

Optimal gamut volume design for three primary and multiprimary display systems

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ABSTRACT

Primary selection plays a fundamental role in display design. Primaries affect not only the gamut of colors the systems is able to reproduce, but also, they have an impact on the power consumption and other cost related variables. Using more than the traditional three primaries has been shown to be a versatile way of extending the color gamut, widening the angle view of LCD screens and improving power consumption of displays systems. Adequate selection of primaries requires a trade-off between the multiple benefits the system offers, the costs and the complexity it implies, among other design parameters.

The purpose of this work is to present a methodology for optimal design for three primary and multiprimary display systems. We consider the gamut in perceptual spaces, which offer the advantage of an evaluation that correlates with human perception, and determine a design that maximize the gamut volume, constrained to a certain power budget, and analyze the benefits of increasing number of primaries, and their effect on other variables of performance like gamut coverage.

Keywords: Primary Selection, Gamut Volume, Display Color Optimization

1. INTRODUCTION

The development of different technologies for color display devices, like DLP and LCD based on LED backlight, offers not only variety for the consumer market, but also significant design flexibility over prior technologies, like CRT phosphors. This flexibility allows design of systems with lower power consumption, higher dynamic range and wider gamut, etc. In all cases, an adequate selection of primaries is a fundamental step.

The use of a certain strategy for selecting the primaries depends on the system requirements and characteristics. Two dimensional approaches, where the characteristics of the display, such as gamut and coverage, are described in the CIE $x-y$ chromaticity diagram, have been commonly used for long time given their simplicity. A more complete description is obtained when the complete gamut is considered. In this sense, the volume of the display gamut in a perceptual space is a natural characterization of the gamut and a useful parameter to describe the performance of a specific configuration, and therefore a parameter to consider for optimization. The use of a perceptual space warrants that color differences and volumes correlate with the observer's perception. Several three dimensional-based strategies for selection of primaries have been proposed recently. One example¹ introduces a multi-objective optimization procedure, where a general framework for display comparison based on colorimetry model of the system, is also described. The selection of particular technologies can benefit the overall process, by providing a description of the primaries, usually in terms of parameters that relate to the physical characteristics of the system and its colorimetry properties.² Searching algorithms in the parameter space, also proposed in recent work,³⁻⁵ can be used to compare the performance of different designs and find trade-offs between objective functions.

In this paper we present a methodology for optimal primary selection. Using a general description of the primaries in terms of their spectral power distributions, we proposed a design methodology based on the maximization of the gamut volume in perceptual spaces. The methodology can be applied for different color technologies

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like, LED, LCD and DLP. The methodology is applied to find optimal instances of three primary systems, as well as optimal configuration in multi-primary displays. The paper is organized as follows. Section 2 presents models for display systems. A spectral model allows the representation of the primaries in terms of spectral parameters, while a colorimetric model facilitates the definition of useful concepts like gamut. Based on these representations, the optimization problem is introduced in Section 3, where the objective function and the constraints are defined. Section 4 presents some optimal configurations and discusses the results, while Section 5 summarizes the findings and future work.

2. DISPLAY MODELING

Most of the color displays available nowadays are built as the composition of two optical systems: a source of light and a filter layer. In this sense, the spectral power distribution of one of the primaries of the system, namely $p(\lambda)$, can be expressed as the multiplication of the spectra of a source of light, $s(\lambda)$, and the spectra of a filter or transmittance layer, denoted by $t(\lambda)$, i.e., $p(\lambda) = s(\lambda)t(\lambda)$. Although these spectral quantities change depending on the technology and the materials involved, in this work we approximate the spectra of the primaries by Gaussian functions.

The idea of the approximation is, first of all, to present a general framework that can be applied to different technologies, for instance, LED, LCD, DLP. The approximation is reasonable, specially with the increasing tendency of using narrow band sources of light or sharp filters, offering saturated primaries. As a second benefit, the approximation simplifies the computation and manipulation of the primaries.

