Inductance Effects in *RLC* Trees

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Abstract **– A closed form solution for characterizing voltage-based signals in an *RLC* tree is presented. This closed form solution is used to derive figures of merit to characterize the effects of inductance at a specific node in an *RLC* tree. The *effective* damping factor of the signal at a specific node in an *RLC* tree is shown to be a useful figure of merit. As the effective damping factor of a signal increases, an *RC* model is sufficiently accurate to characterize that waveform. The rise time of the input signal driving an *RLC* tree is another factor characterizing the importance of inductance. As the rise time of the input signal becomes much larger than the effective *LC* time constant at a specific node within an *RLC* tree, the signal at this node does not exhibit the effects of inductance. Evidence is provided showing that using a single line analysis to determine the importance of including inductance to characterize a tree structured interconnect line is invalid in many cases and can lead to erroneous conclusions.**

I. Introduction

Inductance is becoming increasingly important with faster on-chip rise times and longer wire lengths. Wide wires are frequently encountered in clock distribution networks and in upper metal layers. These wires are low resistive lines that can exhibit significant inductive effects. Furthermore, performance requirements are pushing the introduction of new materials for low resistance interconnect [1] and new dielectrics to reduce interconnect capacitance. These technological advances significantly reduce the *RC* time constants of the interconnect which, as will be shown, increases the importance of on-chip inductance.

The importance of on-chip inductance for single lines has been characterized in [2]-[5]. However, the nets in a VLSI circuit are often structured as trees rather than as single lines. It is shown here that the branches of a tree can not be treated as single lines for the purpose of evaluating the importance of inductance. Rather, the entire tree should be examined for inductance effects as a single entity since there is a large interaction among the different branches.

The focus of this paper is the introduction of simple figures of merit that can be used as criteria to determine which nets (and trees in general) require more accurate *RLC* models. A second order approximation for a signal at a certain node of an *RLC* tree is described in section II. The effective damping factor of a signal at a particular node of a tree and the rise time of the input signal are used to derive two figures of merit that describe the relative importance of inductance for the signal at this node. These figures of merit are presented in section III. In section IV, examples of *RLC* trees are used to illustrate the error encountered in treating a branch of a tree as a single line. Finally, some conclusions are offered in section V.

II. Second Order Approximation for *RLC* Trees

A second order transfer function that approximates a higher order transfer function at a certain node of an *RLC* tree is introduced in this section. Wyatt [6] developed a first order approximation for *RC* trees based on the Elmore delay [7] assuming that the system has only one dominant pole. To characterize a non-monotonic response, at least a second order approximation is needed because a non-monotone response involves complex poles which appear in conjugate pairs for a real system. Thus, a second order system of the form described by

$$g(s) = \frac{\omega_n^2}{s^2 + s2\zeta\omega_n + \omega_n^2}, \tag{1}$$

can approximate a system with a non-monotonic response. Therefore, it is necessary to find a value of ζ and ω_n that makes the second order approximation as accurate as possible as compared to the exact transfer function.

Matching the moments of a transfer function to the moments of a higher order system permits the transfer function to approximate the system [8]-[13]. The greater the number of moments that are matched, the better the transfer function approximates the system. Applying the moment matching method, the transfer function in (1) is expanded in powers of s where the first two moments of the transfer function are equated to the first two moments of the non-monotonic system, m_1 and m_2. The expansion of the transfer function in (1) is

$$g(s) = 1 - s\left(\frac{2\zeta}{\omega_n}\right) + s^2\left(\frac{-1+(2\zeta)^2}{\omega_n^2}\right) - \ldots = 1 + m_1 s + m_2 s^2 + \ldots\ldots \quad . \tag{2}$$

The parameters that characterize the second order approximation of a non-monotone system, ζ and ω_n, can be calculated in terms of the moments of the non-monotonic system and are

$$\zeta = \frac{-m_1}{2}\frac{1}{\sqrt{m_1^2 - m_2}} \qquad \text{and} \qquad \omega_n = \frac{1}{\sqrt{m_1^2 - m_2}}. \tag{3}$$

Hence, for a system with a non-monotone response, a second order approximation can be found if the first and second moments of the system are known.

