Mathematical Discontinuities in CIEDE2000 Color Difference Computations

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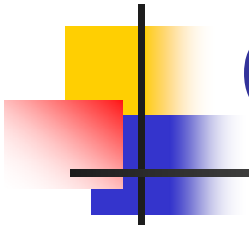
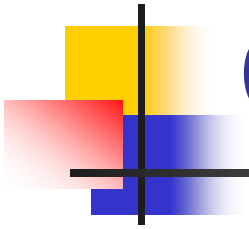


Outline

- Color Difference Equations
- CIEDE2000 Computation
- Sources of Discontinuity
- Discontinuity Visualization
- Discontinuity Magnitude Characterization
  - Maximum (reasonable) magnitude
- Conclusions + workarounds



Color Difference Equations

- Quantitative evaluation of color differences
- Main uses:
  - Quantitative color error evaluation
  - Algorithm/parameter optimization

Color Difference Equations: Desirable Attributes

- Perceptual uniformity
  - Equal numerical differences correspond to equal perceived differences
- Mathematical properties:
  - Continuity and differentiability
    - Taylor series/small-error approximation
    - Gradient based optimization
  - Symmetry
    - reference/sample distinction un-necessary
  - Correspondence to a distance metric
    - Underlying “uniform” color space

CIE 1976 CIELAB Color Space

- "Uniform" color space
  - Based on ANLAB, in turn on Munsell
- Transformation of 1931 CIEXYZ tristimulus coordinates
- Nonlinearity: Cube-root with linear end segment

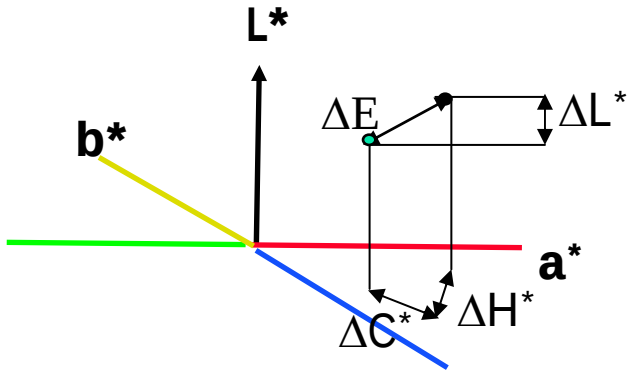
$$f(x) = \begin{cases} x^{\frac{1}{3}} & x > .008856 \\ 7.787x + \frac{16}{116} & x \leq .008856 \end{cases}$$

- Transformation carefully designed
  - Continuous first derivatives [Pauli1976]

CIELAB Based Color Difference Fomulae

- 1976: ΔE^*_{ab} Color difference
  - Euclidean distance betw. points in CIELAB space

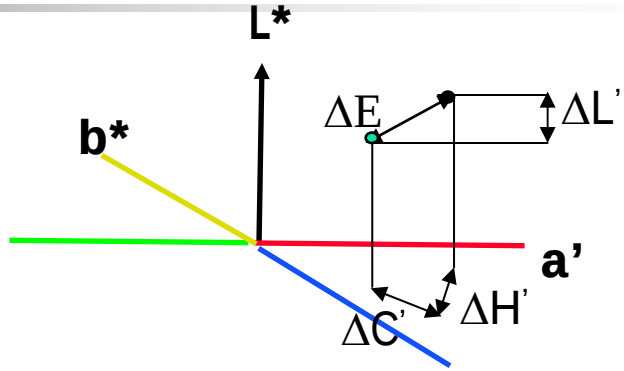
$$\Delta E^*_{ab} = \sqrt{\Delta L^{*2} + \Delta a^{*2} + \Delta b^{*2}} \quad = \quad \sqrt{\Delta L^{*2} + \Delta C^{*2} + \Delta H^{*2}}$$



- CMC and CIE '94 color difference Eqns.
  - Chroma/Hue dependent weights for ΔL^*, ΔC^*, ΔH^*
  - Greater uniformity w.r.t. experimental data
  - Retain continuity of first derivatives