For a system with K primaries, the spectra for the primary i , $1 \leq i \leq K$ can be approximated by the Gaussian function with parameters a_i, λ_i, σ_i in general as,

$$p_i(\lambda) = a_i \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp \left\{ -\frac{(\lambda - \lambda_i)^2}{2\sigma_i^2} \right\}. \quad (1)$$

The use of a normalized Gaussian in (1), although not required, simplifies the computation of optical power $Power_i$ for the primary i , defined as,

$$\begin{aligned} Power_i &= \int p_i(\lambda) d(\lambda) \\ &= a_i. \end{aligned} \quad (2)$$

Note also, that the approximation presented in (1) assumes that all primaries are independent of each other. In that sense, any spectrum emitted by the device can be expressed by a linear combination as,

$$\mathbf{f}(\lambda, \boldsymbol{\alpha}) = \sum_{i=1}^K \alpha_i p_i(\lambda) + \beta(\lambda) \quad 1 \leq i \leq K \quad (3)$$

where $\beta(\lambda)$ represents the black light spectrum for the system, and $\boldsymbol{\alpha}$ is the vector of relative intensities of the primaries for the combination, $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_K]$.

A vector representation of (3) in the additive color space CIEXYZ, can be expressed as,

$$\begin{aligned} \mathbf{f_P}(\boldsymbol{\alpha}) &= \sum_{i=1}^K \alpha_i \mathbf{p}_i + \boldsymbol{\beta} \\ &= \mathbf{P}\boldsymbol{\alpha} + \boldsymbol{\beta}, \end{aligned} \quad (4)$$

where \mathbf{p}_i , $\boldsymbol{\beta}$ and $\mathbf{f}(\boldsymbol{\alpha})$ are, respectively, the CIEXYZ tristimulus values for the i -primary $p_i(\lambda)$, the display black $\beta(\lambda)$, and the color determined by the spectrum emitted by the display when the relative amplitudes $\boldsymbol{\alpha}$ are applied to the primaries. The matrix \mathbf{P} is the primary matrix, $\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_K]$. Note that, it is implicit that each of the primaries depends on the selection of the spectral parameters. In cases when that dependency needs to be highlighted, the matrix of primaries will be denoted by $\mathbf{P}_{\mathbf{a}, \lambda, \sigma}$ as alternative notation, where $\mathbf{a} = [a_1, \dots, a_K]$, $\lambda = [\lambda_1, \dots, \lambda_K]$, $\sigma = [\sigma_1, \dots, \sigma_K]$ are the parameter vectors for the primaries.

2.1 Gamut and Gamut Volume for Display Systems

The gamut of a system, or the set of colors the device is able to reproduce, gives information about the performance of the display.

For a system with a matrix of primaries \mathbf{P} , the gamut in CIEXYZ color space, $\mathcal{G}_{\mathbf{P}}^{XYZ}$, is defined as,

$$\begin{aligned}\mathcal{G}_{\mathbf{P}}^{XYZ} &= \{\mathbf{f}_{\mathbf{P}}(\boldsymbol{\alpha}) | \boldsymbol{\alpha} \in [0, 1]^K\} \\ &= \{\mathbf{P}\boldsymbol{\alpha} + \boldsymbol{\beta} | \boldsymbol{\alpha} \in [0, 1]^K\}.\end{aligned}\quad (5)$$

The volume of the display gamut is a natural characterization of the gamut and a useful parameter to compare different configurations. The relevance of this parameter, increases when the color are described in a perceptual spaces. The use of a perceptual space warranties that color differences correlate with the assessment provided by an observer. Representation in perceptual spaces are obtained by means of nonlinear and space variant transformations, denoted in this paper by $\mathcal{F}(\cdot)$, from additive color spaces, like CIEXYZ. CIELAB and CIELUV are the two standards perceptual spaces mostly used. It has been shown that CIELAB exhibits some deficiencies, specially when representing colors with very low luminance.⁶ For this reason, we center this work on the use of CIELUV color space.

