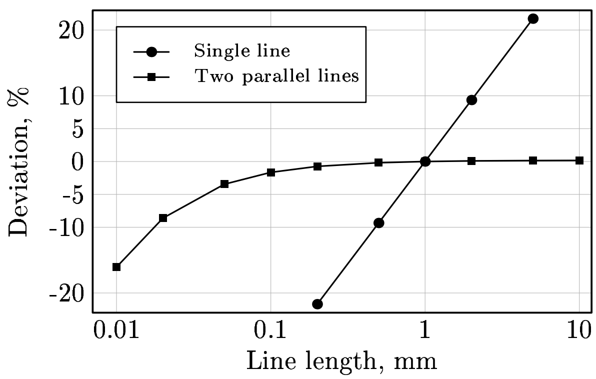

Figure 3: Inductance per length versus line length in terms of the per cent difference of the inductance per length for a 1 mm long line. The line crossection is $1\,\mu\text{m} \times 1\,\mu\text{m}$. The partial inductance per length of a single line is represented by the circles, and the loop inductance per length of a two line return circuit is represented by the squares.

To gain further insight into why the inductance of a complete loop increases linearly with length while the inductance of a single line increases nonlinearly, consider a complete loop as two loop segments connected in series, as shown in Fig. 4. The inductance of the left side loop segment (formed by line segments one and two) is

$$L_{1+2} = L_{11} + L_{22} - 2L_{12}, \tag{5}$$

while the inductance of the right side segment of the loop (formed by line segments three and four) is, analogously,

$$L_{3+4} = L_{33} + L_{44} - 2L_{34}. \tag{6}$$

The inductance of the entire loop is

$$\begin{aligned} L_{loop} &= \sum_{i,j=1}^{4} L_{ij} \\ &= L_{11} + L_{22} + L_{33} + L_{44} - 2L_{12} - 2L_{34} \\ &\quad + 2L_{13} - 2L_{14} + 2L_{24} - 2L_{23}. \end{aligned} \tag{7}$$

Considering that $L_{13} = L_{24}$ and $L_{14} = L_{23}$ due to the symmetry of the structure and substituting (5) and (6) into (7), this expression reduces to

$$L_{loop} = L_{1+2} + L_{3+4} + 2M, \tag{8}$$

where $M = L_{13} - L_{14} + L_{24} - L_{23} = 2(L_{13} - L_{14})$ is the coupling between the two loop segments. This expression for a complete loop is completely analogous to (2) for a single line. Similar to (2), the nonlinear term within the parenthesis augments the inductance beyond a linear value of $L_{1+2} + L_{3+4}$. An important difference, however, is that the nonlinear part is a difference of two terms close in value, because $L_{13} \approx L_{14}$ and $L_{24} \approx L_{23}$.

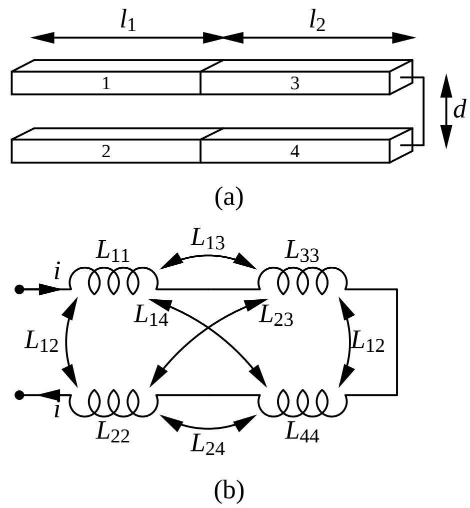

Figure 4: A complete current loop formed by two straight parallel lines consists of two loop segments in series; (a) a physical structure and (b) a circuit diagram of the partial inductance.

This behavior can be intuitively explained as follows. Segments three and four are physically (and inductively) much closer to each other than these segments are to segment one. The effective distance (the distance to move one segment so as to completely overlap the other segment) between line segment three and line segment four is small as compared to the effective distance between segment three and segment one. The inductive coupling is a smooth function of distance. The magnitude of the inductive coupling of segment one to segment three is, therefore, quite close to the magnitude of the coupling of segment one to segment four (but of opposite sign due to the opposite direction of the current flow).

A mathematical treatment of this phenomenon confirms this intuitive insight. The mutual partial inductance of two parallel line segments, for example, segments one and four shown in Fig. 4b, is

$$L_M = 0.1\left(l_1 \ln\frac{l_1 + l_2}{l_1} + l_2 \ln\frac{l_1 + l_2}{l_2} - d\right)\ \mu\text{H}, \tag{9}$$

where l_1 and l_2 are the segment lengths, and d is the distance between the center axes of the segments, as shown in Fig. 4a. Consider, for example, the case where the line segments are of equal length, $l_1 = l_2 = l/2$. The mutual inductance as a function of the axis distance d is

