Characterization of Scanner Sensitivity

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Abstract

Color scanners are becoming quite popular as input devices for desktop publishing. In many applications it is desirable to obtain calibrated color from them. In order to properly calibrate the device for a variety of illuminants, it is necessary to estimate the spectral sensitivity of the scanner. This paper describes a set theoretic approach to this problem. This method is shown to have increased accuracy compared to present methods.

1 Introduction

The output of a color scanner is a three band image. The value of each of the three bands at a pixel is given by

$$t_i = \int_{-\infty}^{\infty} f_i(\lambda)\,\theta(\lambda)\,r(\lambda)\,l(\lambda)\,d\lambda \; + \epsilon_i \qquad i = 1,2,3 \quad (1)$$

where $f_1(\lambda)$, $f_2(\lambda)$ and $f_3(\lambda)$ are the transmittances of the three color filters, $\theta(\lambda)$ is the sensitivity of the detector used in the measurements, $l(\lambda)$ is the illuminant spectrum, $r(\lambda)$ is the reflectance spectrum of the pixel and ϵ_i is the measurement noise. In practice, the spectra in the equation above can be represented in terms of their samples, and the integral may be approximated by a summation. If samples are available at N equi-spaced wavelengths (typically spectra are sampled from 400 to 700 nm at 10 nm increments so that $N = 31$), the scanning process can be represented algebraically as

$$\mathbf{t} = \mathbf{M}^T\,\mathbf{L}\,\mathbf{r} + \text{ffl} \quad (2)$$

where $\mathbf{M}$ is an $N \times 3$ matrix which includes the effect of the filters and the detector sensitivity, $\mathbf{L}$ is an $N \times N$ diagonal matrix representing the spectrum of the illuminant, $\mathbf{r}$ is the vector of reflectance samples, $\mathbf{t}$ is a 3×1 vector of tristimulus values and fflis the 3×1 noise vector. For the treatment in this paper it is convenient to write this equation in the form

$$\mathbf{t} = \mathbf{S}^T\,\mathbf{w} + \text{ffl} \quad (3)$$

where $\mathbf{S} = \mathbf{L}\,\mathbf{M}$, $\mathbf{w} = \mathbf{r}$ if the illuminant spectrum is not known *a priori* and $\mathbf{S} = \mathbf{M}$, $\mathbf{w} = \mathbf{L}\,\mathbf{r}$ if the illuminant is known *a priori*.

In order to characterize the scanner completely, one needs to know $\mathbf{S} = [\mathbf{s}_1\,\mathbf{s}_2\,\mathbf{s}_3]$. For initial calibration, the individual quantities may be measured before assembly, and the scanner sensitivity computed from those. However, such a calibration would deteriorate with time due the aging of the components. Besides, this would represent a significant expense for these low cost devices and is rarely done. A straightforward approach to *in situ* measurement of the scanner sensitivity is to scan a number of samples with known reflectance spectra, $\{\mathbf{r}_k\}_{k=1}^{K}$ $(K \geq N)$ and solve the least squares problem

$$\mathbf{S}_{opt} = \arg\min_{\mathbf{S}} \sum_{k=1}^{K} \parallel \mathbf{t}_k - \mathbf{S}^T\mathbf{w}_k \parallel^2 \quad (4)$$

where the minimization is performed over all possible $N \times 3$ matrices, and $\mathbf{w}_k$ is computed from $\mathbf{r}_k$ as indicated earlier.

The least squares approach suffers from a serious practical problem in that the spectra of natural objects do not have sufficient dimensionality to yield a good estimate of $\mathbf{S}$. Typically, the matrix of reflectance spectra $\mathbf{R} = [\mathbf{r}_1\,\mathbf{r}_2 \ldots \mathbf{r}_K]$ is highly ill-conditioned and has only seven to eight significant singular values [11]. As a result, the matrix $\mathbf{W} = [\mathbf{w}_1\,\mathbf{w}_2 \ldots \mathbf{w}_K]$ is also ill-conditioned and the least squares solution to Eqn. (4) is highly sensitive to noise and yields poor estimates of $\mathbf{S}$ at noise levels typical in desktop scanners.

