Equivalent Rise Time for Resonance in Power/Ground Noise Estimation

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***Abstract*— The non-monotonic behavior of power/ground noise with respect to the rise time t_r is investigated for an inductive power distribution network with a decoupling capacitor. A time domain solution is provided for the rise time that produces resonant behavior, thereby maximizing the power/ground noise. The sensitivity of the ground noise to the decoupling capacitance C_d and parasitic inductance L_g is evaluated as a function of the rise time. Increasing the decoupling capacitance is shown to efficiently reduce the noise for $t_r \leq 2\sqrt{L_g C_d}$. Alternatively, reducing the parasitic inductance L_g is shown to be effective for $t_r \geq 2\sqrt{L_g C_d}$.**

I. INTRODUCTION

The distribution of robust power supply and ground voltages is a challenging task in modern integrated circuits due to scaled power supply voltages and the increased switching activity of the load circuit [1], [2]. The parasitic resistance and inductance of the power and ground distribution networks produce $IR + L\,\partial i/\partial t$ voltage drops, reducing the overall voltage at the load.

The changing voltages increase the delay uncertainty while reducing the noise margin in high performance integrated circuits, possibly causing a timing violation or logic failure. Decoupling capacitors are often used to reduce power/ground noise by decreasing the overall impedance of the power distribution network [3], [4], [5].

The parallel combination of the decoupling capacitor C_d with the parasitic inductance L_g of the power distribution network, however, produces a peak impedance at the resonant frequency $f = 1/(2\pi\sqrt{L_g C_d})$ due to the LC tank circuit. The impedance at the resonant frequency should, therefore, be smaller than the target impedance to satisfy noise constraints. The impedance characteristics of a power distribution system have been investigated with particular focus on the resonant behavior [6], [7]. The corresponding rise time that produces the maximum noise in the time domain, however, has not received much attention.

The non-monotonic behavior of power/ground noise with respect to the rise time is investigated in this paper. A worst case rise time producing the maximum noise in the presence of a decoupling capacitor is presented. The sensitivity of the noise to the decoupling capacitance and parasitic inductance is also evaluated.

The rest of the paper is organized as follows. The equivalent circuit model to estimate the peak-to-peak power/ground noise is described in Section II. The non-monotonic noise behavior with respect to the rise time is investigated in Section III. The paper is concluded in Section IV.

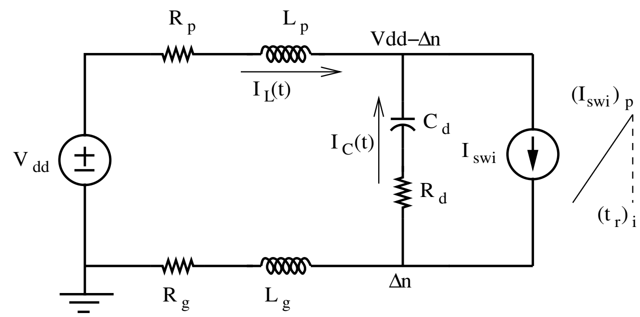

Fig. 1. Equivalent circuit model to estimate power supply noise and ground bounce. R_p, L_p, and R_g, L_g represent the power and ground rail impedances, respectively. C_d is the decoupling capacitor and R_d is the effective series resistance (ESR) of the capacitor. The load circuit is represented by a current source with a rise time $(t_r)_i$ and peak current $(I_{swi})_p$.

II. POWER/GROUND NOISE MODEL

The equivalent circuit model to investigate the noise behavior with respect to the rise time is shown in Fig. 1, where R_p, L_p, and R_g, L_g represent the power and ground rail impedances, respectively. C_d is the decoupling capacitor and R_d is the effective series resistance (ESR) of the capacitor. The load circuit is represented by a current source with a rise time $(t_r)_i$ and peak current $(I_{swi})_p$. The current provided by the decoupling capacitance $I_C(t)$ and the current flowing through the parasitic inductance $I_L(t)$ from the power supply are, respectively,

$$I_C(t) = -C_d \frac{\partial V_C}{\partial t}, \tag{1}$$

$$I_L(t) = \frac{1}{L_g} \int_0^t V_L(t) \partial t, \tag{2}$$

This research is supported in part by the Semiconductor Research Corporation under Contract No. 2004-TJ-1207, the National Science Foundation under Contract Nos. CCR-0304574 and CCF-0541206, grants from the New York State Office of Science, Technology & Academic Research to the Center for Advanced Technology in Electronic Imaging Systems, and by grants from Intel Corporation, Eastman Kodak Company, Intrinsix Corporation, and Freescale Semiconductor Corporation.