$$L_M(d) = (l \ln 2 - d)\ \mu\text{H}, \tag{10}$$

where $L_M(d)$ is a weak function of d if $d \ll l$. The mutual inductance of two segments forming a straight line, *i.e.*, L_{12} in Fig. 1, and L_{13} and L_{24} in Fig. 2, is $L_M(0)$. The mutual inductance L_{14} and L_{23} is $L_M(d)$. The effective inductive coupling of two loop segments, therefore, simplifies to the following expression,

$$4(L_{13} - L_{14}) = 4(L_M(0) - L_M(d)) = 0.4d\ \mu\text{H}. \qquad (11)$$

Note that this coupling is much smaller than the coupling of two straight segments, $L_M(0)$, and is independent of the loop segment length l. As the loop length l exceeds several loop widths d (as $l \gg d$), the coupling becomes negligible as compared to the self inductance of the loop segments. As compared to the coupling between two single line segments, the effective coupling is reduced by a factor of

$$\frac{M_{line}}{M_{loop}} = \frac{L_M(0)}{2(L_M(0) - L_M(d))} = \frac{\ln 2}{2}\frac{l}{d}. \qquad (12)$$

In general, it can be stated that at distances much larger than the effective separation between the forward and return currents, the inductive coupling is dramatically reduced as the coupling of the forward current and return current is mutually cancelled. Hence, the inductance of a long loop ($l \gg d$) depends linearly upon the length of the loop.

4. Inductive coupling between two parallel loop segments

As shown in the previous section, the relative proximity of the forward and return current paths is the reason for the cancellation of the inductive coupling between two loop segments connected in series (*i.e.*, different sections of the same current loop).

The same argument can be applied to show that the effective inductive coupling between two sections of parallel current loops is also reduced. As in the case of the collinear loop segments considered above, this behavior is due to cancellation of the coupling if the distance between the forward and return current paths (the loop width) is much smaller than the distance between the parallel loop segments. The physical structure and circuit diagram of two parallel loop segments are shown in Fig. 5.

The mutual loop inductance of the two loop segments is

$$M_{loop} = L_{13} - L_{14} + L_{24} - L_{23}. \qquad (13)$$

The mutual inductance between two parallel straight lines of equal length is [2]

$$M_{loop} = 0.2l\left(\ln\frac{2l}{d} - 1 + \frac{d}{l} - \ln\gamma + \ln k\right)\ \mu\text{H}, \qquad (14)$$

where l is the line length, and d is the distance between the line centers. This expression is an approximation for the case where $l \gg d$. The mutual inductance of two straight lines is a weak function of the distance between the lines. Therefore, if the distance between lines one and three d_{13} is much greater than the distance between lines three and

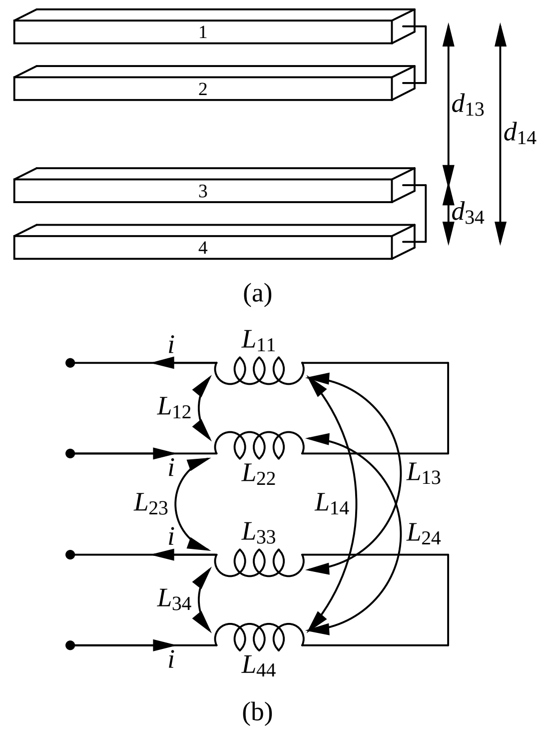

Figure 5: Two parallel loop segments where each loop segment is formed by two straight parallel lines; (a) a physical structure and (b) a circuit diagram of the partial inductance.

four d_{34}, such that $d_{13} \approx d_{14}$, then $L_{13} \approx L_{14}$. Analogously, $L_{23} \approx L_{24}$. The coupling of the loops M_{loop} is a difference of two similar values, which is small as compared to the self inductance of the loop segments. The loop segments can be considered weakly coupled in this case. This effective decoupling means that the reluctance of the loop segments wired in parallel is the sum of the reluctances of the individual loop segments. Alternatively, the inductance of the parallel connection is the inductance of two parallel uncoupled inductors, $L_{||} = \frac{L_{11}L_{22}}{L_{11}+L_{22}}$, similar to the resistance of parallel elements. The circuit inductance, therefore, varies linearly with the circuit "width": as more identical circuit elements (*i.e.*, loop segments) are connected in parallel, the inductance of the circuit decreases inversely linearly with the number of parallel elements. This linear property of inductance is demonstrated in [11] with specific application to high performance power grids.

