

Optimal Filter Design for Multi-illuminant Color Correction

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Abstract

We address the design of optimal scanning filters for applications where colorimetric information under multiple viewing illuminants is desired from measurements of an object with a single color scanner. Objective criteria accounting for colorimetric accuracy and measurement noise are introduced and procedures for designing realizable filters using these criteria are presented.

1 Introduction

It is well known that the colorimetry of a reflective object is dependent on the viewing illuminant. In many applications, color under several different viewing-illuminants must be estimated from measurements obtained with a single device. Examples of such applications include: image scanning and printing systems, where a reproduction of an original may be desired under several different viewing conditions; and textile color measurements, where fabric from different lots may be used in the fabrication of a garment only if the colors match under the commonly used illumination sources.

In the absence of measurement noise, a three channel scanner can be used to obtain colorimetric information under a single viewing-illuminant, provided the scanner channel sensitivities are linear combinations of the viewing-illuminant color-matching-function products. This is the well known Luther-Ives condition [1, 2]. A straightforward generalization of the Luther-Ives condition to multiple viewing illuminants would require $3K$ channels for colorimetry under K illuminants. Spectrophotometric measurements provide an alternative method for obtaining color information under any viewing illuminant. In typical applications, the large number of channels required for either of these approaches makes them expensive and impractical. In addition, in the presence of noise, filters satisfying the Luther-Ives condition are not necessarily optimal [3].

In this paper, the problem of obtaining colorimetric information under multiple viewing illuminants from noisy scanner measurements is formulated as an estimation problem. A linear minimum mean-squared-error (MSE) estimator is used for the estimation of color tristimuli. Using a numerical optimization procedure, color scanning filter sets containing between 3 and 7 filters are designed so as to min-

imize the estimation error. The problem formulation used here is identical to that used earlier by Vrhel et al. [4], where a sub-optimal solution to the problem was also presented. However, the approach presented here determines the optimal solution by transforming the problem into a simpler optimization problem. Results in this paper demonstrate significant improvement over the sub-optimal solution.

2 Scanner Colorimetry

Typical color scanners record spectral information by filtering the light reflected from the scanned object into spectral-bands and recording the light energy in each band. Most color spectra can be represented accurately by equi-spaced samples over the visible range from 400 to 700 nm with a sampling interval of 10 nm. Thus each spectrum will be represented here as a $N = 31$ component vector. If a K channel scanner is used, the measurement of an object whose reflectance is specified by the N -vector \mathbf{r} can be algebraically represented as [4]

$$\mathbf{t}_s = \mathbf{M}^T \mathbf{L}_s \mathbf{r} + \boldsymbol{\eta} = \mathbf{G}^T \mathbf{r} + \boldsymbol{\eta}, \quad (1)$$

where \mathbf{t}_s is a $K \times 1$ vector of scanner measurements, \mathbf{L}_s is the $N \times N$ diagonal matrix with samples of the scanner-illuminant spectrum along the diagonal, $\boldsymbol{\eta}$ is the $K \times 1$ measurement noise vector, $\mathbf{G} = \mathbf{L}_s \mathbf{M}$, and $\mathbf{M} = [\mathbf{m}_1, \mathbf{m}_2, \dots, \mathbf{m}_K]$ is the $N \times K$ matrix of scanner filter transmittances, where \mathbf{m}_i represents the spectral transmittance of the i th filter (including detector sensitivity and the transmittance of the scanner optical path).

The color of an object, under a given viewing illuminant, is specified by its CIE XYZ tristimulus value [5]. If there are J viewing illuminants, in a manner analogous to the scanner measurements, the tristimuli of the object with spectral reflectance \mathbf{r} can be written as

$$\mathbf{t}_i = \mathbf{A}^T \mathbf{L}_i \mathbf{r} = \mathbf{A}_{\mathbf{L}_i}^T \mathbf{r}, \quad i = 1, 2, \dots, J; \quad (2)$$

where \mathbf{t}_i is the 3×1 vector of CIE XYZ tristimulus values under the i^{th} viewing-illuminant, \mathbf{A} is the $N \times 3$ matrix of CIE XYZ color-matching functions [5], \mathbf{L}_i is the $N \times N$ diagonal matrix with samples of the i^{th} viewing-illuminant spectrum along the diagonal, and $\mathbf{A}_{\mathbf{L}_i} = \mathbf{L}_i \mathbf{A}$.

