An Introduction To Distributed Source Coding

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1 DISTRIBUTED SOURCE CODING (DSC)

- What is DSC ?
- Nested Quantizer: A Simple Approach
- DSC using LDPC

2 DSC For Noisy Observation

- A Separative Structure
- Connection With Transform Coding

Nested Quantizer: A Simple Approach

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DISTRIBUTED SOURCE CODING

- Compression of multiple *correlated* sources (e.g. sensor networks)
- sources do not communicate with each other.



Question: will the coding performance degrade due to mutual *blindness*

THEORETICAL RESULTS



- Slepian-Wolf theorem [Slepian and Wolf, 1973]
 - lossless transmission of discrete, memoryless sources
- Wyner-Ziv theorem [Wyner, 1975]
 - lossy transmission of continuous-valued sources under a fidelity criterion.

SIDE-INFORMED SOURCE CODING

Y available at the decoder as the *side information*



the encoder exploits the SI,

 $\mathbf{R} \geq H(\mathbf{X} \mid \mathbf{Y}).$

A special case of DSCChannel is rate constrained, however error-free

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DSC BY NESTED QUANTIZER [Pradhan and Ramchandran, 2003]

Consider
$$\mathbf{y} = \mathbf{x} + \mathbf{n}$$
 as the side information
 $\mathbf{x} \sim \mathcal{N}(0, \sigma_x)$,
 $\mathbf{n} \sim \mathcal{N}(0, \sigma_n)$, $\mathbf{n} \perp \mathbf{x}$, $\sigma_n < \sigma_x$ (relevant side information)
 $\begin{array}{c} \mathbf{x}^{(101)} \\ \mathbf{x}^{$

DSC BY NESTED QUANTIZER [Pradhan and Ramchandran, 2003]



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DSC BY NESTED QUANTIZER [Pradhan and Ramchandran, 2003]



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DSC VS. CHANNEL CODING

improvements to nested quantizer:

- better quantization (TCQ)
- channel coding in DSC



- **y** viewed as a *virtual* channel output,
- extra information generated at encoder performs error correction

DSC resembles channel coding: Trellis code, Turbo code, LDPC code can be properly applied.

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DSC USING LDPC: BINARY CASE

Slepian-Wolf Code: Discrete Source, lossless transmission



The initialization step incorporates the side information:

$$q_i^{init} = \log(\frac{\Pr(\mathbf{v}_i^1 = 0 | \mathbf{w}_i)}{\Pr(\mathbf{v}_i^1 = 1 | \mathbf{w}_i)})$$

 q_i^{init} is determined by statistical correlation between **v** and **w**.

DSC USING LDPC: CONTINUOUS CASE

Wyner-Ziv Code: Non-discrete Source, lossy transmission

- Quantizer + Slepian-Wolf Code
- Side information propagation
 - Decoded higher bit-planes serve as side-information for lower bit-planes



DSC USING LDPC: CONTINUOUS CASE



$$\Pr(\mathbf{v}_{i}^{1}=0|\mathbf{w}_{i}) = \sum_{m=2}^{M} \sum_{b_{m}=0}^{1} \Pr(\mathbf{v}_{i}^{1}=0, \mathbf{v}_{i}^{2}=b_{2}, \dots, \mathbf{v}_{i}^{M}=b_{M}|\mathbf{w}_{i})$$

$$\begin{aligned} \mathsf{Pr}(\mathbf{v}_i^m = \mathbf{0} \quad | \quad \hat{\mathbf{v}}_i^1, \dots, \hat{\mathbf{v}}_i^{m-1}, \mathbf{w}_i) = \\ & \sum_{n=m+1}^M \sum_{b_n=0}^1 \mathsf{Pr}(\mathbf{v}_i^m = \mathbf{0}, \mathbf{v}_i^n = b_n, \mathbf{v}_i^M = b_M | \hat{\mathbf{v}}_i^1, \dots, \hat{\mathbf{v}}_i^{m-1}, \mathbf{w}_i] \end{aligned}$$

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TRANSMISSION OF NOISY OBSERVATION WITH SI

The observation at the encoder is indirect observation



Utilization of DSC

- Potential: The existence of **y**₀ motivates DSC
- Challenge: irrelevant information in **y**₁

A SEPARATIVE STRUCTURE

It can be shown:

$$\arg\min_{m} E\left[\left\| \mathbf{X} - \tilde{\mathbf{x}}_{m}(\mathbf{Y}_{0}) \right\|^{2} \mid \mathbf{y}_{1} \right] = \arg\min_{m} E\left[\left\| \mathbf{v} - \tilde{\mathbf{v}}_{m}(\mathbf{Y}_{0}) \right\|^{2} \mid \mathbf{y}_{1} \right]$$

where

$$\mathbf{v} = \mathbf{R}_{\mathbf{xs}}\mathbf{R}_{\mathbf{s}}^{-1}\mathbf{y}_1, \mathbf{s} = \mathbf{y}_1 - \hat{\mathbf{y}}_1(\mathbf{y}_0)$$



optimal transmission of v guarantees optimal estimation of x.
DSC techniques utilized for v as a practical scheme.

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2 DSC FOR NOISY OBSERVATION A Separative Structure

Connection With Transform Coding

CONNECTION WITH TRANSFORM CODING

Transform coding as a practical scheme after the proposed decomposition:



The proposed decomposition generalize [Sakrison, 1968] into side-informed scenario.

- The redundancy of correlated data motivates DSC
- DSC resembles channel coding
- LDPC/Turbo adapt well for DSC
- SI propagation for continuous Symbols
- DSC for noisy observations: decomposition
- Connection between proposed decomposition with transform coding

Reference

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