COMPARISON OF MEASURES OF GOODNESS OF SETS OF COLOR SCANNING FILTERS

H. J. Trussell, G Sharma, P. Chen and S. A. Rajala

Electrical & Computer Engineering Dept.
North Carolina State University, Raleigh, NC 27695-7911

ABSTRACT

A recently introduced measure of goodness which measures the quality of an arbitrary number of scanning filters is compared to a color quality factor used in the color printing industry. The results show that for high values of both measures the new measure is a better predictor of the color fidelity of filter sets.

1. INTRODUCTION

The color of an object is specified by its CIE tristimulus values:

$$t_i = \alpha \int_{-\infty}^{\infty} a_i(\lambda) l(\lambda) r(\lambda) d\lambda \quad i = 1,2,3 \tag{1}$$

where α is a normalizing constant, $\{a_1(\lambda), a_2(\lambda), a_3(\lambda)\}$ are the CIE color matching functions, $l(\lambda)$ is the spectral radiance of the viewing illuminant and $r(\lambda)$ is the reflectance of the object.

The summation can be expressed in matrix notation as:

$$\mathbf{t} = \alpha \mathbf{A}^T \mathbf{L} \mathbf{r} \tag{2}$$

where $\mathbf{A}$ is an $N \times 3$ matrix of the CIE color matching functions, $\mathbf{L}$ is an $N \times N$ diagonal matrix representing the spectrum of the illuminant, $\mathbf{t}$ is a 3×1 vector of the tristimulus values and $\mathbf{r}$ is the $N \times 1$ vector of the reflective spectrum of the object.

The desired tristimulus values are estimated by

$$\mathbf{t} = \mathbf{A}_L^T \mathbf{r} \approx \mathbf{B} \mathbf{M}^T \mathbf{O} \mathbf{D} \mathbf{L}_0 \mathbf{r} \tag{3}$$

where $\mathbf{A}_L = \mathbf{LA}$ combines the color matching functions and the viewing illuminant, $\mathbf{M}$ is the filter set, $\mathbf{L}_0$ is the diagonal matrix whose elements define the instrument illumination, $\mathbf{D}$ is the diagonal matrix whose elements define the detector sensitivity, $\mathbf{O}$ is the diagonal matrix whose elements represent the transmission of the optical path and $\mathbf{B}$ is the $3 \times K$ transformation to obtain the estimate of the CIE tristimulus values under illuminant $\mathbf{L}$.

The approximation is a result of the fact that the scanning filters, together with the other instrument responses, may not span the space defined by the color matching functions and the viewing illuminant. The measures of goodness attempt to measure the quality of the approximation without simply testing the filter set on a known ensemble of data. The advantage of an easily computed measure is its use in optimization programs for filter design.

2. MEASURES OF GOODNESS

The classical measure of goodness is the Neugebauer measure or Q-Factor. If $\mathbf{m}$ represents a color filter and $P_V(\mathbf{m})$ its orthogonal projection onto the range space of $\mathbf{A}_L$, denoted $R(\mathbf{A}_L)$, the q-factor of $\mathbf{m}$ is defined as:

$$q(\mathbf{m}) = \frac{||P_V(\mathbf{m})||^2}{||\mathbf{m}||^2} \tag{4}$$

where $||.||$ is the 2-norm in N-dimensional vector space. Notice that

$$0 \leq q(\mathbf{m}) \leq 1 \tag{5}$$

and the closer the value of q($\mathbf{m}$) to unity, the 'better' the color scanning filter $\mathbf{m}$. A major disadvantage of the q-factor is that it is designed to be used with only a single filter.

A generalization of the Q-factor was developed to handle a set of filters [2]. This measure considers the distance between subspaces and can handle a problem of any dimensionality. The ν measure is defined by

$$\nu(\mathbf{V}, \mathbf{M}) = \frac{\sum_{i=1}^{\alpha} \lambda_i^2(\mathbf{G}^T \mathbf{N})}{\alpha} \tag{6}$$

where $\lambda_i(\mathbf{G}^T\mathbf{N})$ denotes the i^{th} singular value of $(\mathbf{G}^T\mathbf{N})$, α is the dimension of the color space, usually three, $\mathbf{G}$ is an orthonormal basis for $R(\mathbf{M}^T\mathbf{O}\mathbf{D}\mathbf{L}_0)$ and $\mathbf{N}$ is an orthonormal basis for $R(\mathbf{A}_L)$.