The gamut of color perception for a display systems with matrix of primaries \mathbf{P} is obtained by transforming the display gamut from an additive space,

$$\mathcal{G}_{\mathbf{P}}^{LUV} = \{\mathcal{F}(\mathbf{f}_{\mathbf{P}}(\boldsymbol{\alpha})) | \boldsymbol{\alpha} \in \mathcal{C}^K\}.\quad (6)$$

Thus, the volume The most general definition for gamut volume is given by,

$$V(\mathcal{G}_{\mathbf{P}}^{LUV}) = \int \int \int_{[L,u,v] \in \mathcal{G}^{LUV}} dL du dv.\quad (7)$$

Given the characteristics of the the function $\mathcal{F}(\cdot)$, the gamut for a color device in a perceptual space is usually a non-convex solid. However, the volume computation can be simplified by the use of adequate gamut representations developed in recent work,⁷ where several strategies to compute gamut volume in perceptual spaces are also proposed. Specifically, for multiprimary displays, the proposed gamut representations⁷ express the entire gamut volume in a unique, non-degenerate fashion, thereby, simplifying the computation of volume for multiprimary systems with arbitrary primaries/primary parameters.

3. CONSTRAINED OPTIMAL VOLUME DISPLAY DESIGN

We use the gamut volume as a metric to evaluate and compare display designs. The idea is to find the spectral parameters (A, λ, σ) of the primaries that maximize the volume of the gamut. Increasing the volume suggests, at an initial state, that more colors can be reproduced. To obtain meaningful designs additional constraints are also necessary, which we outline next.

Power Constraint: All designs are constrained to operate within a limited budget of power. Given that the computation of electrical power depends on the selected technology, we consider optical power as the constraint variable. In this sense, and based on the definition in (2), the maximum power consumed by a K -primary system is obtained by adding the optical power of individual primaries,

$$\begin{aligned}TotalPower &= \sum_{i=1}^K \int_{\lambda} p_i(\lambda) d\lambda \\ &= \sum_{i=1}^K a_i\end{aligned}\quad (8)$$

White Balance: The white point of the display, denoted $\mathbf{w_P}$, is a tristimulus value obtained by the maximum combination of all primaries, and represents an important parameter, since the eye always adapts to brightens stimuli, which in turn, allows to discern colors regardless ambient illumination. A white balance procedure is usually included for display system to ensure that the chromaticity coordinates of the white point, match the values of a certain standard reference, referred here as $\mathbf{w_r}$. Given a set of primaries, the white balance can be obtained by modifying the relative amplitude of the primaries. In this problem, we are interested in search for designs for which primaries are already balanced. To include this constraint in the optimization process, we define the vector function $g(\mathbf{P}) = \mathbf{w_{P,xy}} - \mathbf{w_{r,xy}}$, where $\mathbf{w_{P,xy}}$ and $\mathbf{w_{r,xy}}$ are the CIE x - y coordinates of the white point $\mathbf{w_P}$ and the white reference $\mathbf{w_r}$. A feasible matrix of primaries \mathbf{P} satisfies the constraint whenever $g(\mathbf{P}) = \mathbf{0}$.

Flare: Flare is a common characteristic in many of the display systems and therefore is worth considering. Flare by itself, may not represent a constraint to the system, but it may affect the selection of primaries by affecting the volume of the perceptual gamut.⁸ In this work, we consider two scenarios and evaluate the impact of including flare in the design process, by evaluating two different gamuts that differ one from the other, in the presence of the flare parameter β in the gamut model presented in (4). The first gamut, $\mathcal{G}_{1\mathbf{P}}^{LUV}$, is free of flare, while the second one, $\mathcal{G}_{2\mathbf{P}}^{LUV}$, is considered with to have a constant flare. The parameter β for the latter one, is chosen to be 1% of the white point of the configuration which primaries share the same bandwidth parameter (σ), same amplitude (a) and have a white point that matches the chromaticity of the white reference.

