

# Color Scanner Performance Trade-offs

G. Sharma and H.J. Trussell

ECE Dept., North Carolina State University, Raleigh, NC 27695-7911

## Abstract

The goal of a general purpose color scanner is to determine the color of an object in a device independent color space, such as the CIE XYZ tristimulus space. Raw sensor measurements from scanners rarely correspond to CIE XYZ due to design and realizability constraints. The conversion of data from scanner RGB to a device independent color space introduces errors due to the non-colorimetric nature of the scanner and due to the noise present in measurements. This paper analyzes the relative contributions of these two components to color errors. This analysis allows the designer to determine the most cost effective device meeting required specifications.

**Keywords:** color scanners, scanner design, device independent color, color filters.

## 1 Introduction

The color of an object is determined by the projection of its spectrum under the viewing illuminant onto the human visual subspace (HVSS). The goal of a general purpose color scanner is to accurately determine this projection for the scanned samples. A scanner that attains this goal is said to be colorimetric.<sup>1</sup>

Several problems make it difficult to realize a colorimetric scanner in practice. In particular, considerations of power efficiency and heat dissipation often predicate the use of fluorescent lamps in scanners. Since these lamps differ from the viewing illuminant (often incandescent light/ daylight), it is difficult to design realizable filters that meet the requirement of being colorimetric. In addition, once filters have been designed, errors in fabrication and measurement noise in the scanner sensors will contribute to deviations from colorimetric behavior. The errors resulting from the filters *per se* can be minimized by using proper design procedures and accurate control in fabrication. A careful choice of scanner sensors, optics, and electronics can reduce the measurement noise. These choices however involve an increased cost, and therefore it is essential that the improvement of one component should not be carried past the point at which the other becomes a limiting factor yielding diminishing returns. In this paper, the tradeoff between the

filters and the measurement noise is investigated through simulations of the scanning process at different noise levels for several filter sets.

## 2 Scanner Colorimetry

The process of color scanning can be represented as<sup>6</sup>

$$\mathbf{t}_s = \mathbf{M}_s^T \mathbf{L}_s \mathbf{r} + \boldsymbol{\epsilon} = \mathbf{A}_s^T \mathbf{r} + \boldsymbol{\epsilon} \quad (1)$$

where  $\mathbf{t}_s$  is the  $K \times 1$  vector of scanner measurements,  $\mathbf{M}_s$  is a matrix with the samples of the spectral transmittances of the  $K$  color filters (and associated optical path and detector sensitivity) as its columns,  $\mathbf{L}_s$  is a diagonal matrix with samples of the scanner illuminant spectrum along its diagonal,  $\mathbf{r}$  is a vector containing samples of the spectral reflectance of the scanned object, and  $\boldsymbol{\epsilon}$  is the  $K \times 1$  measurement noise vector, and  $\mathbf{A}_s = \mathbf{L}_s \mathbf{M}_s$ .

The color of the same object is determined by its CIE XYZ tristimulus values<sup>2</sup> given by

$$\mathbf{t} = \mathbf{A}^T \mathbf{L} \mathbf{r} = \mathbf{A}_L^T \mathbf{r} \quad (2)$$

where  $\mathbf{A}$  is a matrix with the CIE XYZ color matching functions as its columns,  $\mathbf{L}$  is a diagonal matrix with samples of the viewing illuminant spectrum, and  $\mathbf{A}_L = \mathbf{A}^T \mathbf{L}$ .

In terms of the above notation, the scanner is colorimetric if there is a linear transformation  $\mathbf{B}$  which satisfies  $\mathbf{L}_s \mathbf{M}_s \mathbf{B} = \mathbf{L} \mathbf{A}$ . In the absence of noise, the same transformation transforms scanner measurements to CIE XYZ.

