Signal Processing Methods in Color Calibration

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Abstract

With the increasing use of desktop color scanners for digitizing color images it has become desirable to obtain device independent color information from these scanners. In order to achieve this goal the scanner spectral sensitivity needs to be estimated. This paper describes the application of signal processing techniques to the problem of estimating the scanner sensitivity. Results obtained by applying the methods described to an actual commercial scanner are presented and the performance of two different techniques is compared.

1 Introduction

A color scanner outputs a three band image. Mathematically the process of scanning may be represented by a linear model, the value of each of the three bands at a pixel being given by

$$t_i = \int_{-\infty}^{\infty} f_i(\lambda)\,\theta(\lambda)\,r(\lambda)\,l(\lambda)\,d\lambda \;+\; \epsilon_i \qquad i = 1,2,3 \quad (1)$$

where $f_1(\lambda)$, $f_2(\lambda)$ and $f_3(\lambda)$ are the transmittances of the three color filters, $\theta(\lambda)$ is the product of the detector sensitivity and the optical path transmittance (comprising possibly of ultra-violet and infra-red filters), $l(\lambda)$ is the illuminant spectrum, $r(\lambda)$ is the reflectance spectrum of the pixel and ϵ_i is the measurement noise. In practice, the spectra in the equation above can be represented in terms of their samples, and the integral may be approximated by a summation. If samples are available at N equi-spaced wavelengths the scanning process can be approximated as

$$\mathbf{t} = \mathbf{M}^T \mathbf{L}\,\mathbf{r} + \boldsymbol{\epsilon} \qquad (2)$$

where $\mathbf{M}$ is an $N \times 3$ matrix which includes the effect of the filters, the optical path and the detector sensitivity, $\mathbf{L}$ is an $N \times N$ diagonal matrix representing the spectrum of the illuminant, $\mathbf{r}$ is the vector of reflectance samples, $\mathbf{t}$ is a 3×1 vector of tristimulus values and $\boldsymbol{\epsilon}$ is the 3×1 noise vector.

Typically in colorimetric work a sampling rate of 10nm has been used but recently [1] it has been noted that a 10nm sampling rate is inadequate for computations involving fluorescent illuminants which require a sampling rate of at least 2 nm for accurate colorimetric computations. Since the scanner in consideration employed a fluorescent lamp equispaced samples at 2 nm increments in the range 390 to 730 nm were used for all spectral quantities[1].

In Eqn. (2) the overall spectral sensitivity is given by the product $\mathbf{M}^T \mathbf{L}$. The fluorescent illuminant used in the scanner is shown in Fig. 1. The sharp spectral peaks make the problem of estimating this net sensitivity accurately rather difficult [2] and hence it is assumed that the illuminant spectrum is measured independently (which can be readily done in most cases). For the treatment in this paper it is convenient to write this equation in the form

$$\mathbf{t} = \mathbf{M}^T \mathbf{w} + \boldsymbol{\epsilon} \qquad (3)$$

where $\mathbf{w} = \mathbf{L}\,\mathbf{r}$ is radiant spectrum incident on the filters.

In order to characterize the scanner completely, one needs to know $\mathbf{M} = [\mathbf{m}_1\,\mathbf{m}_2\,\mathbf{m}_3]$. A direct measurement of M using narrow band reflectances is possible in theory but such a calibration would have a significant impact on the price of these low cost devices and is rarely done. A straightforward approach to *in situ* measurement of the scanner sensitivity is to scan a number of samples with known reflectance spectra and perform a least squares fit. Such an approach, however, encounters a serious practical problem since spectra of natural objects do not have sufficient dimensionality to yield a good estimate of $\mathbf{M}$. Typically, the matrix of radiant spectra $\mathbf{W} = [\mathbf{w}_1\,\mathbf{w}_2 \ldots \mathbf{w}_K]$ is highly ill-conditioned and has only

[1]Simulations show that even a 2 nm sampling can give rise to significant errors in computed tristimulus values and a truly accurate computation would use the summation in Eqn. (2) only for the continuous part with a pointwise term added for the spectral spikes in the illuminant.

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seven to eight significant singular values [3, 4, 15]. As a consequence, the least squares solution is highly sensitive to noise and yields poor estimates of $\mathbf{M}$ at noise levels typical in desktop scanners[2].