Colorimetric information about the object is determined from the scanner measurements by estimating the tristimulus values. Using a linear estimator, the estimates

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can be written as $\hat{\mathbf{t}}_i = \mathbf{B}_i \mathbf{t}_s$, where \mathbf{B}_i is an $3 \times K$ matrix. The average magnitude of “color error” in the estimate $\hat{\mathbf{t}}_i$ in comparison with the true color \mathbf{t}_i can be used as an error metric for quantifying the scanner performance under the transformations specified by the \mathbf{B}_i . Different color spaces can be used in the computation of the “color error”. For several of these, the mean squared color error can be expressed as $\sum_{i=1}^J E\{\|\mathcal{F}(\mathbf{t}_i) - \mathcal{F}(\hat{\mathbf{t}}_i)\|^2\}$, where $E\{\cdot\}$ denotes the expectation (over the ensemble of objects to be scanned and the measurement noise) and $\mathcal{F}(\cdot)$ denotes the transformation from the CIE XYZ space into the appropriate color space. The mean squared color error defined above depends on the scanner sensitivity \mathbf{G} and the transformations $\{\mathbf{B}_i\}_{i=1}^J$. If the optimal transformations that minimize the mean-squared color error are used, the error represents the best performance achievable by the scanner, and the scanner sensitivity \mathbf{G} can then be designed to minimize this error.

Ideally, in order to have good agreement with color perception, a uniform color space such as CIELAB [5] should be chosen for the computation of the color error. Due to the nonlinear nature of the transformation $\mathcal{F}(\cdot)$ for typical uniform color spaces, such a choice does not lend itself to further analysis without simplifying approximations. A local linearization of $\mathcal{F}(\cdot)$ can be utilized to obtain closed-form approximations for the optimal transformations \mathbf{B}_i and the mean squared color error [6, 7]. However, the cost of computing the mean squared color error in this locally linearized CIELAB space is considerably higher [7] than the corresponding computation in a tristimulus space (i.e., a color space for which the transformation $\mathcal{F}(\cdot)$ is a linear 3×3 transformation). Since the major computational cost is incurred in the optimization of the scanner sensitivity, a hybrid approach is considered here. The optimization is carried out to minimize the MSE in a tristimulus space and the optimal transformations are then determined so as to minimize the MSE in linearized CIELAB space.

Since the CIE XYZ tristimulus values are highly correlated, and because significant magnitude differences can exist between different illuminants, each of the illuminant color-matching matrices, $\{\mathbf{A}_{\mathbf{L}_i}\}_{i=1}^J$, is orthonormalized to obtain $\{\mathbf{A}_{\mathbf{L}_i} \mathbf{F}_i\}_{i=1}^J$. The vector $\mathbf{F}_i^T \mathbf{A}_{\mathbf{L}_i} \mathbf{r}$ then represents a tristimulus in an orthogonal tristimulus space.

The problem of scanner design is now formulated as an optimization problem, where the color filter/recording illuminant matrix, \mathbf{G} , is chosen so as to minimize the MSE in the orthogonal tristimulus space,

$$\epsilon = \sum_{i=1}^K E\{\|\mathbf{F}_i^T (\mathbf{B}_i \mathbf{t}_s - \mathbf{A}_{\mathbf{L}_i} \mathbf{r})\|^2\}. \quad (3)$$

If the noise $\boldsymbol{\eta}$ is assumed to be uncorrelated with the signal $\mathbf{G}^T \mathbf{r}$, then the optimal \mathbf{B}_i ’s are readily determined, and it can be seen that the linear minimum MSE (LMMSE) can be written as $\epsilon_{LMMSE} = \alpha - \tau(\mathbf{G})$, where

$$\alpha = \text{tr}(\mathbf{S} \mathbf{S}^T \mathbf{K}_{\mathbf{r}}) \text{ and}$$