In the more practical situation, when the scanner is not colorimetric and the measurements are corrupted with noise,<sup>3</sup> one would like to determine the “best” linear transformation  $\mathbf{B}$  that transforms scanner data to CIE XYZ. A suitable definition of “best” would be the transformation that minimizes the perceptual color error. However, such a formulation<sup>9</sup> does not yield a closed form solution and is therefore not very suitable for design problems or for analyzing the minimum error. Hence, the linear transformation yielding minimum mean squared error in CIE XYZ space is considered here,

$$\begin{aligned} \mathbf{B}_{opt} &= \arg \min_{\mathbf{B}} E \{ \| \mathbf{t} - \mathbf{B} \mathbf{t}_s \|^2 \} \\ &= \mathbf{A}_L^T \mathbf{K}_r \mathbf{A}_s (\mathbf{A}_s^T \mathbf{K}_r \mathbf{A}_s + \mathbf{K}_\epsilon)^{-1} \end{aligned} \quad (3)$$

where  $E\{\}$  denotes the expectation operator and  $\mathbf{K}_y \stackrel{\text{def}}{=} E\{\mathbf{y}\mathbf{y}^T\}$  denotes the correlation matrix for the vector  $\mathbf{y}$ . The CIE XYZ tristimulus values for a sample with scanner measurement vector  $\mathbf{t}_s$  can now be estimated as  $\hat{\mathbf{t}} = \mathbf{B}_{opt} \mathbf{t}_s$ . The expected (minimum) mean squared error in CIE XYZ space obtained with this estimate is given by

$$e_{min} = \text{tr} \{ \mathbf{A}_L^T \mathbf{K}_r \mathbf{A}_L - \mathbf{A}_L^T \mathbf{K}_r \mathbf{A}_s (\mathbf{A}_s^T \mathbf{K}_r \mathbf{A}_s + \mathbf{K}_\epsilon)^{-1} \mathbf{A}_s^T \mathbf{K}_r \mathbf{A}_L \} \quad (4)$$

where  $\text{tr}\{\cdot\}$  denotes the trace operator.

This expression indicates the tradeoff between the colorimetric quality of the scanner and the noise level. In the absence of any noise,  $\mathbf{K}_\epsilon = \mathbf{0}$  and the error is determined by the colorimetric quality of the scanner filter illuminant combination. In the presence of excessive noise the error tends to its maximum value  $\text{tr}\{\mathbf{A}_L^T \mathbf{K}_r \mathbf{A}_L\}$  irrespective of the color filters in the scanner. While the above discussion using  $e_{min}$  is intuitively appealing, it is well known that the mean squared error in CIE XYZ space does not correlate well with perceptual measures of color error. Hence, in the next section simulations are considered in order to determine the nature of the above tradeoff in terms of mean squared error in CIE  $L^*a^*b^*$  space.

In the scenario discussed above, only one viewing illuminant was considered. For true device independence, it is desirable to obtain colorimetric data under several viewing illuminants from a single scan of the image. In order to minimize metamerism effects, more than three channels may be employed in the color scanner. This problem was addressed by Vrhel and Trussell<sup>8-10</sup> using several different techniques. However, the design of optimal non-negative color scanning filters in the presence of noise was left as an open problem. Recently, the problem has been solved.<sup>4</sup> While the full details of the solution are beyond the scope of this paper, the tradeoff between the number of filters and the error in  $L^*a^*b^*$  space is presented here for completeness.

### 3 Simulation Results

Three sets of simulations were performed. The first was aimed at determining the variation in scanner performance with change in noise level, the second investigated the impact of colorimetric quality on scanner performance at a constant noise level, and the third considered the utility of adding additional channels to obtain multi-illuminant data.

For the first set of simulations, the use of a three channel scanner with a fluorescent mercury lamp, for colorimetry under CIE illuminant D65 was considered. The scanner illuminant spectrum is shown in Fig. 1. Two sets of scanning filters were considered in the simulations. The first was a set of red, green and blue filters chosen from the Kodak Wratten filter set in combination with UV and IR cutoff filters from Schott. The transmittances of the filters are shown in Fig 2. This set of filters is not colorimetric. The second set of filters was a set of specifically designed spectral transmittances described by Gaussian functions that minimized the error in eq. (4) (i.e., were nearly colorimetric). The transmittances for the filters in this set are shown in Fig. 3.