Problem formulation: Having defined the setting and the constraints, we can now formulate the problem of primary design as a constrained optimization problem, as follows,

$$\begin{aligned}
& \max_{\mathbf{a}, \lambda, \sigma} V(\mathcal{G}_{\mathbf{P}_{\mathbf{a}, \lambda, \sigma}}^{LUV}) \\
& \text{subject to: } g(\mathbf{P}_{\mathbf{a}, \lambda, \sigma}) = \mathbf{0} \\
& \sum_{i=1}^K a_i = 1 \\
& 0 \leq \mathbf{a} \\
& 380nm \leq \lambda_i \leq 680nm, \quad i = 1, 2, \dots, K \\
& 2nm \leq \sigma_i \leq 100nm, \quad i = 1, 2, \dots, K
\end{aligned} \tag{9}$$

The problem is not convex in general, which implies the presence of multiple local maxima across the solution space. To search for a global maximum, the differential evolution⁹ algorithm is applied. The idea is to determine a population of feasible points over the solution space, and in each iteration, their location is modified exploring the space toward an optimal solution. Though differential evolution was originally designed for unconstrained scenarios, different approaches to handle constraint have also been proposed.¹⁰⁻¹² The method has been successfully applied to a number of benchmark problems with performance competitive or better than the best known global optimization methods.⁹

4. RESULTS

The problem defined in (9) was solved for the particular case of three primary systems. The parameters for the optimal designs found for the two different gamuts $\mathcal{G}_{1\mathbf{P}}^{LUV}$ and $\mathcal{G}_{2\mathbf{P}}^{LUV}$, defined in Section 3, are described in Table 1.

For both cases, the final configurations provide intuitive solutions, generating clearly a red, a green, and a blue primaries, while covering an important section of the CIE uv diagram, as seen in Fig. 1a. The spectra for the primaries obtained are narrow-band, reaching the lower bound for the σ parameters. Despite the similarity in a two dimensional analysis, there are differences when considering the luminance. As Table 1 shows, the amplitudes for the first case are concentrated on the red primary, reducing the capability of producing high

luminance colors. In Fig. 1b, where the gamut in CIELUV space for the gamut $\mathcal{G}_{1\mathbf{P}}^{LUV}$ is plotted as a surface, while the mesh represent the configuration when applying $\mathcal{G}_{2\mathbf{P}}^{LUV}$. In a relative comparison for the gamut, both configurations generates a similar volume, giving some advantage to the gamut $\mathcal{G}_{1\mathbf{P}}^{LUV}$. However, when compared under an absolute criteria, the disadvantage of ignoring flare becomes evident.

The objective is to maximize the function $V(\mathcal{G}_{\mathbf{P}}^{LUV})$, as defined in (7). However, as the computation in perceptual spaces depends on the white point adaptation conditions, it is possible to obtain a configuration with very low luminance and still obtain a big gamut. The power constraint guarantee that at least one of the primaries has nonzero amplitude. This leads to some undesirable possibilities, like the one in which the power is concentrated in one or few primaries while the others use the necessary power to balance the white point. These kind of configurations tend to present poor performance when compared to other systems that are able to display brighter colors.

Figure 2 shows a comparison between the gamut $\mathcal{G}_{2\mathbf{P}}^{LUV}$ and some standard gamuts. It can be appreciated that gamut is able to cover a significant proportion of these standard gamuts.