2 Principal Eigenvector (PE) Method

One way of reducing the sensitivity of the pseudoinverse solution to noise is to use only the singular vectors corresponding to the significant singular values in the solution. Consider the singular value decomposition of $\mathbf{W}$ [5]

$$\mathbf{W} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^t \quad (5)$$

$$\mathbf{U}^t\mathbf{U} = \mathbf{U}\,\mathbf{U}^t = \mathbf{I}_{N\times N} \quad (6)$$

$$\mathbf{V}^t\mathbf{V} = \mathbf{V}\,\mathbf{V}^t = \mathbf{I}_{K\times K} \quad (7)$$

$$\mathbf{\Lambda} = [\mathbf{\Sigma}_{N\times N} \quad \mathbf{0}_{N\times K-N}] \quad (8)$$

where $\mathbf{0}_{N\times K-N}$ is an $N \times K - N$ matrix of all zeros and $\mathbf{\Sigma}$ is the diagonal matrix of the singular values $\{\sigma_i\}_{i=1}^{N}$

$$\mathbf{\Sigma} = \text{diag}\,(\sigma_1,\,\sigma_2,\,\ldots\,\sigma_N) \quad (9)$$

$$\sigma_1 \quad \geq \quad \sigma_2 \geq \ldots \geq \sigma_N \geq 0 \tag{10}$$

In terms of these vectors the least squares solution can be written as

$$\hat{\mathbf{s}}_j = \sum_{i=1}^{P} \frac{(\mathbf{v}_i^T \mathbf{t}_j)}{\sigma_i} \mathbf{u}_i \qquad j = 1 \ldots 3 \tag{11}$$

where P ($\leq N$) is the rank of $\mathbf{W}$, i.e., the largest value of i such that $\sigma_i > 0$; $\{\mathbf{u}_i\}_{i=1}^N$ and $\{\mathbf{v}_i\}_{i=1}^N$ are the columns of $\mathbf{U}$ and $\mathbf{V}$ respectively (the left and right singular vectors of $\mathbf{W}$) and $\mathbf{t}_j = [t_{j_1}, t_{j_2}, \ldots t_{j_K}]^T$ is the vector of the K measurements made using the j^{th} filter.

If the noise is assumed to be uncorrelated and signal independent, the mean squared estimation error can be written as

$$\mathrm{E}\left\{\| \mathbf{s}_j - \hat{\mathbf{s}}_j \|^2\right\} = \sum_{i=P+1}^{N} \mathrm{E}\left\{(\mathbf{u}_i^T \mathbf{s}_j)^2\right\} + \sum_{i=1}^{P} \frac{\sigma_\epsilon^2}{\sigma_i^2} \tag{12}$$

where E denotes the expectation operator, and σ_ϵ^2 is the noise variance.

It is clear from this equation that the mean squared estimation error is large if any of the singular values $\{\sigma_i\}_{i=1}^P$ is small. From the expression one can also see that the mean squared estimation error is reduced if the singular vectors for which σ_i is small are not included in the solution. Thus if the singular values beyond $\sigma_P 0$ are insignificant the estimate of the sensitivity becomes

$$\hat{\mathbf{s}}_j = \sum_{i=1}^{P0} \frac{(\mathbf{v}_i^T \mathbf{t}_j)}{\sigma_i} \mathbf{u}_i \qquad j = 1, 2, 3 \tag{13}$$

This solution, referred to as the 'Principal Eigenvector (PE) solution', is far less sensitive to noise than the least-squares solution. However, this method still suffers from several limitations. The physical situation affords considerable *a priori* knowledge, and the method fails to take this into account. For instance, except for the illuminant (which may be a fluorescent or gas discharge lamp with sharp peaks in its spectrum) the scanner sensitivity is a fairly smooth function of wavelength. Hence if the illuminant is known *a priori* the functions $\{\mathbf{s}_j\}_{j=1}^3$ to be estimated are smooth functions. However, if the illuminant has sharp peaks in its spectrum the principal eigenvectors of $\mathbf{W} = \mathbf{L}\,\mathbf{R}$ will also have sharp peaks and will therefore yield estimates of $\{\mathbf{s}_j\}_{j=1}^3$ that have sharp spectral peaks. Other *a priori* information such as non-negativity and boundedness is also not incorporated in the estimation process discussed so far.