For both filter sets, noisy scanner measurements were simulated for a Kodak Q60 target<sup>5</sup> by using measured reflectance spectra for the target in eq. (1). The noise was assumed to be uncorrelated with the signal, white, and Gaussianly distributed, with the variance determined

by the signal-to-noise ratio (SNR), defined on a per-channel basis as,

$$\text{SNR (dB)} = 10 \log_{10} \left( \frac{\| \mathbf{m}_i^T \mathbf{L}_s \mathbf{K}_r \mathbf{L}_s \mathbf{m}_i \|^2}{\sigma_\epsilon^2} \right). \quad (5)$$

where  $\mathbf{m}_i$  is the  $i^{\text{th}}$  column of  $\mathbf{M}$ , i.e., the filter transmittance of the  $i^{\text{th}}$  channel.

Simulations were conducted for SNR's in the region 30 dB to 60 dB. CIE XYZ tristimulus values were estimated from the noisy “measurements” using the transformation of eq. (3). The actual tristimulus values from eq. (2) and the estimates were transformed to CIE  $L^*a^*b^*$  space. The Euclidean distance between them was computed to obtain the  $\Delta E_{ab}^*$  error for each block on the target. The errors were averaged over all the blocks on the target to obtain average  $\Delta E_{ab}^*$  errors.

The results of the simulations are shown in Fig. 4. The graph indicates the tradeoff between the colorimetric quality of the scanner and the noise level. At an SNR of 30 dB, the performance of both filter sets is close with the noise being the limiting factor in performance. At 60 dB, the impact of noise is negligible and the performance of either filter set is limited by its colorimetric quality. Both curves display a knee region beyond which there are marginal gains with increasing SNR. The knee occurs around 40 dB for the Wratten filter set and around 45 dB for the “optimally” designed filter set. Since the SNR due to quantization noise is roughly 6 dB per bit, these observations indicate that 8 bits per channel are enough for representing color for the scenario simulated here (in which the scanner uses a fluorescent lamp and attempts to determine color under daylight). Greater colorimetric accuracy may be obtained with 12 bits per channel only if the scanning filters and illuminant are made more colorimetric.

For the second set of simulations, the colorimetric quality of filters was quantified using the measure proposed by Vora<sup>7</sup>

$$\nu = \frac{\text{trace}(P_{A_L} P_{A_s} P_{A_L})}{3} \quad (6)$$

where  $P_{A_L}$  and  $P_{A_s}$  are the orthogonal projection matrices that projects onto the columns of  $\mathbf{A}_L$  and  $\mathbf{A}_s$ , respectively. The motivation for this measure of goodness for a color filter set is given in an earlier paper by Vora and Trussell.<sup>7</sup> For our purposes, it suffices that the measure correlates well with mean squared error in tristimulus space, for noiseless measurements made with the color filter set.

For the second set of simulations, both the scanning and viewing illuminants were assumed to be the CIE illuminant D65. Under these conditions the CIE XYZ color matching functions define a filter set with measure  $\nu = 1$ . To obtain filter sets with other values of the measure, a set of filters with Gaussian transmittances was designed with means and variances chosen to maximize the measure in eq. (6). The means and variances were then varied around their optimal values to get a number of filter sets with measures ranging from 0.75 to 0.996. In order to maintain numerical stability almost linearly dependent filter sets arising in the process above were rejected.

Once again noisy “measurements” for the Q60 target were simulated for each filter set at SNR’s of 30 dB and 60 dB. For each filter set, the linear minimum mean squared error estimator of eq. (3) was used to estimate CIE XYZ tristimulus values from the noisy measurements. Average  $\Delta E_{ab}^*$  errors were then computed for each filter set and SNR combination, as in the previous simulation. The measure of each filter set and the average  $\Delta E_{ab}^*$  were then plotted against each other in a scatter diagram. This diagram is shown in Fig. 5. Superimposed on the scatter plots for each SNR are smooth curves obtained by fitting piecewise cubic polynomials to the data with continuity and differentiability constraints. Table 1 tabulates the normalized mean squared residual error (NMSR) between the scatter points and the fitted smooth curve at different SNR’s. From the NMSR and the spread in points for filter sets with the same measure, it is clear that the measure predicts the performance (in terms of  $\Delta E_{ab}^*$  errors) better at higher SNR’s than at lower SNR’s. Thus at a 30 dB SNR, the points are scattered so far apart that no functional relationship is apparent between the measure  $\nu$  and the average  $\Delta E_{ab}^*$ , whereas at an SNR of 60 dB, the points group close together almost forming a curve. From the plots, it can also be seen that the (negative) correlation between the measure and the  $\Delta E_{ab}^*$  error is better for higher measure filters than for lower measure filters. This is in agreement with the observation that at high values of the measure the errors are small and a first order Taylor series gives a good approximation to the  $\Delta E_{ab}^*$  error.<sup>11</sup>

Table 1: Normalized Mean Squared Residual for Smooth fit to Scatter Plots.