$$\tau(\mathbf{G}) = \text{tr} \left(\mathbf{S} \mathbf{S}^T \mathbf{K}_{\mathbf{r}} \mathbf{G} [\mathbf{G}^T \mathbf{K}_{\mathbf{r}} \mathbf{G} + \mathbf{K}_{\boldsymbol{\eta}}]^{-1} \mathbf{G}^T \mathbf{K}_{\mathbf{r}} \right), \quad (4)$$

where $\mathbf{S} = [\mathbf{A}_{\mathbf{L}_1} \mathbf{F}_1, \mathbf{A}_{\mathbf{L}_2} \mathbf{F}_2, \dots, \mathbf{A}_{\mathbf{L}_J} \mathbf{F}_J]$, $\mathbf{K}_{\mathbf{r}}$ is the correlation matrix of the reflectance spectra, $\mathbf{K}_{\boldsymbol{\eta}}$ is the correlation matrix of the noise and $\text{tr}(\cdot)$ denotes the trace operator. Since the sensor measurements are performed independently on the K channels, it will be assumed that $\mathbf{K}_{\boldsymbol{\eta}} = \sigma^2 \mathbf{I}$, where σ^2 denotes the variance of the noise in the individual measurement channels.

It is clear that ϵ_{LMMSE} is minimized if the filter/illuminant matrix \mathbf{G} is chosen so as to maximize $\tau(\mathbf{G})$. In practice, the recording device is subject to additional limitations of illuminant intensity and integration time. In order to incorporate these, it is assumed, as in [4], that the expectation of the total signal power is constrained to be a finite positive number, κ . Mathematically, this assumption is stated as $E\{\|\mathbf{G}^T \mathbf{r}\|^2\} = \kappa$. Since this is a nonlinear constraint, it is not readily incorporated in the optimization.

3 Constraint Simplification

Instead of the absolute quantities κ and σ^2 it is more useful to specify the signal to noise power ratio (SNR) defined as the ratio $\kappa/\sigma^2 \equiv \Gamma$ for the design. By eliminating σ^2 , the filter design problem can be stated as

$$\max_{\mathbf{G}} f(\mathbf{G}) \quad \text{subject to} \quad \mathbf{G}^T \geq \mathbf{0}, \text{tr}(\mathbf{G}^T \mathbf{K}_{\mathbf{r}} \mathbf{G}) = \kappa, \quad (5)$$

where $f(\mathbf{G}) \equiv$

$$\text{tr} \left(\mathbf{S} \mathbf{S}^T \mathbf{K}_{\mathbf{r}} \mathbf{G} \left[\mathbf{G}^T \mathbf{K}_{\mathbf{r}} \mathbf{G} + \frac{\text{tr}(\mathbf{G}^T \mathbf{K}_{\mathbf{r}} \mathbf{G})}{\Gamma} \mathbf{I} \right]^{-1} \mathbf{G}^T \mathbf{K}_{\mathbf{r}} \right)$$

Note that the objective function is invariant to a scaling of \mathbf{G} . Hence, if the solution to this problem is denoted as $\mathbf{G}_{opt}(\kappa, \Gamma)$ then the solution to the corresponding problem where the signal power is scaled by a positive constant α is simply $\mathbf{G}_{opt}(\alpha\kappa, \Gamma) = \sqrt{\alpha} \mathbf{G}_{opt}(\kappa, \Gamma)$. Hence given a solution to the problem,

$$\max_{\mathbf{G}} f(\mathbf{G}) \quad \text{subject to} \quad \mathbf{G}^T \geq \mathbf{0}. \quad (6)$$

then the solution to problem (5) can be immediately found by scaling with a positive scalar to satisfy the constraint $\text{tr}(\mathbf{G}^T \mathbf{K}_{\mathbf{r}} \mathbf{G}) = \kappa$.

Since (6) involves only a nonnegativity constraint, it is more readily handled by numerical optimization programs. If the covariance matrix of the reflectance spectra and the parameters of the scanning illuminant in \mathbf{S} are known, the expressions for the function and the gradient can be readily used in gradient-projection optimization schemes [8] to determine the (locally) optimal solution to (6).