3 The Method of Projections onto Convex Sets (POCS)

The problem of estimating the scanner sensitivity can alternately be formulated using set theory. Based on each constraint that the scanner sensitivity must satisfy, a constraint set may be defined in which the true value of the sensitivity must lie. Any element in the intersection of the constraint sets is called a feasible solution and may be used as an estimate of the sensitivity. Based on the physical nature of the problem, it can be said that the sensitivity function $\mathbf{s}_j$ (for each j) probably lies in the following constraint sets:

1. The set of non-negative vectors

$$A_n = \{\mathbf{y} \in \mathcal{R}^N | y_i \geq 0, \quad \forall 1 \leq i \leq N\} \tag{14}$$

2. The noise variance set

$$A_\epsilon = \{\mathbf{y} \in \mathcal{R}^N | \parallel \mathbf{t}_j - \mathbf{W}^T \mathbf{y} \parallel^2 \leq \nu\} \tag{15}$$

where usually the value of ν is set to $N\,\sigma_\epsilon^2$

3. The noise outlier sets

$$A_o^i = \{\mathbf{y} \in \mathcal{R}^N | \quad |t_{j_i} - \mathbf{W}_i^T \mathbf{y}| \leq \xi\} \quad i = 1, 2, \ldots K \tag{16}$$

where $\xi = 3\,\sigma_\epsilon$ is used for Gaussian noise.

Additionally, if the illuminant is known *a priori*, the combined effect of the passive filter and detector responses is to be determined. The estimate should therefore lie in the set of passive spectral responses

$$A_p = \{\mathbf{y} \in \mathcal{R}^N | y_i \leq 1, \quad \forall 1 \leq i \leq N\} \tag{17}$$

In either of the cases when the illuminant is known *a priori* or when it is known to be smooth the sensitivities $\{\mathbf{s}_j\}_{j=1}^3$ are known to be smooth functions of wavelength. This can be incorporated in the estimation process by placing a bound on the second order difference of the components of $\mathbf{s}_j$ (for each j). Let $\mathbf{h} = (1, -2, 1)^T$ represent the Laplacian filter impulse response. Then the filtered output can be represented as [10]

$$\mathbf{f}_j = \mathbf{H}\,\mathbf{s}_j \tag{18}$$

where $\mathbf{H}$ represents the convolution operator for convolution kernel $\mathbf{h}$. The set of smooth spectra can then be defined in terms of an upper bound on the energy in the filtered output

$$A_s = \{\mathbf{y} \in \mathcal{R}^N | \parallel \mathbf{H}\,\mathbf{y} \parallel^2 \leq \mu\} \tag{19}$$

where $\mu > 0$ is suitably chosen so as to impose the desired degree of smoothness.

The sets defined here are all closed, convex sets. Hence, a point in the intersection can be found by the method of successive projections, i.e., starting from any arbitrary point in $\mathcal{R}^N$ a point in the intersection of these sets can be determined by successively projecting onto each of them. This is called the method of Projections Onto Convex Sets (POCS). Since the sets are closed and convex,

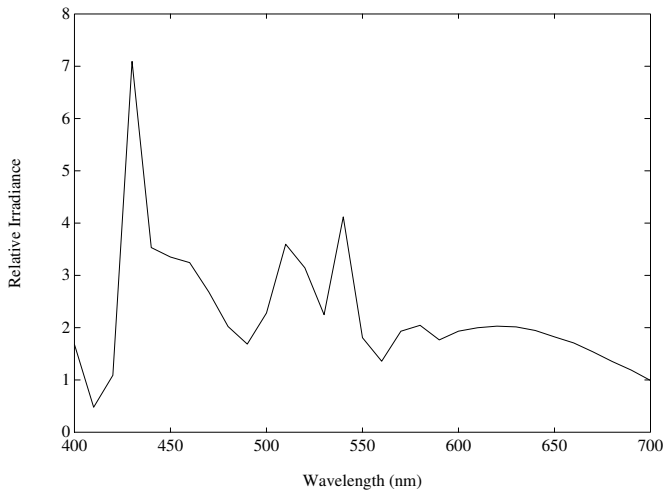


Figure 1: Spectrum of Illuminant used in Simulations

the iterative process of successive projections is guaranteed to converge to a point in the intersection provided the intersection is non-empty [6, 7]. If the sets have been defined properly and the model is accurate, the fact that the measurements arise from a physical experiment implies that to a high degree of probability the intersection of the constraint sets is non-empty and hence the algorithm will converge.