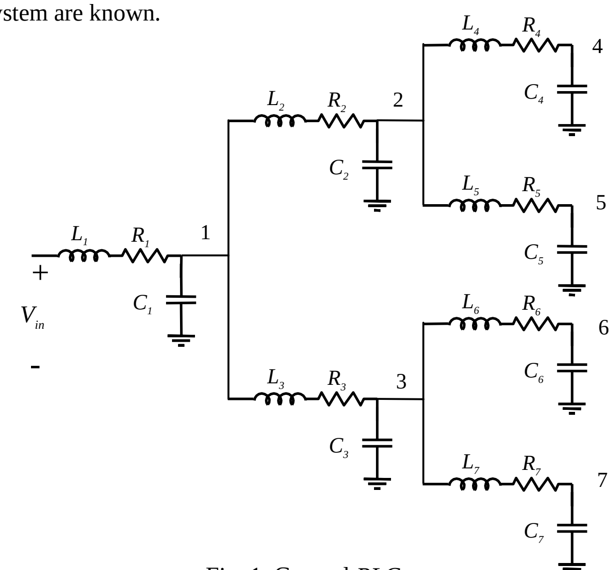

Fig. 1. General *RLC* tree.

For the general *RLC* tree shown in Fig. 1, the first and second moments of a general *RLC* tree at node *i* are

$$m_1^i = -\sum_k C_k R_{ik} \quad \text{and} \quad m_2^i = \sum_k \sum_j C_k R_{ik} C_j R_{kj} - \sum_k C_k L_{ik}, \tag{4}$$

where R_{ik} (L_{ik}) is the common resistance (inductance) from the input to

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nodes i and k. For example, $R_{77} = R_1 + R_3 + R_7$, $R_{67} = R_1 + R_3$, and $R_{27} = R_1$. The summation variable k operates over all the capacitors in the circuit. The first term in m_2^i can be approximated by $\left(\sum_k C_k R_{ik}\right)^2$. This approximation is particularly accurate for balanced trees. Thus, the second moment of $g_i(s)$ can be approximated by

$$m_2^i = \left(\sum_k C_k R_{ik}\right)^2 - \sum_k C_k L_{ik}\,. \tag{5}$$

Substituting the first and second moments for a general *RLC* tree into (3), ζ_i and ω_{ni} to characterize a second order approximation of the transfer function at node i are

$$\zeta_i = \frac{1}{2}\frac{\sum_k C_k R_{ik}}{\sqrt{\sum_k C_k L_{ik}}} \qquad \text{and} \qquad \omega_{ni} = \frac{1}{\sqrt{\sum_k C_k L_{ik}}}\,. \tag{6}$$

This second order approximation of an *RLC* tree has the same accuracy characteristics of the Wyatt approximation for an *RC* tree [6].

The second order approximation is compared in Fig. 2 to AS/X [14] simulations of the output node 7, V_7, of the tree shown in Fig. 1. A balanced tree is used. The supply voltage is 2.5 volts. A step input is applied to the *RLC* tree. Note the accuracy that the second order approximation exhibits as compared to AS/X simulations for the case of a balanced tree. If the tree is unbalanced, the second order approximation is less accurate. Wyatt's approximation is also shown in Fig. 2. Note that Wyatt's approximation fails to match the response of an *RLC* tree with significant inductance effects.