CIEDE2000

- a^* Axis Scaling
  - a^* -> a'
- Decomposition
- Hue, Chroma Dependent Weighting
- Cross Term (blue hue nonlinearity)



$$\Delta E_{00}^{12} = \sqrt{\left(\frac{\Delta L'}{k_L S_L}\right)^2 + \left(\frac{\Delta C'}{k_C S_C}\right)^2 + \left(\frac{\Delta H'}{k_H S_H}\right)^2 + R_T \left(\frac{\Delta C'}{k_C S_C}\right)\left(\frac{\Delta H'}{k_H S_H}\right)}$$

- CIEDE2000 Color Difference is discontinuous

CIEDE2000 Hue & Hue Weighting Functions

$$\Delta H' = 2\sqrt{C'_1 C'_2}\sin\left(\frac{\Delta h'}{2}\right)$$

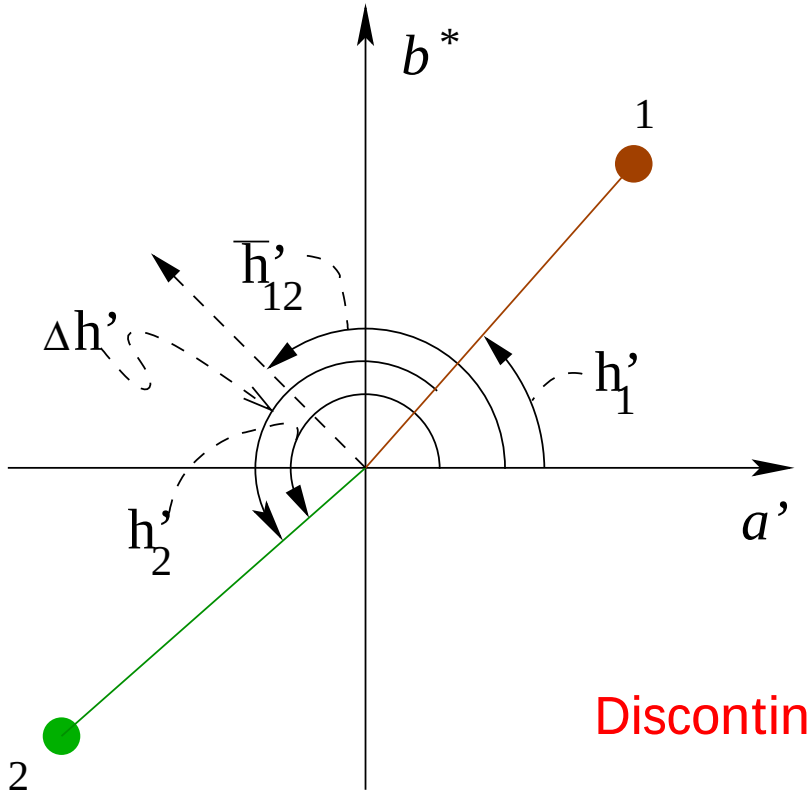
$$T = 1 - 0.17\cos(\bar{h}' - 30°) + 0.24\cos(2\bar{h}') + 0.32\cos(3\bar{h}' + 6°) - 0.20\cos(4\bar{h}' - 63°)$$

$$S_H = 1 + 0.015\bar{C}'T$$

- C'_1, C'_2 sample chroma values
- $\Delta h'$ hue angle difference
- $\bar{h}'$ mean hue angle
- $\bar{C}'$ mean chroma value (arithmetic)