POCS is a powerful estimation technique that combines *a priori* information with the measurements to obtain the estimates. Since estimates obtained using POCS conform to all the known constraints that the true vector obeys it is expected that the estimates will be better than those obtained by other methods. It may also be noted that if the intersection of the constraint sets is non-empty it will rarely be a singleton and hence the POCS estimate is non-unique. In particular, the POCS estimate can depend considerably on the initial point chosen to start the iterations.

4 Experimental Results

For illustrating the performance of the estimation techniques developed here simulations were conducted for a calibration problem. A set of red, green and blue filter transmittances were chosen from the 'Wratten' filter set. In particular, the WR-25 red, WR-40 green and WR-39 blue filters were chosen for comparing the estimation methods. The spectrum of a lamp used in an existing desktop scanner was used as the illuminant. The illuminant spectrum is shown in Fig. 1

To minimize the dimensionality problem a special data set was chosen from a collection of actual measured spectra. The set was chosen to maximize the smallest singular value of $\mathbf{R}$. A set of simulated noisy measurements were made using the model of Eqn. (2). A Gaussian random number generator was used to generate noise samples ϵ_i. The value of the noise variance σ_ϵ^2 was calculated assuming a Signal-to-Noise Ratio (SNR) of 40 dB in conjunction with the definition

$$\text{SNR (dB)} = 10 log_{10} \left(\frac{\| \mathbf{R}^T \mathbf{L} \mathbf{s}_j \|^2}{N \sigma_\epsilon^2} \right) \qquad (20)$$

For POCS it was assumed that the filter function under consideration is known to be red, green or blue. Accordingly, the initial estimates were taken to be nearly rectangular functions with transmittance windows positioned approximately in the red, green or blue region. The detector response was assumed to be uniform.

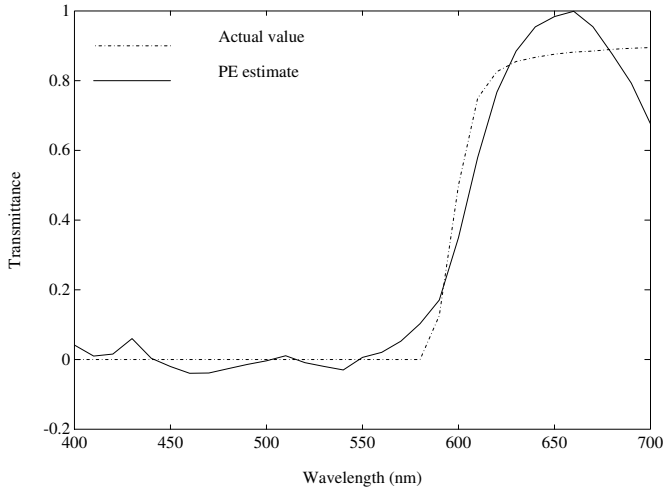
For the dataset reflectance vectors, the singular values of the reflectance matrix drop sharply after the first few singular values and values beyond the 8^{th} are negligibly small as compared to the first singular value[1]. Hence, in the principal eigenvector method $P' = 8$ was used.

In the simulations conducted, the illuminant spectrum was assumed to be a part of the *a priori* knowledge so that the sensitivity matrix $\mathbf{S}$ was the same as the matrix $\mathbf{M}$ of filter transmittances. For this case both the smoothness and passivity constraints can be imposed.