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where $V_C(t)$ and $V_L(t)$ are, respectively,

$$V_C(t) = V_{dd} - 2\Delta n + I_C(t)R_d, \tag{3}$$

$$V_L(t) = \Delta n - I_L(t)R_p. \tag{4}$$

Assuming a ramp function for the noise $\Delta n(t) = [(V_{gnd})_p/(t_r)_v]t$, replacing (3) in (1) and (4) in (2), and taking the derivative with respect to time results in the following differential equations,

$$I_C(t) = \frac{2C_d(V_{gnd})_p}{(t_r)_v} - R_dC_d\frac{\partial I_C(t)}{\partial t}, \tag{5}$$

$$\frac{\partial I_L(t)}{\partial t} = \frac{(V_{gnd})_pt}{L_g(t_r)_v} - \frac{R_p}{L_g}I_L(t). \tag{6}$$

Solving these differential equations with the initial conditions $I_C(0) = 0$ and $I_L(0) = 0$ produces the inductive and capacitive current, respectively,

$$I_C(t) = (V_{gnd})_p\frac{2C_d}{t_r}(1 - e^{-t/(R_dC_d)}), \tag{7}$$

$$I_L(t) = (V_{gnd})_p[\frac{t}{t_rR_g} - \frac{L}{t_rR_g^2}(1 - e^{-t/\frac{L}{R_g}})]. \tag{8}$$

These currents can be rewritten as

$$I_C(t) = G_C(t)(V_{gnd})_p, \tag{9}$$

$$I_L(t) = G_L(t)(V_{gnd})_p, \tag{10}$$

where $G_C(t)$, the conductance of the capacitance path, and $G_L(t)$, the conductance of the inductance path, are given, respectively, by

$$G_C(t) = \frac{2C_d}{(t_r)_v}(1 - e^{-t/(R_dC_d)}), \tag{11}$$

$$G_L(t) = \frac{t}{(t_r)_vR_g} - \frac{L_g}{(t_r)_vR_g^2}(1 - e^{-t/\frac{L_g}{R_g}}). \tag{12}$$

Note that these conductances are both a function of the rise time $(t_r)_v$. Specifically, as the rise time becomes smaller, $G_C(t)$ increases and $G_L(t)$ decreases. The capacitive current, therefore, increases with decreasing rise time. Alternatively, the inductive current increases with longer rise times. Intuitively, a smaller rise time corresponds to a higher frequency, where the impedance of the capacitance is smaller and the inductance is higher. The capacitance is, therefore, more effective at smaller rise times and becomes less effective as the rise time increases.

Assuming the peak noise occurs when the switching current reaches the maximum current, *e.g.*, $(t_r)_v = (t_r)_i = t_r$, the peak ground noise at $t = t_r$ can be expressed as

$$\frac{1}{(V_{gnd})_p} = \frac{G_C(t_r)}{(I_{swi})_p} + \frac{G_L(t_r)}{(I_{swi})_p}. \tag{13}$$

Replacing (11) and (12) in (13) produces

$$(V_{gnd})_p = \frac{(I_{swi})_pR_g^2t_r}{2C_dR_g^2(1 - e^{-t_r/(R_dC_d)}) - L(1 - e^{-t_r/\frac{L_g}{R_g}}) + R_gt_r}. \tag{14}$$

Note that if the capacitive current is much greater than the inductive current, *e.g.*, (9) $\gg$ (10) or (11) $\gg$ (12), the second term in (13) can be neglected without a significant loss in accuracy, guaranteeing the pessimism of the expression. In this case, the peak ground noise is approximated by

$$(V_{gnd})_p \approx (I_{swi})_p/G_C(t_r). \tag{15}$$

Alternatively, if the inductive current is much greater, the first term in (13) can be neglected and the peak noise is estimated as

$$(V_{gnd})_p \approx (I_{swi})_p/G_L(t_r). \tag{16}$$

If the circuit is underdamped, *i.e.*, the damping factor is smaller than one, oscillations occur due to a parallel combination of the parasitic inductance and decoupling capacitor. In this case, the peak-to-peak ground noise voltage is

$$(V_{gnd})_{pp} = (V_{gnd})_p[1 + e^{-\pi\zeta/\sqrt{1-\zeta^2}}], \tag{17}$$

where $\zeta = [(2R_g + R_d)/2]\sqrt{C_d/2L_g}$ is the damping factor.