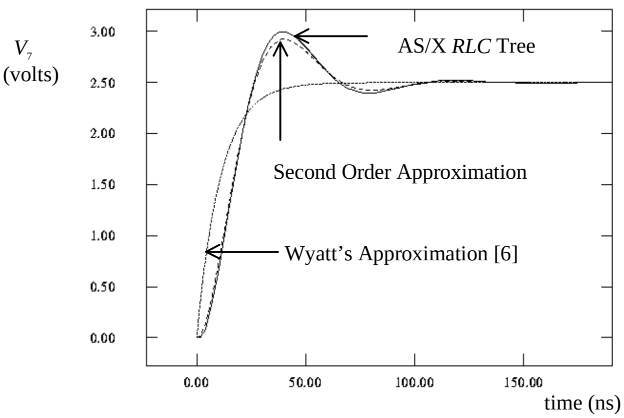

Fig. 2. AS/X simulations as compared to the second order approximation and the Wyatt model.

III. Effect of Damping Factor and Input Rise Time

In this section a second order approximation of the signals in an *RLC* tree is used to determine if the signal at a certain node exhibits significant inductive effects. In subsection *A*, the effective damping factor ζ_i at node i of an *RLC* tree is used to characterize when an *RC* model is sufficiently accurate as compared to an *RLC* model, permitting inductance to be neglected. In subsection *B*, the effect of the input rise time on the importance of inductance is discussed.

A. Damping Factor

A step signal is used as the input to the second order approximation of the transfer function at node i of an *RLC* tree to investigate the relationship between the effective damping factor ζ_i and the significance of inductance on the transient behavior of the signal at node i. A step input is used since it eliminates the effect of the rise time and maximizes the significance of the inductance, permitting the effect of the damping factor to be investigated. For a step input and a supply voltage of V_{DD} volts, the signal at node i is

$$S_i(t) = V_{DD} + V_{DD}\left[\frac{\exp[\omega_{ni}t(-\zeta_i + \sqrt{\zeta_i^2 - 1})]}{-\zeta_i + \sqrt{\zeta_i^2 - 1}} - \frac{\exp[\omega_{ni}t(-\zeta_i - \sqrt{\zeta_i^2 - 1})]}{-\zeta_i - \sqrt{\zeta_i^2 - 1}}\right]. \tag{7}$$

As the damping factor increases, the importance of the inductance on the circuit decreases. Thus, the following approximation can be made assuming large ζ_i,

$$\sqrt{\zeta_i^2 - 1} \approx \zeta_i[1 - \frac{1}{2\zeta_i^2}], \quad \text{with a relative error} < \frac{1}{4\zeta_i^4}\,. \tag{8}$$

With $\zeta_i > 2.5$, the error due to this approximation is less than 0.7%. With this approximation, the signal at node i can be approximated by

$$S_i(t) = V_{DD} + \frac{V_{DD}}{2[\zeta_i - \frac{1}{2\zeta_i}]}\left[-2\zeta_i \exp[\omega_{ni}t(-1/2\zeta_i)] + \frac{\exp[\omega_{ni}t(-2\zeta_i - 1/2\zeta_i)]}{2\zeta_i}\right]. \tag{9}$$

For $\zeta_i > 2.5$, this expression can be approximated by

$$S_i(t) = V_{DD} - V_{DD}\exp[\omega_{ni}t(-1/2\zeta_i)] = V_{DD}(1 - \exp[-t/\sum_k C_k R_{ik}])\,, \tag{10}$$

with an error less than 8%. Note that (10) is precisely Wyatt's approximation for a step response at node i of an *RC* tree. This relation shows that for $\zeta_i > 2.5$, inductance has a minimal effect on the transient response at node i which is similar to the response of an equivalent *RC* tree where inductance is neglected. Thus, the first figure of merit presented in this paper is

$$\zeta_i = \frac{1}{2}\frac{\sum_k C_k R_{ik}}{\sqrt{\sum_k C_k L_{ik}}} > 2.5. \tag{11}$$

If this inequality is satisfied, the effects of inductance at node i are negligible. A plot of AS/X [14] simulations for the tree shown in Fig. 1 at output node 7 as compared to an equivalent tree with all inductances equal to zero is shown in Fig. 3 for several values of ζ_i. The closed form solution in (7) is also shown. Note that for $\zeta_i > 2.5$, the response of the *RLC* tree is almost identical to that of an equivalent *RC* tree in which inductance is neglected.