SNR (dB)	30	35	40	45	50	55	60
NMSR	3.48	2.70	1.85	1.36	1.14	1.04	1.00

Finally, simulations were performed to study the influence of additional scanner channels in a multi-illuminant scenario. The CIE illuminants<sup>2</sup> D65, A, C and two fluorescent lamps designated F2, and F7 were used as the five viewing illuminants in the experiment. A collection of 343 reflectance samples from a color copier was used as the the dataset for determining  $\mathbf{K}_r$  and for computing average  $\Delta E_{ab}^*$  errors. Filter sets containing 3, 4, 5, 6, and 7 non-negative filters were designed<sup>4</sup> to minimize the sum of  $e_{min}$ ’s in eq. (4) over the five illuminants, for SNR values of 30, 35, 40, 45 and 50 dB. Noisy measurements were simulated at these SNR values and average  $\Delta E_{ab}^*$  errors were computed over the reflectance samples and the five illuminants using the designed “best” non-negative filter sets. The results are presented graphically in Fig. 6. From the graphs, one can see that for all values of SNR the addition of a fourth filter provides significant improvement over three filters, however, using more than 4 filters provides very limited improvements at all SNR values. Hence, if multiple viewing illuminants are to be used there is a strong case for using 4 filters instead of 3. This result is a corroboration of an earlier result of Vrhel and Trussell<sup>10</sup> which was based on sub-optimal non-negative filters.

## 4 Conclusions

In this paper, several tradeoffs in scanner design were studied through simulations. From the results, several significant deductions can be made. Firstly, for typical scanners with a fluorescent illuminant and three smooth filters, using more than 8 bits per channel is likely to provide only marginal gains. Secondly, for most color scanners, improvements in SNR over 45 dB make little or no improvements in color accuracy. Thirdly, in order to provide colorimetric data under a variety of illuminants, it would be advantageous to add a fourth channel to present day scanners, however, addition of more than four channels would not provide significant improvements.

## 5 References

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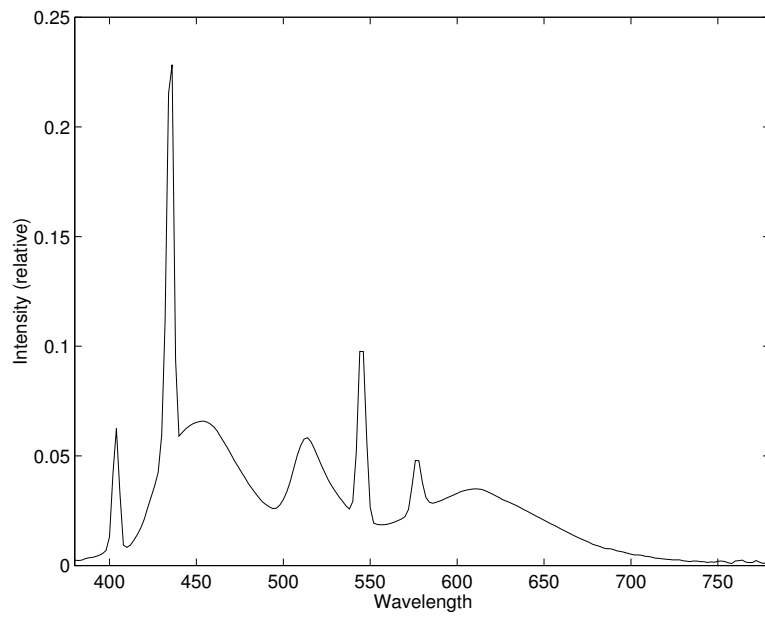


Figure 1: Spectrum of Scanner Illuminant.