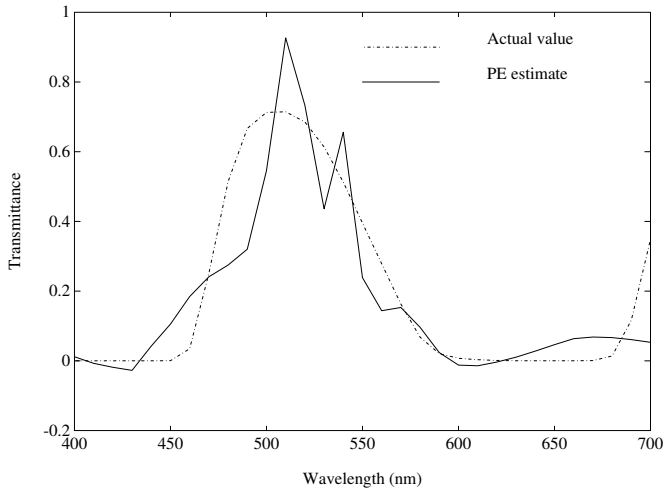
The estimates obtained for the sensitivity functions of the three color bands are shown in Figure 2 for the PE method and in Figure 3 for POCS. The parts (a), (b) and (c) of each of these figures show the results for the red, green and blue filters respectively. It is interesting to note the differences between the PE and POCS estimates. Since the illuminant has sharp peaks in its spectrum the left singular vectors of $\mathbf{W}(= \mathbf{L}\ \mathbf{R})$ are not smooth. Therefore, the estimates of $\{\mathbf{s}_j\}_{j=1}^3$ obtained by the PE method (see Eqn. (13)) also exhibit sharp peaks. However, since POCS imposes an explicit smoothness constraint in this case the POCS estimates are smooth. Since the POCS estimates also meet the other known constraints on the sensitivity, they are much better than the PE estimates. The PE method provides no natural way for incorporating the fact that the sensitivity functions are smooth into the estimation process and therefore yields poor estimates. Using a smooth basis set and attempting to estimate the filter sensitivities in the span of the smooth set also does not yield good results. It can also be seen that the PE estimates fail to meet other physical criteria required of transmittances as they becomes negative in certain region of the spectrum.

In order to quantify the accuracy of the estimates, mean squared tristimulus errors between the actual model

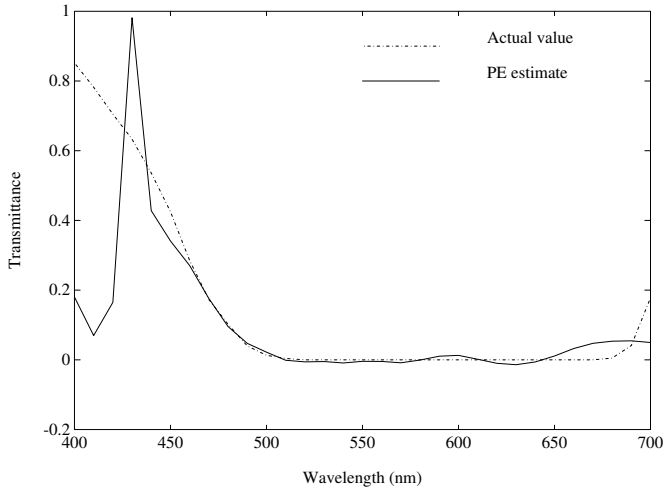
[1] The nature of the results does not change if any integer between 6 and 10 is used instead of 8



(a)

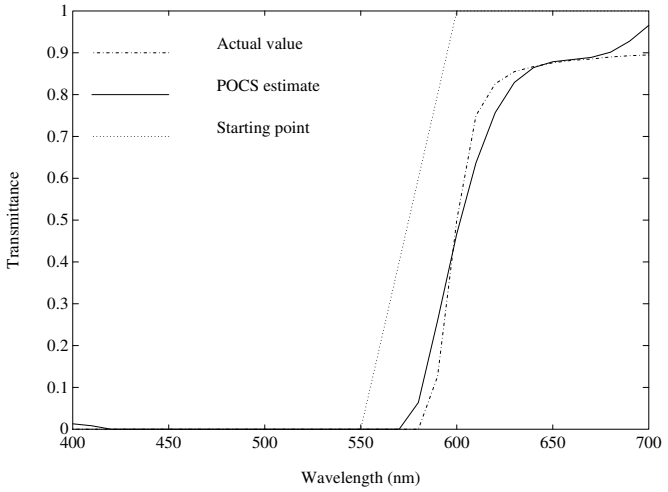


(b)