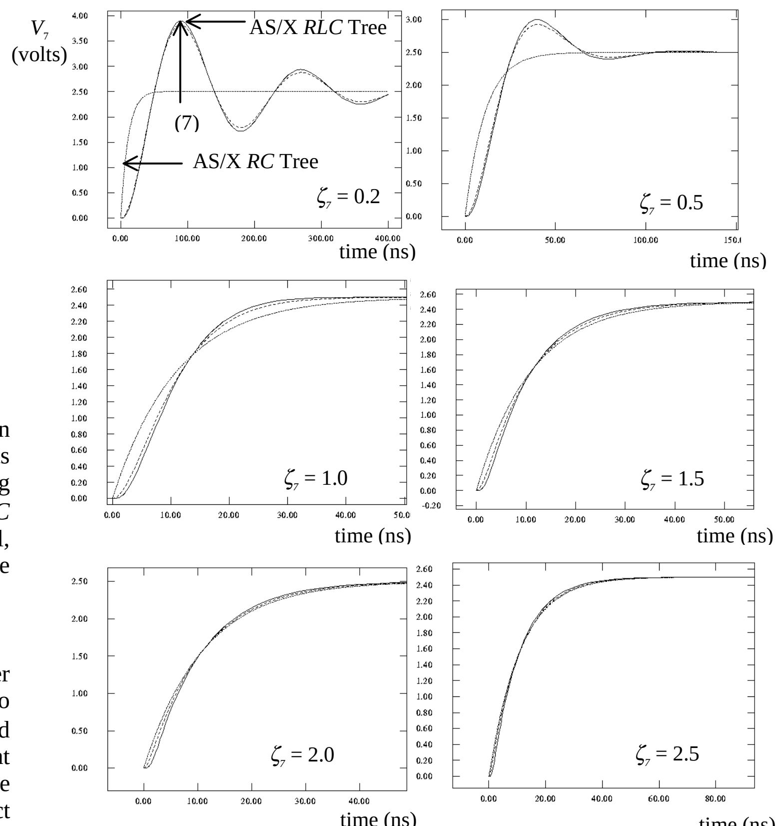

Fig. 3. Effect of the equivalent damping factor on the accuracy of the *RLC* and *RC* models.

B. Input Rise Time

An exponential signal of the form,

$$V_{in}(t) = V_{DD}[1 - \exp(-t/\tau)]u(t)\,, \tag{12}$$

is used as the input to the second order approximation of the transfer function at node *i* of an *RLC* tree to investigate the relation between the input rise time and the effects of inductance on the transient behavior of the signal at node *i*. $u(t)$ is the unit step function, V_{DD} is the supply voltage, and the 90% rise time of this input signal is 2.3τ. With this input signal, the response at node *i* of an *RLC* tree is

$$e_{i_{RLC}}(t) = V_{DD}\left[1 - ke^{-\frac{t}{\tau}} + \frac{e^{-\zeta_i \omega_{ni} t}}{\sqrt{1-\zeta_i^{\,2}}}\left[\sin(\omega_{ni}t - \theta_1) - \sqrt{\frac{1}{k}}\sin(\omega_{ni}t - \theta_2)\right]\right], \tag{13}$$

where

$$\theta_1 = \tan^{-1}\left[\frac{\sqrt{1-\zeta_i^{\,2}}}{\zeta_i}\right] \quad , \quad \theta_2 = \tan^{-1}\left[\frac{\left(\tau/T_{LCi}\right)\sqrt{1-\zeta_i^{\,2}}}{\left(\tau/T_{LCi}\right)\zeta_i - 1}\right], \tag{14}$$

and

$$k = \frac{\left(\tau/T_{LCi}\right)^2}{\left(\tau/T_{LCi}\right)^2 - 2\zeta\left(\tau/T_{LCi}\right) + 1}. \tag{15}$$