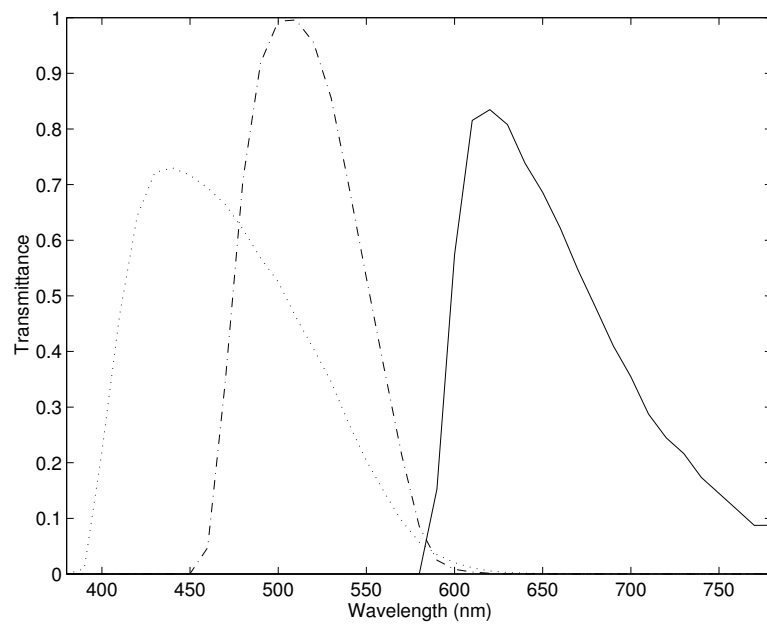


Figure 2: Scanner Filter Set I.

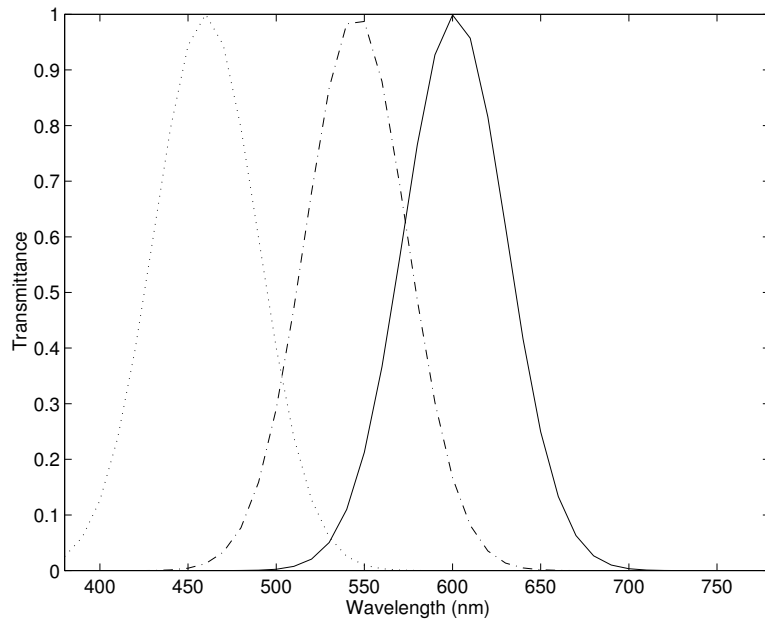


Figure 3: Scanner Filter Set II.