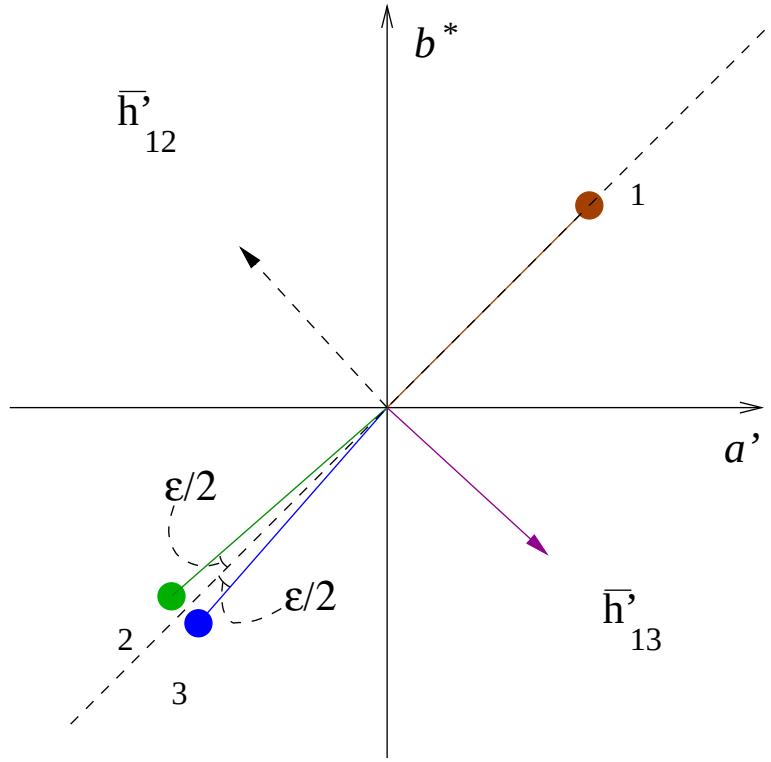
Mean Hue/Hue Difference Computation

- Mean: Bi-sector of smaller angle betw h_1, h_2
- Difference: Smaller angle + direction gives sign



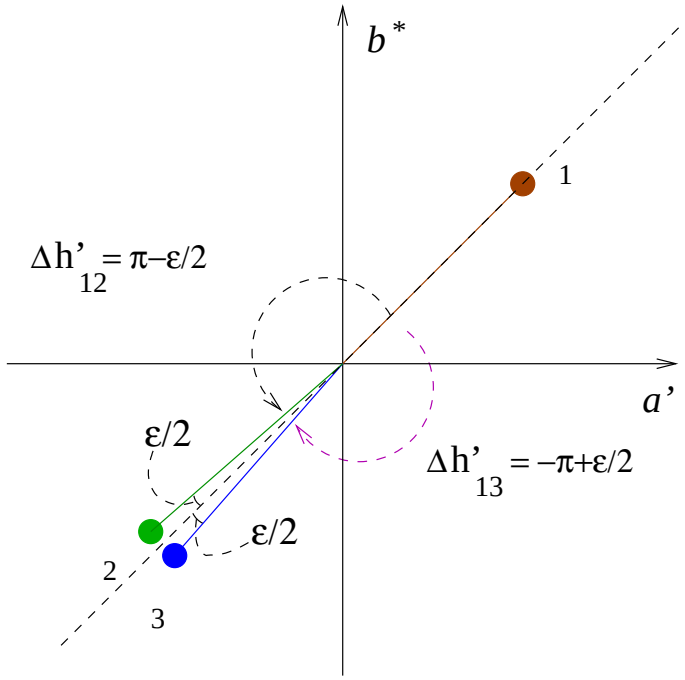
Discontinuous Operations

Mean Hue Discontinuity



- 180^{o} discontinuity in mean hue

Hue-difference Discontinuity



- 180º (Sign) discontinuity in hue difference

Discontinuity Characterization

- Where does it occur ?
- How big is it (magnitude) ?

Discontinuity Locations

- 6-D Space of input values

$$\Delta E_{00}(L_1^*, a_1^*, b_1^*; L_2^*, a_2^*, b_2^*)$$

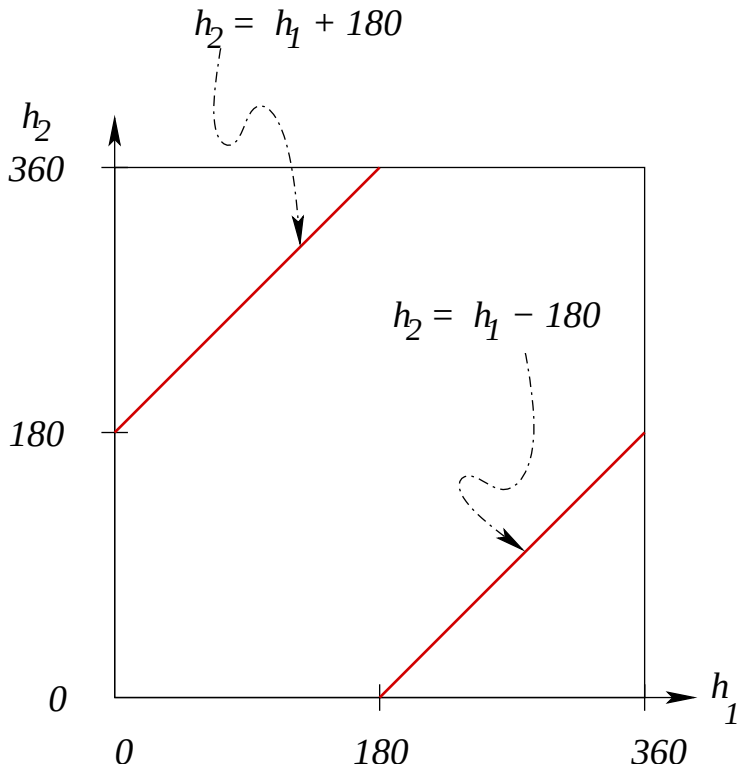
- Discontinuity for points 180^o apart in hue