T_{LCi} is

$$T_{LCi} = \sqrt{\sum_k C_k L_{ik}}\,. \tag{16}$$

According to Wyatt's approximation [6], if the same input is applied to an *RC* tree, the response at node *i* is

$$e_{i_{RC}}(t) = V_{DD}\left[1 - k_2 e^{-\frac{t}{\tau}} + e^{-t/T_{RCi}}[k_2 - 1]\right], \tag{17}$$

where

$$k_2 = \frac{\left(\tau/T_{RCi}\right)}{\left(\tau/T_{RCi}\right) - 1} \qquad \text{and} \qquad T_{RCi} = \sum_k C_k R_{ik}. \tag{18}$$

When the rise time of the input signal increases, (13) approaches (17). This trend can be seen by noting that if τ / T_{LCi} and τ / T_{RCi} are much greater than one, k and k_2 tend to one and θ_2 tends to θ_1. Thus, if τ / T_{LCi} and τ / T_{RCi} are both much greater than one, the response at node *i* of an *RLC* tree does not exhibit any effects caused by inductance and an *RC* tree model can be used to model the interconnect tree. These two conditions, τ / T_{LCi} and τ / T_{RC}, are much greater than one and can be reduced to the first condition if the damping factor figure of merit in (11) is considered. If ζ_i is greater than 2.5, inductance effects are not significant because of the damping factor and there is no need to determine the rise time of the input signal. If ζ_i is less than 2.5, then $T_{RCi} < 5T_{LCi}$. Thus, $\tau / T_{LCi} < 5\tau / T_{RCi}$ is the range where the input rise time should be evaluated ($\zeta_i < 2.5$). Hence, if τ / T_{LCi} is much greater than one and $\zeta_i < 2.5$, then τ / T_{RCi} is also much greater than one.

The second figure of merit can be derived by assuming $(\tau / T_{LCi}) = 10$ to insure sufficient accuracy. With $t_{rin} = 2.3\tau$, the second figure of merit becomes

$$t_{rin} > 23\sqrt{\sum_k C_k L_{ik}}\,. \tag{19}$$

If this inequality is satisfied, the effects of inductance at node *i* can be neglected. A plot of AS/X [14] simulations of the *RLC* tree shown in Fig. 1 at output node 7 as compared to an equivalent tree with no inductances is shown in Fig. 4 for several values of t_{rin}. The closed form solution (13) is also shown. ζ_i is kept constant at 0.5 so that the inductance can not be ignored. Note that for $t_{rin} / T_{LCi} > 23$, the response of the *RLC* tree is the same as that of an equivalent *RC* tree in which inductance is neglected.