2 Principal Eigenvector Method

One way of reducing the sensitivity of the pseudoinverse solution to noise is to use only the singular vectors corresponding to the significant singular values in the solution. Consider the singular value decomposition of $\mathbf{W}$ [9]

$$\mathbf{W} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^t \tag{4}$$

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Sigma}_{r\times r} & \mathbf{0}_{r\times(K-r)} \\ \mathbf{0}_{(N-r)\times r} & \mathbf{0}_{(N-r)\times(K-r)} \end{bmatrix} \tag{5}$$

where r is the rank of W, U and V are orthogonal matrices consisting of the left and right singular vectors respectively, $\mathbf{0}_{m\times n}$ is an $m \times n$ matrix of all zeros and $\mathbf{\Sigma}$ is the diagonal matrix of the non-zero singular values $\{\sigma_i\}_{i=1}^r$

$$\mathbf{\Sigma} = \operatorname{diag}(\sigma_1, \sigma_2, \ldots \sigma_r) \tag{6}$$

$$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r \geq 0 \tag{7}$$

In terms of these vectors the "Principal Eigenvector" (PE) estimate that uses the P ($P \leq r$) most significant singular vectors is given by

$$\hat{\mathbf{m}}_j = \sum_{i=1}^{P} \frac{(\mathbf{v}_i^T \mathbf{t}_j)}{\sigma_i}\mathbf{u}_i \qquad j = 1, 2, 3 \tag{8}$$

This solution, is far less sensitive to noise than the least-squares solution. However, this method still suffers from several limitations. The physical situation affords considerable *a priori* knowledge, which the method fails to take into account. The quantities we are estimating are non-negative owing to their physical nature but the estimation scheme does not use this *a priori* information. More seriously, since the fluorescent lamp has sharp peaks in its spectrum at fixed locations the principal singular vectors of W (the matrix of radiant spectra) will also have sharp peaks at those locations. Since the estimate is a linear combination of these principal eigenvectors it will also have sharp spectral peaks at the same locations in spite of the fact that the true sensitivities would typically be smoother functions of wavelength.

3 The Method of Projections onto Convex Sets (POCS)

The problem of estimating the scanner sensitivity can alternately be formulated using set theory. Based on each constraint that the scanner sensitivity must satisfy, a constraint set may be defined in which the true value of the sensitivity must lie. Any element in the intersection of the constraint sets is called a feasible solution and may be used as an estimate of the sensitivity. Based on the physical nature of the problem, it can be said that the sensitivity function $\mathbf{m}_j$ (for each j) probably lies in the following constraint sets:

1. The set of non-negative vectors

$$A_n = \{\mathbf{y} \in \mathcal{R}^N | y_i \geq 0, \quad \forall 1 \leq i \leq N\} \tag{9}$$

2. The noise variance set

$$A_\epsilon = \{\mathbf{y} \in \mathcal{R}^N | \parallel \mathbf{t}_j - \mathbf{W}^T \mathbf{y} \parallel^2 \leq \nu\} \tag{10}$$

where the value of ν is usually set to $N\,\sigma_\epsilon^2$

3. The noise outlier sets

$$A_o^i = \{\mathbf{y} \in \mathcal{R}^N | \quad |t_{j_i} - \mathbf{W}_i^T \mathbf{y}| \leq \xi\} \quad i = 1, 2, \ldots K \tag{11}$$

where $\xi = 3\,\sigma_\epsilon$ is used for Gaussian noise.

Additionally, it is known that the sensitivities $\{\mathbf{m}_j\}_{j=1}^3$ are continuous (smooth) functions of wavelength. This can be incorporated in the estimation process by placing a bound on the second order difference of the components of $\mathbf{m}_j$ (for each j). Let $\mathbf{h} = (1, -2, 1)^T$ represent the Laplacian filter impulse response. The filtered output can then be represented as [14]

$$\mathbf{f}_j = \mathbf{H}\,\mathbf{m}_j \tag{12}$$

where $\mathbf{H}$ represents the convolution operator for convolution kernel $\mathbf{h}$. The set of smooth spectra can then be defined in terms of an upper bound on the energy in the filtered output

$$A_s = \{\mathbf{y} \in \mathcal{R}^N | \parallel \mathbf{H}\,\mathbf{y} \parallel^2 \leq \mu\} \tag{13}$$

where $\mu > 0$ is suitably chosen so as to impose the desired degree of smoothness.

The sets defined here are all closed, convex sets. Hence, a point in the intersection can be found by the method of successive projections, i.e., starting from any arbitrary point in $\mathcal{R}^N$ a point in the intersection of these sets can be determined by successively projecting onto each of them. This is called the method of Projections Onto Convex Sets (POCS). Since the sets are closed and convex, the iterative process of successive projections is guaranteed to converge to a point in the intersection provided the intersection is non-empty [10, 11]. If the sets have been defined properly and the model is accurate, the fact that the measurements arise from a physical experiment implies that to a high degree of probability the intersection of the constraint sets is non-empty and hence the algorithm will converge.