$$a_1 b_2 = -a_2 b_1$$

- 5-D manifold in 6-D space

Discontinuity Locations

- Discontinuity loci in h_1, h_2 plane



Visualization

Discontinuity Magnitude

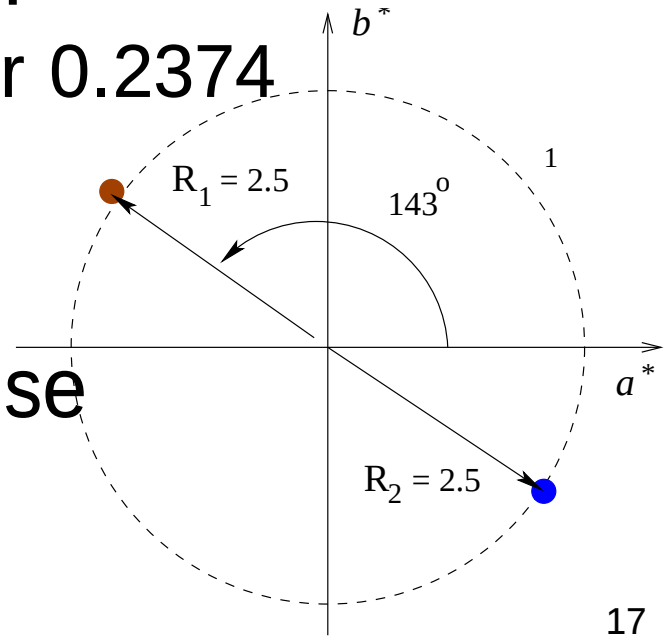
- Main contribution mean hue discontin. in

$$\left(\frac{\Delta H'}{k_H S_H}\right)^2$$

- Minor contribution from hue diff. discontin.
  - Sign change of $\Delta H'$
  - Contributes through rotation term

Discontinuity Magnitude Bounds

- CIEDE2000 intended for small color differences
- Colors under 5 $\triangle E^*_{ab}$ units apart
  - Discontinuity magnitude under 0.2374
    - Non-negligible, not too large
  - Occurs for 143^o hue sample
- Increasing distance: sharp rise



Conclusions

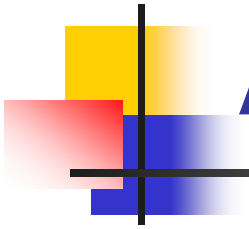
- CIEDE2000 color difference is a discontinuous function
- Discontinuity for colors 180^{o} apart in hue
- Discontinuity magnitude small in small error practical applications
  - Under 0.238 for color under 5 ΔE^*_{ab} units apart
- Serious limitation for
  - Taylor series/small error approximations
  - Gradient based optimization

Potential workarounds/fixes

- Use formula asymetrically
  - Major discontinuity due to mean hue eliminated
- Symmetrize if nesc by averaging color differences
- Discontin in Rotation term remains
  - Harder to fix
    - Probably requires different functional format and re-optimization of parameters

Additional Information

- Upcoming paper in Color Research and Application (Feb 2005)
  - includes detailed algorithmic statement of CIEDE2000 computation
  - Additional test data
    - Several available implementations
      + Agreement over CIE draft test data, disagreement over other data!!



Acknowledgements

- Thanks for suggestions/comments to:
  - Mike Brill
  - Anonymous reviewers

Questions