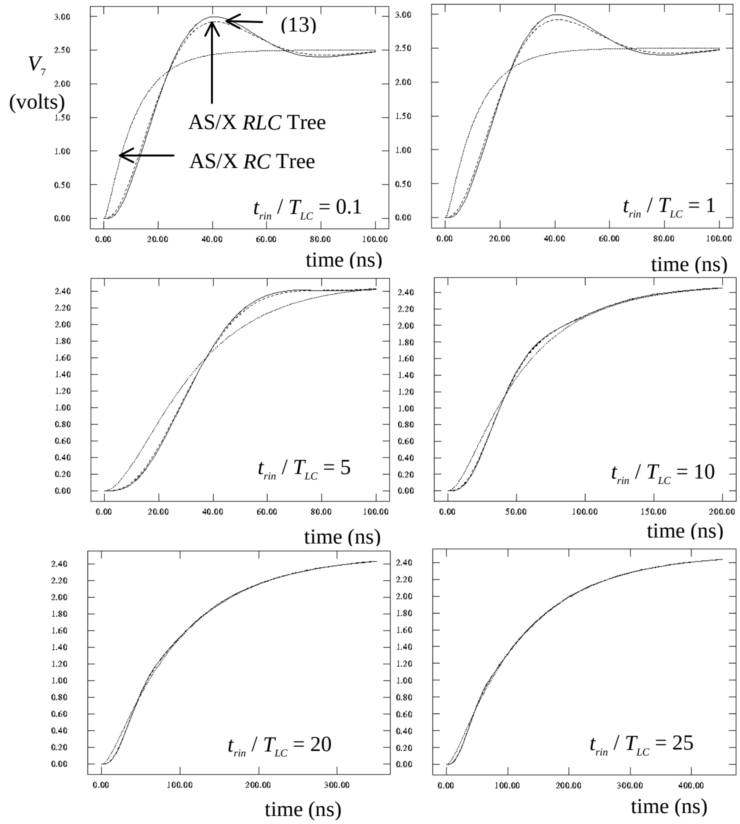

Fig. 4. Effect of the rise time on the inductance effects in an *RLC* tree. t_{rin} / T_{LC} is varied from 0.1 to 25 and AS/X simulations are shown for an *RC* tree and an *RLC* tree. (13) is also shown to illustrate the accuracy of the closed form solution introduced here. Note that as t_{rin} / T_{LC} increases, the *RC* model approaches the *RLC* model. $\zeta_i = 0.5$.

IV. Examples and Results

The analysis of single lines to characterize the importance of on-chip inductance has been previously evaluated [2]-[5]. However, analyzing single lines to characterize the importance of inductance in *RLC* trees can be invalid. To illustrate this point, values for the branch resistances, inductances, and capacitors for the *RLC* tree shown in Fig. 1 are listed in Table 1. According to [2]-[5], if a single line analysis is used for each branch, the damping factor for branch *i* is

$$\zeta_i = \frac{1}{2}\frac{R_i C_i}{\sqrt{L_i C_i}}\,. \tag{20}$$

The damping factor of branch *i* affects the signal at node *i*. The single line analysis and the *RLC* tree analysis introduced here are compared in Table 2 to the tree shown in Fig. 1. The branch impedance values listed in Table 1 are used. Note the large difference in the values of the damping factors according to an *RLC* single line analysis as compared to an *RLC* tree analysis. For example, at node 7, the *RLC* single line analysis anticipates no significant inductance effects ($\zeta_7 = 1.58$) while an *RLC* tree analysis anticipates large inductive effects ($\zeta_7 = 0.529$). Simulations of the voltage signal at node 7 of the *RLC* tree shown in Fig. 1 with the branch impedance values listed in Table 1 are shown in Fig. 5. The voltage at node 7 exhibits high inductive effects as anticipated by the *RLC* tree analysis introduced here. This simple example demonstrates that an *RLC* single line analysis can lead to erroneous conclusions in certain cases. Note also that for node 1, the *RLC* single line analysis anticipates greater inductance effects ($\zeta_1 = 0.176$) as compared to the *RLC* tree analysis ($\zeta_1 = 0.306$).

The *RLC* single line analysis generates a significant difference between the maximum and minimum damping factors ($0.176 < \zeta < 1.58$) as compared to the difference between the maximum and minimum damping factors in the more accurate *RLC* tree analysis ($0.306 < \zeta < 0.529$). This behavior is caused by each line being analyzed individually while in reality all the lines in the tree interact significantly, distributing the effects of inductance throughout the tree. Alternatively, the lines with higher inductive effects and the lines with lower inductive effects influence each other, making the effect of inductance less on those lines with higher inductance effects and more on those lines with lower inductance effects. This phenomenon is accurately captured by the *RLC* tree analysis introduced here.