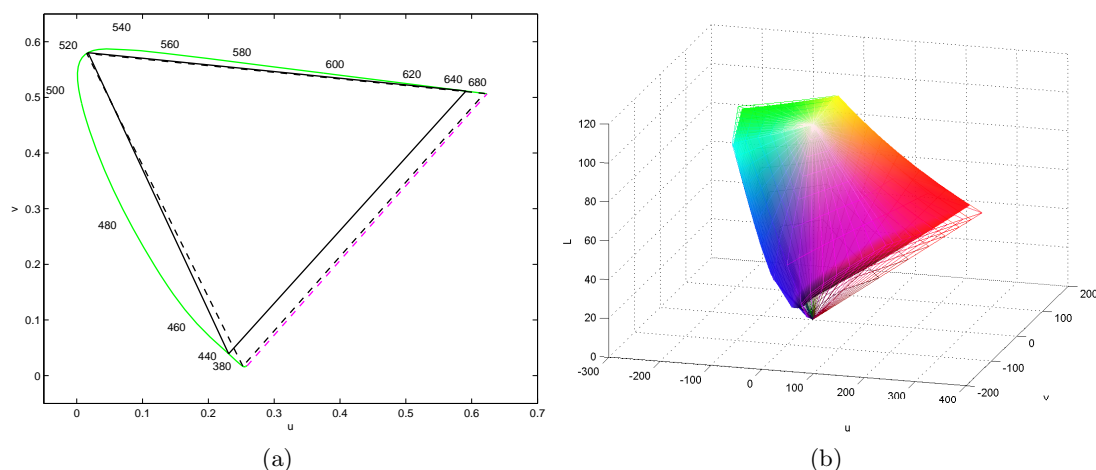


Figure 1: Gamut for optimal configurations $\mathcal{G}_{1\mathbf{P}}$ and $\mathcal{G}_{2\mathbf{P}}$ in 1a the CIEuv chromaticity diagram. 1b the CIELUV space, wherer $\mathcal{G}_{1\mathbf{P}}$ and $\mathcal{G}_{2\mathbf{P}}$ are represented by the surface and the mesh diagram, respectively.

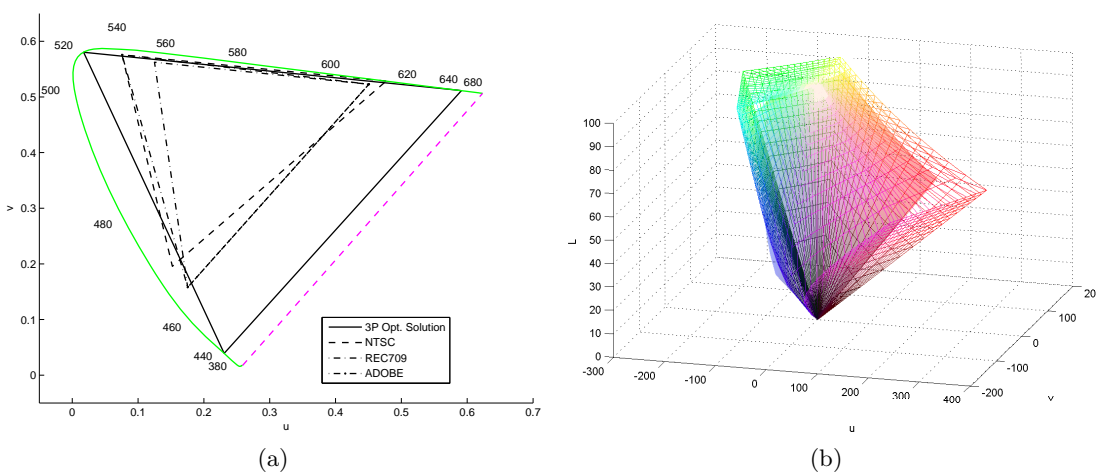


Figure 2: 2a Comparison between the gamut optimal gamut $\mathcal{G}_{2\mathbf{P}}$ and a set of standard gamuts in the the CIEuv chromaticity diagram. 2b A comparison between the NTSC gamut volume and $\mathcal{G}_{2\mathbf{P}}$ in the LUV space.

	\mathcal{G}_1			\mathcal{G}_2		
	P_1	P_2	P_3	P_1	P_2	P_3
λ	418.98	516.34	678.60	442.47	517.27	641.40
σ	2	2	2	2	2	2
a	0.0992	0.0701	0.8305	0.1682	0.3266	0.5052
Y_w	41.280			205.923		
$V(\mathcal{G}^{LUV})$	3.0120e6			2.9158e6		

Table 1: Table 1. Optimal designs for three primary systems.