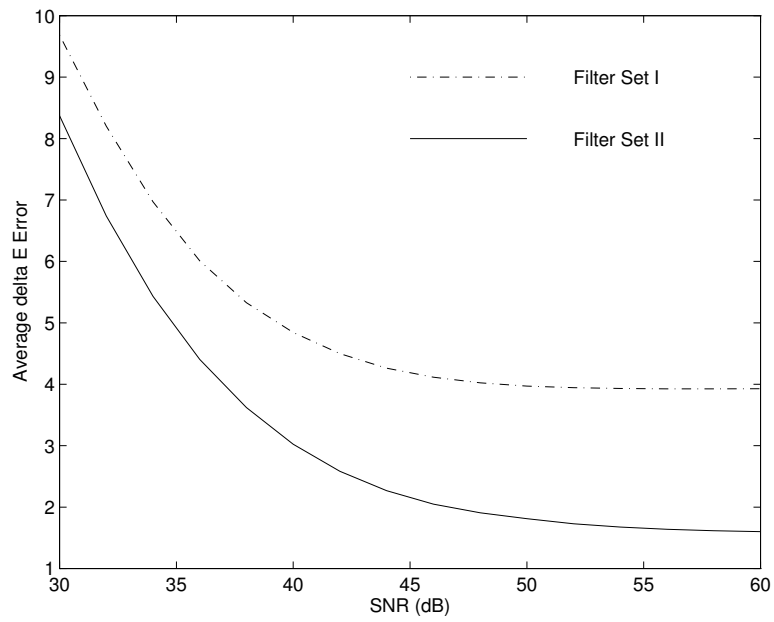


Figure 4: SNR vs. Avg.  $\Delta E$  for the two filter sets.



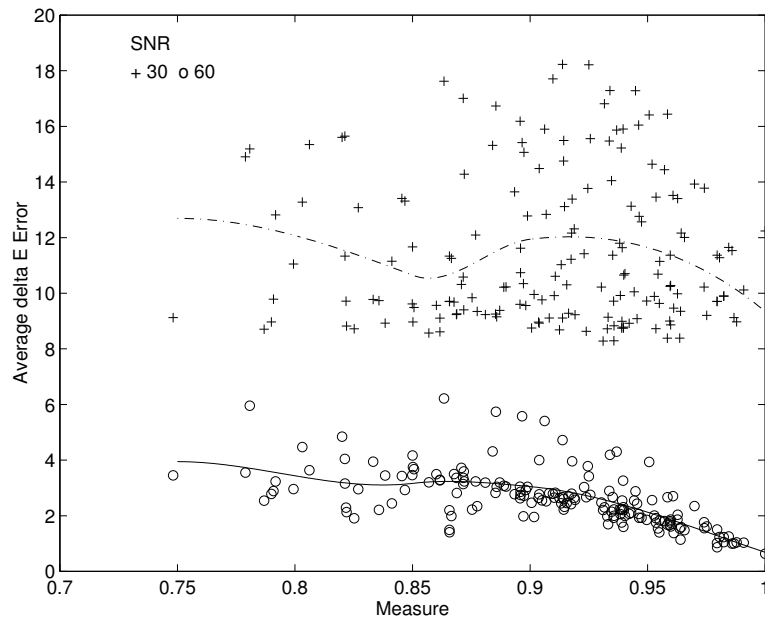


Figure 5: Vora measure vs. Avg.  $\Delta E$  for filter sets.

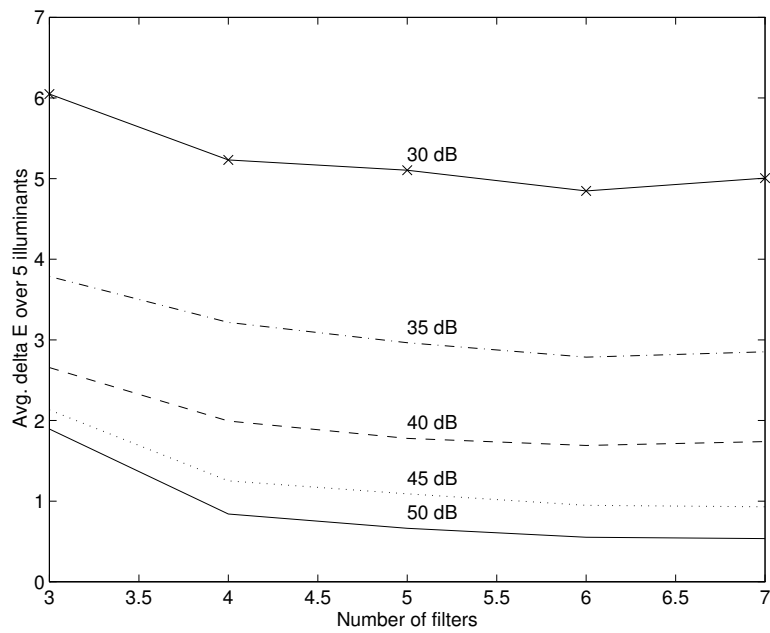


Figure 6: Avg.  $\Delta E$  vs. number of filters.