Table 1. Branch impedances for the *RLC* tree shown in Fig. 1.

Branch	R (Ω)	L (nH)	C (pF)
1	25	10	2
2	50	10	1
3	50	10	1
4	100	0.5	0.5
5	100	0.5	0.5
6	100	0.5	0.5
7	100	0.5	0.5

Table 2. Damping factors for the nodes of both the *RLC* single line and the *RLC* tree analyses shown in Fig. 1.

Node	ζ_i (*RLC* single line analysis)	ζ_i (*RLC* tree analysis)
1	0.176	0.306
2	0.25	0.441
3	0.25	0.441
4	1.58	0.529
5	1.58	0.529
6	1.58	0.529
7	1.58	0.529

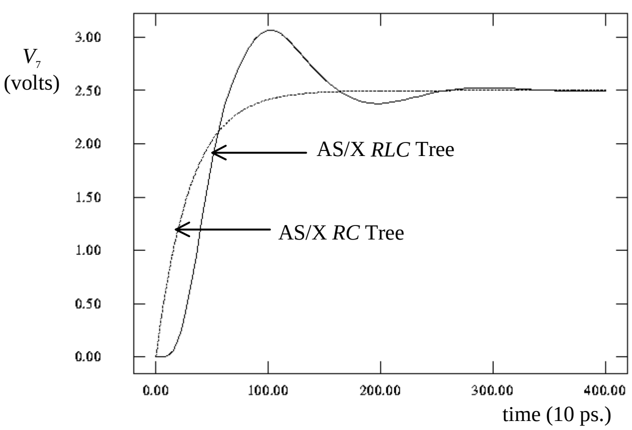

Fig. 5. AS/X simulations of the output voltage at node 7 of the *RLC* tree shown in Fig. 1 with the branch impedance values listed in Table 1 and the equivalent *RC* tree.

The effect of increasing the size of the tree is to increase the damping factors at the nodes of the tree (and thus decrease the importance of the inductance). If the size of a tree increases, both of the summations $\sum_k C_k R_{ik}$ and $\sum_k C_k L_{ik}$ increase. As described by (6), ζ_i is half the first summation over the square root of the second summation. Thus, if the two summations increase at the same rate while increasing the size of the tree, the net result is an increase in ζ_i. For example, the damping factor at node 1 for the *RLC* tree shown in Fig. 1 is

$$\zeta_1 = \frac{1}{2}\frac{R_1 C_T}{\sqrt{L_1 C_T}} = \frac{R_1}{2}\sqrt{\frac{C_T}{L_1}}, \tag{21}$$

where C_T is the total capacitance of the tree. If the size of the tree increases, C_T increases which increases the damping factor at node 1.

Alternatively, if the size of the tree is smaller, the rise time of the input signal can be much greater than T_{LCi} which, according to the second figure of merit in (19), eliminates the effects of inductance. Thus, there is a range of the size of an *RLC* tree where inductance effects are significant. For the special case of a single *RLC* line the size is simply represented by the length of the line which is consistent with the results described in [5], in which there is a range of interconnect line length where inductance effects are significant.

V. Conclusions

A second order approximation of an *RLC* tree with the same accuracy characteristics as the Wyatt approximation for an *RC* tree has been introduced. This second order approximation is used to derive two simple figures of merit to evaluate the significance of inductance on the transient behavior of an *RLC* tree. The first figure of merit is the damping factor of a signal at a specific node of a tree. It is shown that as the damping factor increases, inductance effects decrease. The second figure of merit is the rise time of the input signal as compared to the effective *LC* time constant of the tree at a specific node. It is also shown that as the input rise time increases as compared to the effective *LC* time constant, the importance of inductance decreases. Evidence is provided that using a single *RLC* line analysis for those branches within an *RLC* tree can lead to incorrect conclusions. Finally, it is also shown that there is a range of the size of an *RLC* tree where a tree can exhibit significant inductive effects.

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