5. Application to circuit analysis

Although rectangular lines with a unity height to width aspect ratio are used in the case studies described above, the conclusions are quite general and hold for different wire shapes and aspect ratios. At low frequencies, where the current density is uniform throughout a wire cross section, the self inductance of a wire is determined by the *geometric mean distance* of the wire cross section and the mutual inductance of two wires is determined by the *geometric mean distance* between two cross sections, as first described by Maxwell [12]. Rosa and Grover systematized and tabulated the geometric mean distance data for several practically important cases [4, 5]. Both the self and mutual inductance of a conductor are moderately dependent on the perimeter length of the inductor cross section and are virtually independent of the conductor cross-sectional shape. For example, the self inductance of rectangular conductors with aspect ratios of 1 and 10, but with the same perimeter length-to-line length ratio of 40, differ by only 0.012%, according to (1).

Skin effects reduce the current density as well as the magnetic field within the core of the conductor. This effect slightly reduces the self inductance of a wire and has virtually no effect on the mutual inductance. Proximity effects in two neighboring wires carrying current in the opposite directions (as in a loop formed by two parallel wires) can only reduce the effective distance between the two currents, making the loop effectively "longer." Therefore, a uniform current density distribution is a conservative assumption regarding the linearity of inductance with loop length.

The particular on-chip current return path is rarely known before analysis of the circuit. Nevertheless, if an approximate but conservative estimate of the distance d between the signal wire and the current return path can be made which satisfies the long loop condition $l \gg d$, the inductance can be considered to vary linearly with the length of the structure. Similar to the resistance and capacitance, the inductance of such a structure is effectively a local characteristic, independent of the length of the structure. The analysis of a large interconnect structure is therefore not necessary to obtain local inductive characteristics, thereby greatly simplifying the circuit analysis process.

6. Conclusions

The variation of the partial and loop inductance with line length is analyzed in this paper. Inductive coupling among circuit segments is shown to be the cause of the nonlinear variation of inductance with length. In long loops, the effective coupling between loop segments is small as compared to the self inductance of the segments due to the mutual cancellation between the coupling to the forward current and the coupling to the return current. The inductance of long loops, therefore, increases virtually linearly with length. In a similar manner, the effective inductive coupling between two parallel current loops is reduced through cancellation as compared to the coupling between line segments of the same length, and has a negligible effect on the inductance of the circuit. As a general rule, the inductance of circuits scales *linearly* with circuit dimensions where the distance between the forward and return currents is much smaller than the length and width of the circuit. This linear property can be exploited to simplify the inductance extraction process and the related circuit analysis of on-chip interconnect.

7. References

[1] Y. I. Ismail and E. G. Friedman, *On-Chip Inductance in High Speed Integrated Circuits*, Kluwer Academic Publishers, Norwell, Massachusetts, 2001.

[2] E. B. Rosa, "The Self and Mutual Inductance of Linear Conductors," *Bulletin of the National Bureau of Standards*, Technical paper reprint number 80, Vol. 4, No. 2, pp. 301–344, Government Printing Office, Washington, January, 1908.

[3] E. B. Rosa and L. Cohen, "Formulæ and Tables for the Calculation of Mutual and Self-Inductance," *Bulletin of the National Bureau of Standards*, Technical paper reprint number 93, Vol. 5, No. 1, pp. 1–132, Government Printing Office, Washington, August, 1908.

[4] E. B. Rosa and F. W. Grover, "Formulæ and Tables for the Calculation of Mutual and Self-Inductance," *Bulletin of the National Bureau of Standards*, Technical paper reprint number 169, Vol. 8, No. 1, pp. 1–237, Government Printing Office, Washington, August, 1912.

[5] F. W. Grover, *Inductance Calculations: Working Formulas and Tables*, D. Van Nostrand Company, Inc., New York, 1946.

[6] A. E. Ruehli, "Inductance Calculations in a Complex Integrated Circuit Environment," *IBM Journal of Research and Development*, Vol. 16, No. 5, pp. 470–481, September, 1972.

[7] M. Kamon and M. J. Tsuk and J. White, "FastHenry: A Multipole-Accelerated 3-D Inductance Extraction Program," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 42, No. 9, pp. 1750–1758, September, 1994.

[8] D. Edelstein *et al.*, "Full Copper Wiring in a Sub-0.25 μm CMOS ULSI Technology," *Proceedings of the IEEE International Electron Device Meeting*, pp. 773–776, December, 1997.

[9] S. Venkatesan *et al.*, "A High Performance 1.8 V 0.20 μm CMOS Technology with Copper Metalization," *Proceedings of the IEEE International Electron Device Meeting*, pp. 769–772, December, 1997.

[10] Semiconductor Industry Association, "International Technology Roadmap for Semiconductors, 2001 Update," `http://public.itrs.net`, 2001.

[11] A. V. Mezhiba and E. G. Friedman, "Inductive Characteristics of Power Distribution Grids in High Speed Integrated Circuits," *Proceedings of the IEEE International Symposium on Quality Electronic Design*, March, 2002.

[12] J. M. Maxwell, *A Treatise on Electricity and Magnetism*, Vol. 2, Part IV, Chapter XIII, The Clarendon Press, Oxford, United Kingdom, 1881.