4 Experimental Results

In order to compare results with the previous sub-optimal solution obtained by Vrhel et al. [4], the same multi-illuminant color recording problem was simulated. The reflectance spectra ensemble, consisting of 343 spectral reflectances from a color copier, was used to determine the spectral covariance, \mathbf{K}_r . The CIE incandescent illuminant A, CIE daylight illuminant D65, and CIE fluorescent illuminant F2, were used as the viewing illuminants to determine the matrix \mathbf{S} . Using a commercial scientific-optimization routine [9] based on a modified-Newton method with gradient-projection, sets of 3 to 7 color filters were calculated for SNRs of 30, 35, 40, 45, and 50 dB.

Using simulations, the recording accuracy of the optimal filter-sets obtained from the above procedure was compared with the accuracy of the sub-optimal filter-sets of [4]. In order to perform the comparison, for each filter-set (optimal/sub-optimal and having between 3 and 7 filters) a noisy recording of the copier data-set used in [4] was simulated using the model of (1), where white Gaussian noise, with variance determined by the SNR, was used for $\boldsymbol{\eta}$. For the sub-optimal filters the CIE tristimulus vectors of the spectral reflectance samples under the three viewing illuminants were estimated from the recorded data using an LMMSE estimator in CIE XYZ space (as in Vrhel et al. [4]) and for the and optimal filters an LMMSE estimator in locally linearized CIELAB space (as described in [7, 6]) was used. True tristimuli were also calculated using (2). In order to calculate color errors in perceptually relevant units, the tristimuli were converted to CIE $L^*a^*b^*$ space [5] and the ΔE_{ab}^* error (Euclidean distance in $L^*a^*b^*$ space) was computed for each estimated tristimulus. For each filter set, average ΔE_{ab}^* errors were computed over the copier reflectance-ensemble and the viewing illuminants. The average ΔE_{ab}^* errors for the optimal and the sub-optimal filter sets are compared in Fig. 1, for SNRs of 30, 40, and 50 dB. The number of filters is represented along the abscissa and the average ΔE_{ab}^* errors are plotted along the ordinate for different filter-sets, with the crosses (x) representing averages for the sub-optimal filters from [4], and the circles (o) representing the averages for the optimal filters obtained by the aforementioned procedure.

Several interesting observations can be made from Fig. 1. First observe that the optimal filters perform consistently better than the sub-optimal filters. Given the non-linear relation of the CIE $L^*a^*b^*$ space to the orthogonal-tristimulus space in which the optimal filters are defined, this improvement does not directly follow from the “optimality”. The reduction in error is significant for all filter-sets, but the largest improvements in average ΔE_{ab}^* performance are at low SNRs and for filter sets with large number of filters. Intuitively, one expects the average error to monotonically decrease as additional filters are added at a given SNR. The optimal filters follow this trend, but due to the additional constraints on the sub-optimal filters, they often show an increase in average ΔE_{ab}^* error with in-

crease in the number of filters. These facts indicate that the additional constraints imposed on the filters in defining the sub-optimal solution were inappropriate. One may also note here that in the absence of noise, $f(\mathbf{G})$ is invariant under nonsingular transformations of \mathbf{G} . Therefore, in the absence of noise, nonnegative filters can be obtained by a nonsingular transformation of the optimal unconstrained filters. Since the sub-optimal filters were initialized using such nonsingular transformations, this fact explains the (relative) improvement in performance of the sub-optimal filter-sets at high SNR’s.

The plots also indicate that at all simulated SNRs, going from three to four filters offers the most significant decrease in the average ΔE_{ab}^* error, and the improvement obtained upon using more than 4 filters is incremental. Hence, it is desirable to use a four-channel scanner and four-tuples for obtaining multi-illuminant color information. Vrhel *et al.* [4] first arrived at this conclusion, which is strengthened by these new results.

In addition to the feasibility requirement embodied in the nonnegativity constraint, it is desirable that the filter-set \mathbf{G} be ready fabricable for use in a scanning device. The manufacturability of the sub-optimal 4-filter set from [4], using dichroic materials, was examined in [10], and fairly close approximations to the sub-optimal filter-sets were deemed producible. A similar study has not been performed for the optimal filter-sets determined in this paper. However, it is unlikely that there will be significant differences in manufacturability between the optimal and the sub-optimal filters. This claim is validated by the details of the fabrication procedure described in [10] and by the comparison of the optimal and sub-optimal nonnegative 4-filter sets at 30 dB SNR shown in Fig. 2.