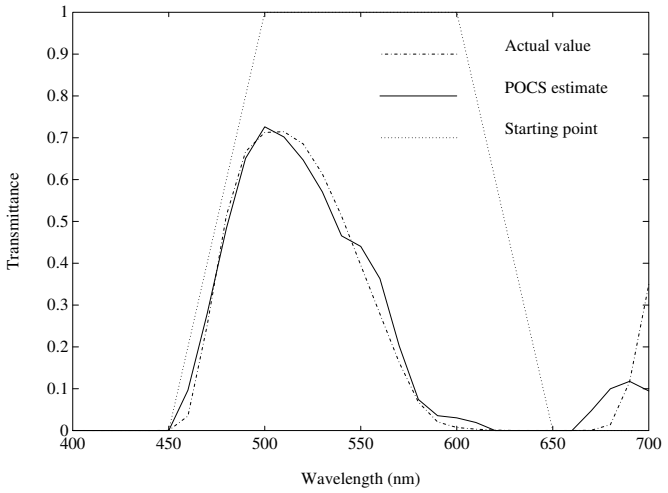


(c)

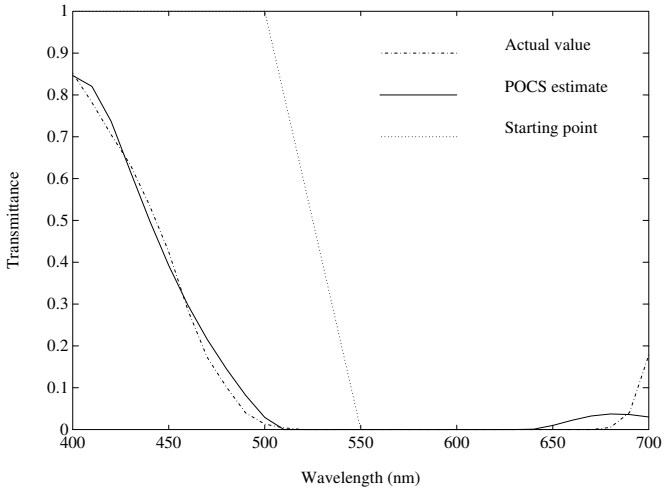
Figure 2: PE Estimate (a) Red (b) Green (c) Blue



(a)



(b)



(c)

Figure 3: POCS Estimate (a) Red (b) Green (c) Blue

Table 1: Mean Squared Error for the Tristimulus Values Obtained from the PE and POCS estimates

	Mean Squared Error (dB)		
Estimate	*Red*	*Green*	*Blue*
PE	-33.31	-31.25	-18.37
POCS	-37.24	-36.78	-37.26

tristimuli and the tristimuli resulting from the estimated sensitivity functions were computed. For the computation a set of reflectances corresponding to 12 Color and Interchange standard color chips, 64 Munsell chips, 120 Dupont paint chips and 170 natural objects were used in the model of Eqn. (2) with the noise term set to zero. Note this is a different set from the set of reflectance vectors used for determining the filter transmittances. 'Measurements' were generated (using the model) with the actual sensitivity, and with the estimated sensitivity. Mean squared error between these values was obtained by averaging over the entire set of reflectances for each of the three filters for both the POCS estimate and the PE estimate. The resulting mean squared errors are summarized in Table 1. From the tabulation it can be seen that the POCS estimate performs significantly better than the PE estimate.

It may be noted here that the true strength of the POCS estimation procedure lies in its exploitation of *a priori* knowledge. An experiment was conducted in an identical manner to the one described above except the illuminant was not assumed to be a part of the *a priori* knowledge and was included in the sensitivity. Since the illuminant has sharp spectral peaks, the sensitivities for the three channels also have sharp spectral peaks. However, since most real world reflectances (including the dataset reflectance vectors) are smooth functions of wavelength, the principal left singular vectors of $\mathbf{W}$ ($= \mathbf{R}$) are also smooth and are not able to capture the sharp peaks in the spectral response in the PE method. Since the filter sensitivities were not smooth functions the smoothness constraint was not used in POCS. However, the statistical information from the measurements leads to smooth estimates of the sensitivities for POCS also. The estimates for the PE and the POCS method are very similar, and both are rather poor approximations to the actual sensitivities. It may be possible to use information about the location of the illuminant spectral peaks (which are usually known) to produce a better estimate. This approach is under investigation.

5 Conclusions

This paper looked at the performance of two estimation methods, viz., PE and POCS, applied to the problem of color scanner characterization. We compared the two methods through simulations. From the results it is apparent that when greater *a priori* information is available the method of POCS outperforms the PE method, yielding significantly better estimates of the scanner sensitivities.

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