POCS is a powerful estimation technique that combines *a priori* information with the measurements to obtain the estimates. Since estimates obtained using POCS conform to all the known constraints that the true vector obeys it is expected that the estimates will be better than those obtained by other methods. It may also be noted that if the intersection of the constraint sets is non-empty it will rarely be a singleton and hence the POCS estimate is non-unique. In particular, the POCS estimate can depend considerably on the initial point chosen to start the iterations.

4 Experimental Results

The two estimation techniques described above were applied to the calibration of an HP ScanjetIIc scanner. The spectrum of the fluorescent illuminant was measured directly by taking the lid off. This is shown is shown in Fig. 1

For making measurements a Kodak Q60 target having 228 different color patches was used. The spectra of these were measured using a spectraphotometer. The target was then scanned on the scanner[2] and the values of pixels within each patch were averaged to obtain the device tristimuli.

For POCS it was assumed that the filter function under consideration is known to be red, green or blue. Accordingly, the initial estimates were taken to be nearly rectangular functions with transmittance windows positioned approximately in the red, green or blue region.

For the reflectance vectors corresponding to the Q60

[2] The scanner employs an internal matrixing of the data to obtain tristimulus values corresponding to the NTSC primaries. However, such a matrixing would invalidate the positivity constraint that we employ in POCS and hence raw device data was used in this work.

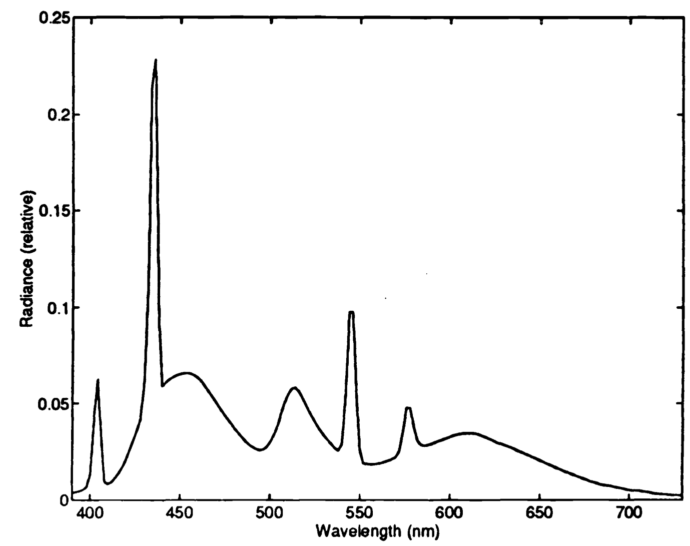

Figure 1: Spectrum of Illuminant used in Simulations

target, the singular values of the reflectance matrix drop sharply after the first few singular values and values beyond the 6^{th} are negligibly small as compared to the first singular value[3]. Hence, in the principal eigenvector method $P = 6$ was used.

The estimates obtained for the sensitivity functions of the three color bands are shown in Figure 2 for the PE method and in Figure 3 for POCS. The parts (a), (b) and (c) of each of these figures show the results for the red, green and blue filters respectively. It is interesting to note the differences between the PE and POCS estimates. Since the illuminant has sharp peaks in its spectrum the left singular vectors of $\mathbf{W}(= \mathbf{L}\ \mathbf{R})$ are not smooth. Therefore, the estimates of $\{\mathbf{m}_j\}_{j=1}^{3}$ obtained by the PE method (see Eqn. (8)) also exhibit sharp peaks. However, since POCS imposes an explicit smoothness constraint the POCS estimates are smooth. Since the POCS estimates also meet the other known constraints on the sensitivity, from a physical validity standpoint they are much better than the PE estimates. The PE method provides no natural way for incorporating the fact that the sensitivity functions are smooth into the estimation process and therefore yields poor estimates. Using a smooth basis set and attempting to estimate the filter sensitivities in the span of the smooth set also does not yield good results. It can also be seen that the PE estimates fail to meet other physical criteria required of

[3] The nature of the results does not change if any integer between 5 and 8 is used instead of 6.

transmittances as they becomes negative in certain region of the spectrum.