III. Non-monotonic Noise Behavior

The impedance of a parallel LC circuit is maximum at the resonant frequency, $\omega = 1/\sqrt{(LC)}$. At this frequency, both the capacitive and inductive paths carry a significant amount of current, giving rise to resonant behavior. Similarly, in the time domain, there exists a rise time at which the capacitive and inductive currents are close and the peak-to-peak noise is maximum. The *worst case rise time* producing the maximum noise is described in Section III-A. The sensitivity of the noise to the decoupling capacitance and parasitic inductance as a function of the rise time is investigated in Section III-B.

A. Worst Case Rise Time

The capacitive and inductive currents and the corresponding ground noise are plotted as a function of rise time in Fig. 2 using (7) and (8) for the currents, and (17) for the noise voltage when $(I_{swi})_p = 11.5$ *mA*, $L_g = 1$ *nH*, $C_d = 10$ *pF*, $R_g = 2.2\ \Omega$, and $R_d = 0.1\ \Omega$. Equation (17) is also compared with SPICE in Fig. 2.

The model accurately captures the non-monotonic dependence of noise on rise time, exhibiting a maximum error of 12.5%. As shown in Fig. 2, for sufficiently small rise times, the capacitive current dominates, and the inductance does not affect the ground noise. As the rise time increases, the capacitive current decreases and the inductive current increases. The peak noise occurs at a rise time where these currents are approximately equal, denoted as the *worst case rise time* (similar to the resonant behavior in the frequency domain). If the rise time increases further, the noise decreases due to a lower $L\ \partial i/\partial t$ noise, making the capacitance ineffective. The assumption of fast transients as the worst case scenario for noise can be overly optimistic in a circuit with sufficient decoupling. This conclusion is similar to reducing the resonant frequency with a larger capacitance. Increasing the decoupling capacitance, therefore, has the drawback of

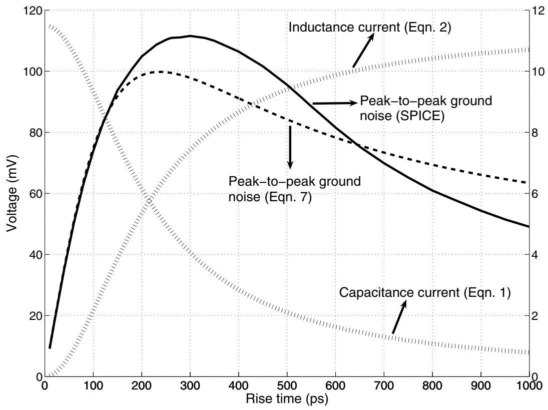

Fig. 2. Comparison of peak-to-peak ground noise as a function of the rise time obtained from SPICE simulations and (17) for $(I_{swi})_p = 11.5\ mA$. The ground network impedances are $L_g = 1\ nH$, $C_d = 10\ pF$, $R_g = 2.2\ \Omega$, and $R_d = 0.1\ \Omega$. The dotted lines depict the estimated capacitive and inductive currents as a function of the rise time.

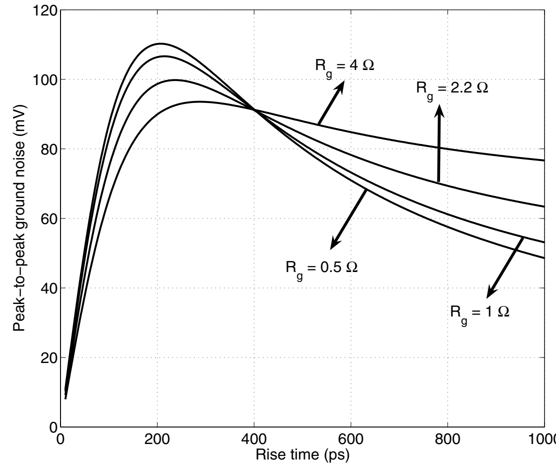

Fig. 3. Peak-to-peak ground noise for different values of R_g when $(I_{swi})_p = 11.5\ mA$, $L_g = 1\ nH$, $C_d = 10\ pF$, and $R_d = 0.1\ \Omega$