A color quality factor (CQF) which has been used in industry is defined by measuring how well the color matching functions defined by $\mathbf{A}_L$ can be fit using the basis vectors defined by $\mathbf{M}$. This measure can be shown to be defined by

$$\tau(\mathbf{A}_L, \mathbf{M}) = \min_i \left\{ \frac{\|\mathbf{P}_G \mathbf{a}_i\|^2}{\|\mathbf{a}_i\|^2} \right\}, \tag{7}$$

where $\mathbf{P}_G$ is the orthogonal projection operator on the the space defined by $\mathbf{M}^T\mathbf{ODL}_0$.

3. COMPARISON OF MEASURES

If the performance of the filters were based on error in the CIEXYZ space the ν measure be the obvious winner since it can be formulated as minimizing a weighted projection error. However, the common measure of color fidelity is distance in the CIELab space which is a nonlinear transformation of the CIEXYZ space. Because of this the behavior of the measures is not easily seen. Since all of the transformations are continuous, both measures will predict zero error for a measure of unity. However, other than that rather general statement little can be said from an analytical point.

In order to test the predictive capabilities of the measures to non-perfect filter sets a large number of sets was needed. This was generated by using parameterized mathematical filters. The parameters were randomly varied so the deviation from a perfect set could be controlled. This allowed us to see how the measures worked over a wide range. A graph of the measures versus the average deltaE error is shown in Figure 1. The mean square fitting error for the average delta E for ν and τ measures is 0.3917 and 0.5485 respectively. This confirms the advantage of the ν measure. The fitting error for the maximum delta E are even more in favor of the measure (8.8677 vs. 23.8004). A practical application is to choose the best filter sets from a combination of commercial filters. Figure 2 shows the measures versus delta E plots for the twenty best filter set obtained using each measure.

As the values decrease the variation of the two measures increases. This would indicate the reliability of the measures for predicting accuracy decreases as the measure decreases. For example, if filters had to be chosen from a set of filters where the measure was above 0.96 in ν, the choice of measure is inconsequential.

4. CONCLUSION

For the highest values, the Vora-Trussell (ν) measure gives a smaller variation in Delta E than the CQF (τ). This indicates that for use in filter design for accurate color scanning the (ν) measure is preferred. The use of the measures in a practical problem confirms this conclusion.

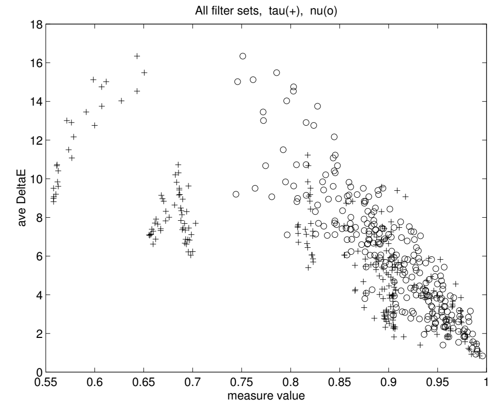

Figure 1: Measure Values vs. Delta E

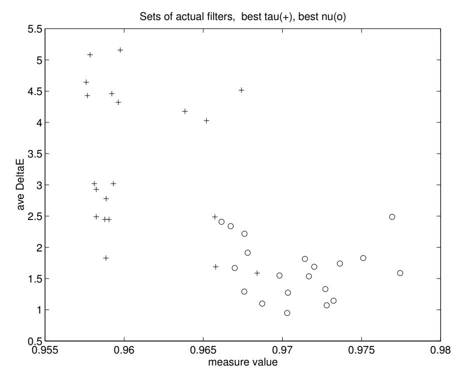

Figure 2: Highest 20 Measure Values vs. Delta E

5. REFERENCES

[1] Neugebauer, H. E. J., "Quality Factor for Filters Whose Spectral Transmittances are Different from Color Mixture Curves, and its Application to Color Photography", Journal of the Optical Society of America, Vol. 46, No. 10, pp 821-824, Oct 1956.

[2] P. L. Vora and H. J. Trussell, "Measure of Goodness of a Set of Colour Scanning Filters," J. of Optical Soc. of Am., Vol. 10, No. 7, pp. 1499-1508, July 1993.