In order to quantify the accuracy of the estimates, mean squared tristimulus errors between the actual model tristimuli and the tristimuli resulting from the estimated sensitivity functions were computed. For the computation two different sets of reflectances were used. The first set consisted of 120 Dupont paint chips with relatively smooth spectra and the second used 12 Color and Interchange (CNI) standard color chips, which have rather sharp spectral curves. Note that the test reflectance sets are different from the reflectances used for determining the sensitivity.These reflectances were used in the model of Eqn. (2) with the noise term set to zero. For each data set, mean squared error (MSE) between the values predicted by the model of Eqn. (2) and the measured values was obtained by averaging over the entire set of reflectances. This was done for each of the three filters for both the POCS and PE estimates. The resulting mean squared errors are summarized in Table 1 and Table 2 for the two sets of test spectra. From the tabulation it can be seen that the two estimates give nearly the same performance as far as prediction of device tristimulus values is (concerned for both data sets).

One can also note that the estimates do reasonably well over the first data set since the principal eigenvectors of the Q60 reflectances cover this set well. The sharp spectral transitions in the reflectances of the CNI chips make them lie well outside the span of the principal eigenvectors of the Q60 reflectances and hence this data set gives much poorer performance.

Overall the MSE values tend to be on the higher side. The average measurement variance over uniform patches was found to be around 36 dB for each of the tristimulus values. Hence the high MSE values cannot be explained in terms of the device variation alone. Possible causes for this poor performance include scanner nonlinearity [8] and noise in the measured reflectance spectra. Evidence of scanner non-linearity was seen in the regions in which the device tristimuli were high in the form of unduly high variation over uniform patches. Simulations carried out using a fluorescent illuminant model with measured reflectances indicate that noise in the measured reflectance can be greatly amplified by the sharp spikes in the fluorescent illuminant (consider the scale of the peaks in Fig. 1 in relation to the continuous background) and this would partly explain why the estimates of tristimulus values are poor. Additionally, one can see that the estimated sensitivities do not drop down to zero beyond the range over which they have been estimated. Thus the truncation of the end regions of the blue and red channel sensitivities would also contribute to the error.

Table 1: Mean Squared Error for the Tristimulus Values Obtained from the PE and POCS estimates over the Dupont data set.

	Mean Squared Error (dB)		
Estimate	*Red*	*Green*	*Blue*
PE	-25.97	-25.53	-20.60
POCS	-26.44	-23.73	-22.86

Table 2: Mean Squared Error for the Tristimulus Values Obtained from the PE and POCS estimates over the CNI chip data set.

	Mean Squared Error (dB)		
Estimate	*Red*	*Green*	*Blue*
PE	-19.75	-14.77	-11.98
POCS	-19.78	-13.09	-14.79

5 Conclusions

This paper looked at the performance of two estimation methods, viz., PE and POCS, applied to the problem of color scanner characterization. The two methods were applied to the calibration of a scanner and the resulting estimates were evaluated from a physical validity and prediction accuracy standpoint. The method of POCS exploits the *a priori* information to give better estimates of the scanner sensitivities than the PE method.

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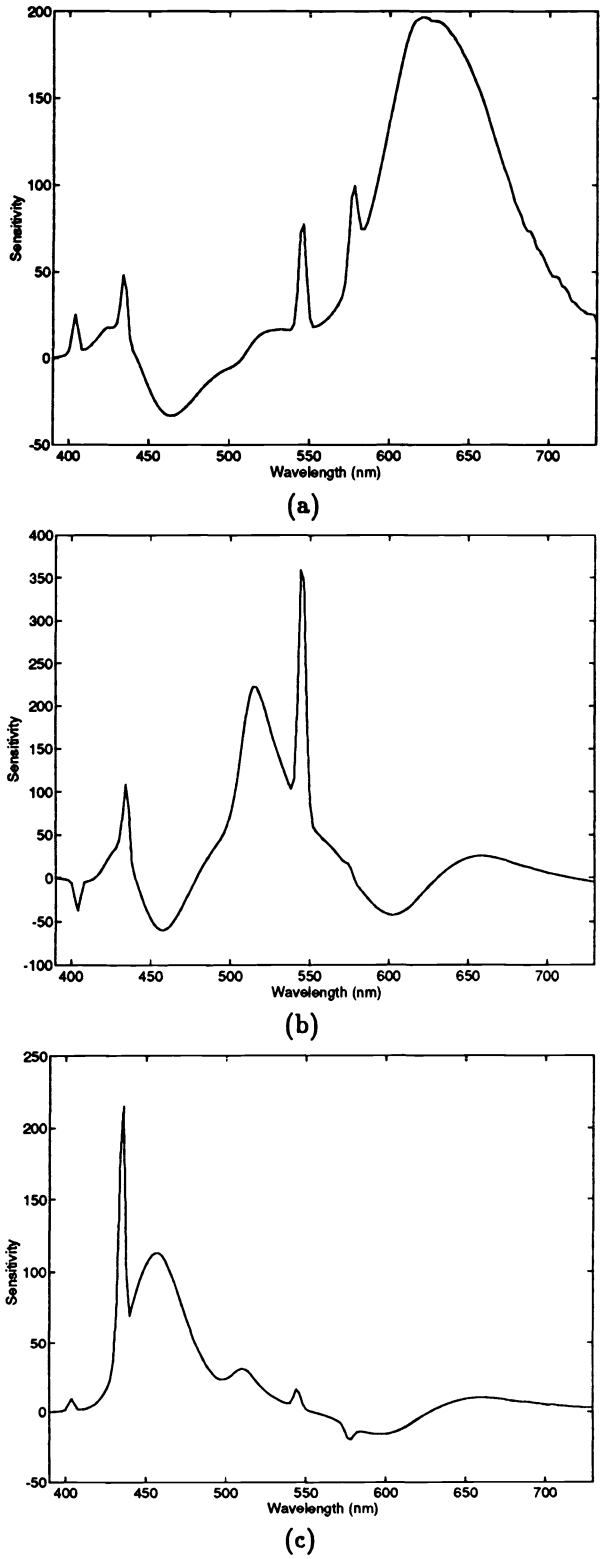