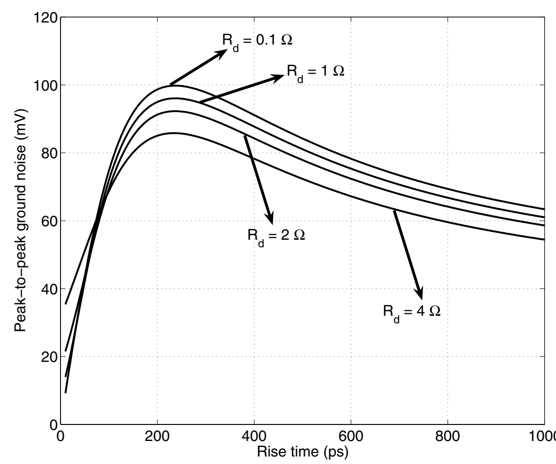

Fig. 4. Peak-to-peak ground noise for different values of R_d when $(I_{swi})_p = 11.5\ mA$, $L_g = 1\ nH$, $C_d = 10\ pF$, and $R_g = 2.2\ \Omega$

reducing the resonant frequency, or similarly, increasing the worst case rise time.

According to Fig. 2, the maximum peak-to-peak noise occurs at the rise time where the inductive and capacitive currents are approximately equal. Thus, an expression for the worst case rise time can be developed by equating (11) with (12) at $t = t_r$ and solving for t_r,

$$G_C(t_r) - G_L(t_r) = 0. \tag{18}$$

A closed form solution, however, does not exist due to the exponential terms in $G_C(t_r)$ and $G_L(t_r)$. Assuming $R_d \to 0$, $G_C(t_r) \approx \frac{2C_d}{t_r}$ since $e^{-t_r/(R_d C_d)} \to 0$. Similarly, assuming $R_g \to 0$, $G_L(t_r) \approx \frac{t_r}{2L_g}$ from a Taylor series expansion. The rise time t_r at which these conductances are approximately equal is, therefore,

$$\frac{2C_d}{t_r} = \frac{t_r}{2L_g} \Longrightarrow t_r = 2\sqrt{(L_g C_d)}. \tag{19}$$

The ground noise is obtained at $t_r = 2\sqrt{(L_g C_d)}$ for different parasitic impedances of the ground network and compared with the maximum noise obtained at the same impedance. These results are listed in Table I. The error of $t_r = 2\sqrt{(L_g C_d)}$ in estimating the maximum noise is greater with increasing R_g and R_d, but is sufficiently small within the practical values of these resistances, as listed in Table I.

The effect of the parasitic resistance R_g and the ESR of the decoupling capacitance R_d on the worst case rise time is further illustrated, respectively, in Figs. 3 and 4. Increasing R_g reduces the noise until a specific rise time is reached due to additional damping. Beyond this rise time, however, the noise increases due to a greater *IR* drop on the ground network, making the decoupling capacitance ineffective. Alternatively, an increase in R_d results in decreased noise at higher rise times due to the increased damping and higher noise at smaller rise times where the decoupling capacitance is effective.

B. Noise Sensitivity as a Function of Rise Time

The normalized sensitivity of the ground noise as a function of the rise time is determined in this section to evaluate the efficacy of reducing the parasitic inductance and increasing the decoupling capacitance on reducing the ground noise. The normalized sensitivity of the ground noise to a parameter p_i is determined by

$$S_{p_i}^{(V_{gnd})_{pp}} = \lim_{\Delta p_i \to 0} \frac{\frac{\Delta (V_{gnd})_{pp}}{(V_{gnd})_{pp}}}{\frac{\Delta p_i}{p_i}} = \frac{p_i}{(V_{gnd})_{pp}} \frac{\partial (V_{gnd})_{pp}}{\partial p_i}. \tag{20}$$

The normalized sensitivity of the ground noise as a function of rise time, as determined by (20), is shown in Fig. 5. The sensitivity of the noise to the decoupling capacitance is high at small rise times and decreases with increasing rise time. Alternatively, the sensitivity of the noise to the parasitic inductance is low at small rise times and increases with longer rise times. Increasing the decoupling capacitance

TABLE I

Comparison of the peak-to-peak ground noise obtained at $t_r = 2\sqrt{(L_g C_d)}$ and the maximum noise for different parasitic ground network impedances.