	\mathcal{G}_1				\mathcal{G}_2			
	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4
λ	414.87	500.50	523.06	680.00	433.25	499.81	520.57	644.28
σ	2	2	2	2	2	2	2	2
a	0.1203	0.0399	0.0339	0.8058	0.1647	0.0908	0.2329	0.5116
Y_w	36.275				186.180			
$V(\mathcal{G}^{LUV})$	3.2402e6				3.0958e6			

Table 2: Table 2. Optimal designs for four primary systems.

The same methodology can be applied to multiprimary systems. A four primary selection is obtained by maximizing the gamut volume. Figure 3 shows the comparison of the resultant configuration with some standards gamuts. Once again the spectra of the primaries tend to be narrow-band, as depicted in the table 2. For the particular configuration obtained. Compared with the design for three primaries, the blue and red primaries are shifted toward the end of the spectrum, increasing the gamut around the violet region. The fourth primary in this case increase the colors around cyan.

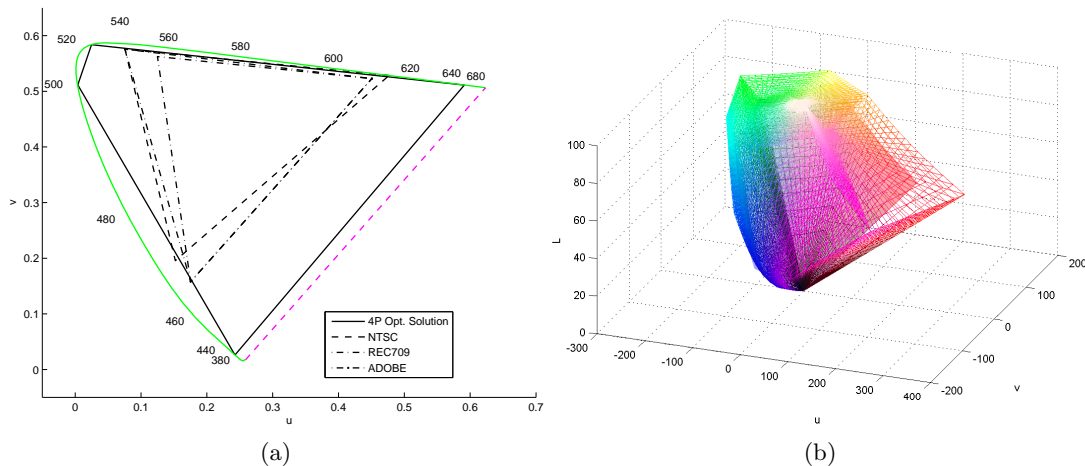


Figure 3: 3a Comparison between the gamut optimal gamut \mathcal{G}_{2P} and a set of standard gamuts in the the CIEuv chromaticity diagram. 3b A comparison between the NTSC gamut volume and \mathcal{G}_{2P} in the LUV space.

5. CONCLUSION

We presented a methodology for optimal primary selection in a display system. The process includes a spectral modeling of the system that can be applied for different display technologies, and a non linear constrained maximization of the gamut volume. Optimizing the gamut volume of a display yielded intuitive and expected designs. For the three primary case, optimal designs offer primaries that can clearly labeled as red, green and blue, and all of them reach (or get very close) the lower bound for the spectral bandwidth. A similar affirmation

applies for the multiprimary case, where the extra primary extends the coverage particular in regions around cyan colors.

The spectral model, as well as the constraints introduced in the document, were formulated in a general scenario, and therefore it is possible to adapt them by to address particular specifications of the selected technology, like the efficiency of LEDs of different wavelengths, and to incorporate considerations of minimal absolute white luminance, observer metamerism¹³ and coverage of surface colors datasets.^{14,15}

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