Figure 2: PE Estimate (a) Red (b) Green (c) Blue

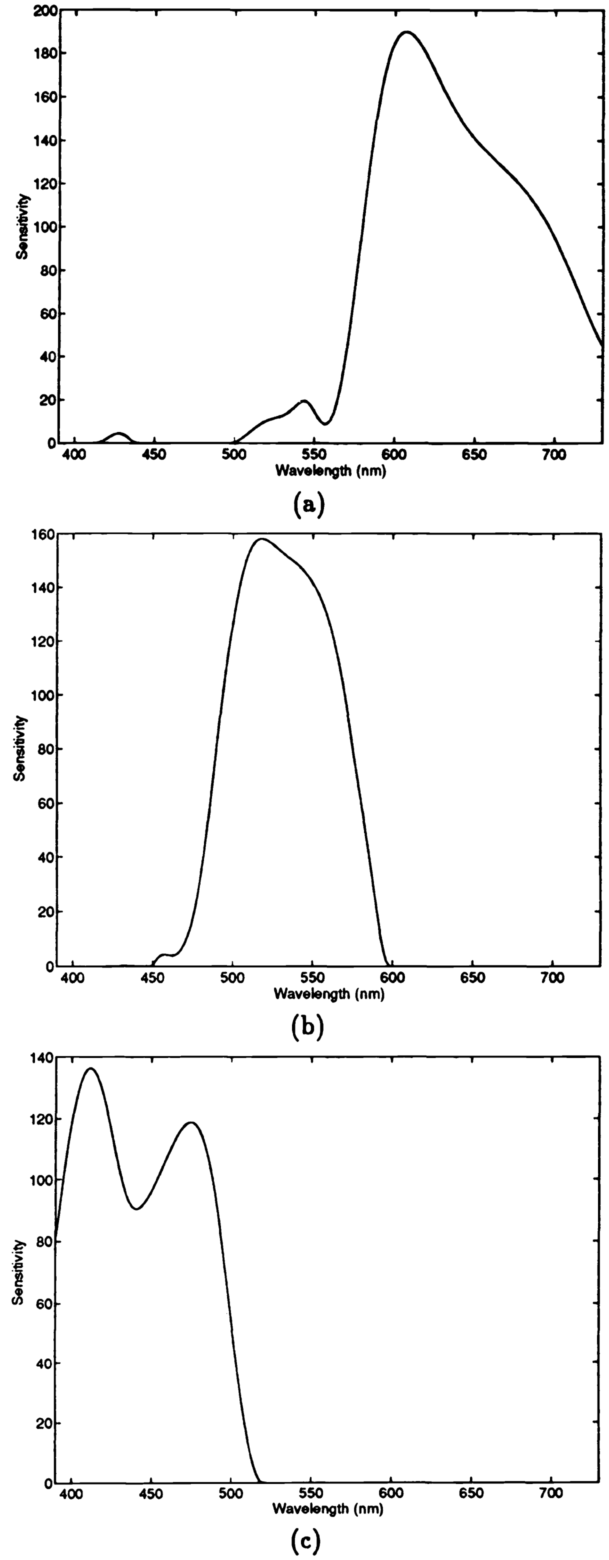

Figure 3: POCS Estimate (a) Red (b) Green (c) Blue

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