$(I_{swi})_p$ mA	L_g (nH)	C_d (pF)	R_g (ohm)	R_d (ohm)	Ground noise at $t_r = 2\sqrt{L_g C_d}$ (mV)	Maximum ground noise (mV)	Error (%)
11.5	0.25	10	2.2	0.1	44	46.2	4.8
11.5	0.5	10	2.2	0.1	66.3	67.8	2.2
11.5	1	10	2.2	0.1	98.5	99.8	1.3
11.5	1	15	2.2	0.1	78.3	79.6	1.6
11.5	1	20	2.2	0.1	66.3	67.8	2.2
11.5	1	20	4	0.1	59.4	64.8	8.3
11.5	1	20	6	0.1	55.1	70.2	21.5
11.5	1	20	6	1	54	68.6	21.3
11.5	1	20	6	2	52.8	67.2	21.4

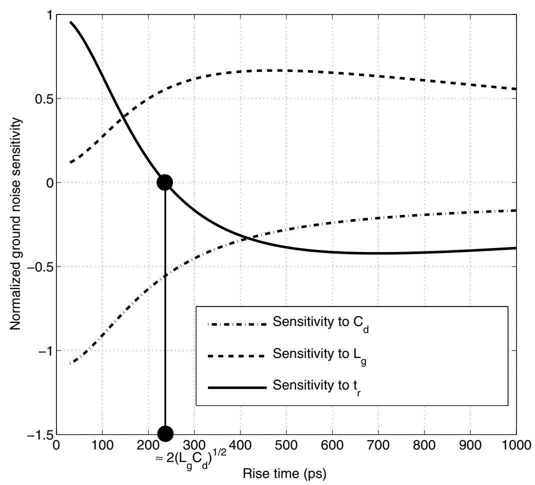

Fig. 5. Normalized sensitivity of the ground noise as a function of rise time when $(I_{swi})_p = 11.5\ mA$, $L_g = 1\ nH$, $C_d = 10\ pF$, $R_g = 2.2\ \Omega$, and $R_d = 0.1\ \Omega$

TABLE II

The effect of the decoupling capacitance on reducing the peak-to-peak ground noise. $L_g = 1\ nH$.

Rise time (ps)	$C_d = 10\ pF$	$C_d = 20\ pF$	Reduction
70	57.8	28.2	51.2%
$200 = 2\sqrt{L_g C_d}$	98.7	59.5	39.7%
400	91.1	67.7	25.7%
800	69.4	58.7	15.4%

is therefore effective in reducing the noise for $t_r \leq 2\sqrt{L_g C_d}$. Alternatively, reducing the parasitic inductance is effective for $t_r \geq 2\sqrt{L_g C_d}$. This behavior is due to the changing ratio of the capacitive and inductive currents with respect to the rise time, as shown in Fig. 2. The effect of the decoupling capacitance and parasitic inductance on the ground noise is listed, respectively, in Tables II and III for different rise times. At $t_r = 70\ ps$, doubling the decoupling capacitance reduces the noise by 51.2%, and only 25.7% when $t_r = 400\ ps$. Halving the parasitic inductance, however, reduces the noise by only 15.4% when $t_r = 70\ ps$, and 35.6% when $t_r = 400\ ps$. Note that the sensitivity to the rise time crosses over at zero when $t_r \approx 2\sqrt{(L_g C_d)}$, demonstrating the non-monotonic dependence, as described in Section III-A.

TABLE III

The effect of the parasitic inductance on reducing the peak-to-peak ground noise. $C_d = 10\ pF$.

Rise time (ps)	$L_g = 1\ nH$	$L_g = 0.5\ nH$	Reduction
70	57.8	48.9	15.4%
$200 = 2\sqrt{L_g C_d}$	98.7	67.7	31.4%
400	91.1	58.7	35.6%
800	69.4	48.2	30.5%

IV. Conclusions

The non-monotonic dependence of the power/ground noise on the rise time is shown for an inductive power distribution network with a decoupling capacitance. The power/ground interconnect is modeled as a series *RL* impedance. The decoupling capacitance is modeled as a capacitance in series with a resistance. The model captures the dependence of noise on the rise time with sufficient accuracy, as compared to SPICE simulations. The worst case rise time producing the maximum peak-to-peak noise is presented based on this model. The sensitivity of the noise on the decoupling capacitance and parasitic inductance is also investigated. A decoupling capacitance is shown to efficiently reduce the noise for $t_r \leq 2\sqrt{L_g C_d}$. Alternatively, reducing the parasitic inductance is effective for $t_r \geq 2\sqrt{L_g C_d}$.

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