5 Conclusion

Optimal nonnegative color-scanning filters (for multi-illuminant color correction) designed in this paper were shown to offer significant improvements in color recording accuracy in comparison with a sub-optimal scheme reported in earlier literature. The simulation results further reinforced the conclusion in [4] that for multi-illuminant color-recording the use of four-tuples is desirable, instead of the tristimuli currently used.

6 Acknowledgment

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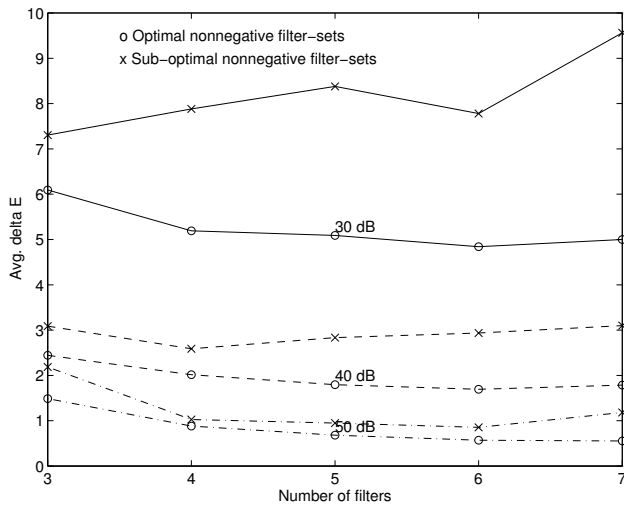


Figure 1: Performance comparison of optimal and sub-optimal filter sets.

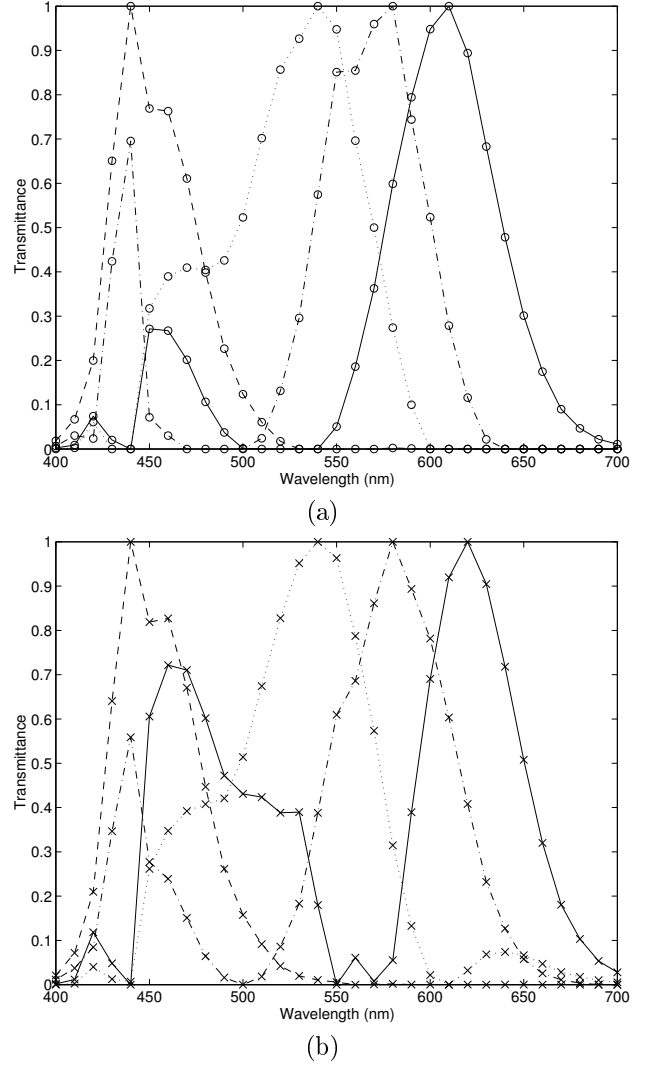


Figure 2: (a) Optimal and (b) sub-optimal 4-filter sets at 30 dB SNR.