Lecture Physical Modeling of a Plucked String (1)

General methods for sound synthesis:

1. Sampling methods, concatenative sampling. Synthesis
2. Structured sampling: sample more fundamental physical quantities instead of acoustic pressure wave, e.g. impulse response.
3. Spectral Models: Match spectral characteristics e.g. LPC. FM synthesis, phase vocoder, etc.
4. Virtual Analog: digital models for analog synthesizers.
5. Physical Modeling: Computational modeling of physical processes.

Wave Equation for 1D ideal string:

$$\underbrace{K}_{\text{string tension}} \cdot \frac{\partial^2}{\partial x^2} y(x,t) = \underbrace{\varepsilon}_{\text{mass density}} \frac{\partial^2}{\partial t^2} y(\underbrace{x}_{\text{position}}, \underbrace{t}_{\text{time}})$$

transverse displacement

$$\frac{\partial^2}{\partial x^2} y(x,t) = \frac{1}{\frac{K}{\varepsilon}} \cdot \frac{\partial^2}{\partial t^2} y(x,t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} y(x,t)$$

where $c = \sqrt{\frac{K}{\varepsilon}}$

arbitrary function 2nd order differentiable

Traveling Wave Formulation: $y(t,x) = y_r(t - x/c) + y_l(t + x/c)$

(Solution) In fact, any linear combination of the two traveling waves would work.

Digital Waveguide Modeling: Sample traveling waves in time and space.

$$y(nT, mX) = y_r(nT - mX/c) + y_l(nT + mX/c)$$

Let $X = cT$

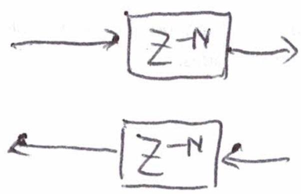
$$= y_r(nT - mT) + y_l(nT + mT)$$

sampling interval in space

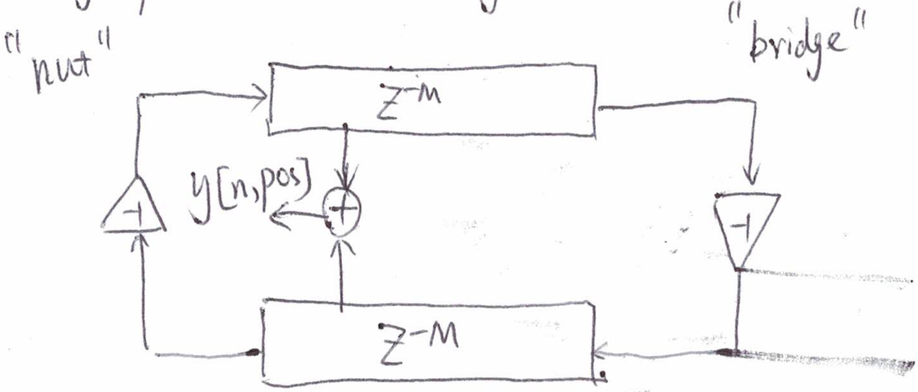
sampling period

$$y[n, m] = y^+[n-m] + y^-[n+m]$$

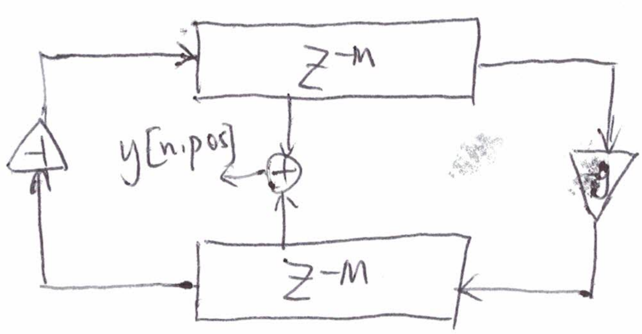
For an N-sample section of an ideal string, the vibration can be simulated as the superposition of a bidirectional delay line



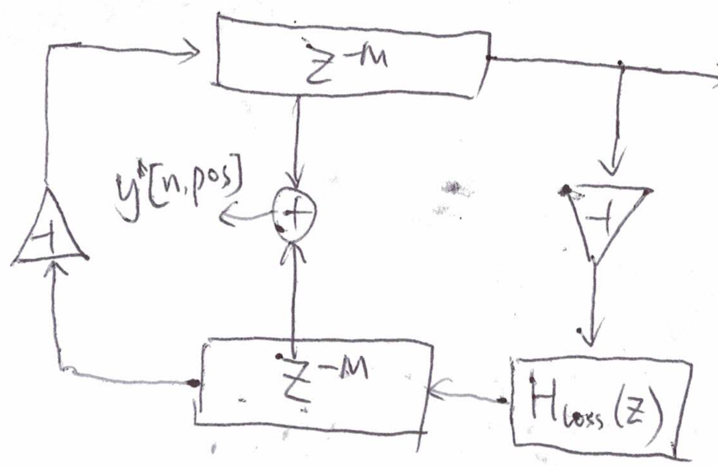
For rigidly terminated strings (lossless)



Add a loss gain: this loss is the same for all frequencies



Replace With a frequency-dependent loss.



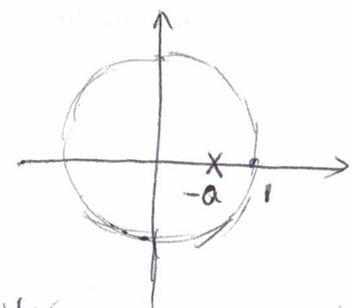
output to bridge
then to body resonance.

loss filter

One-pole filter for Loss filter:

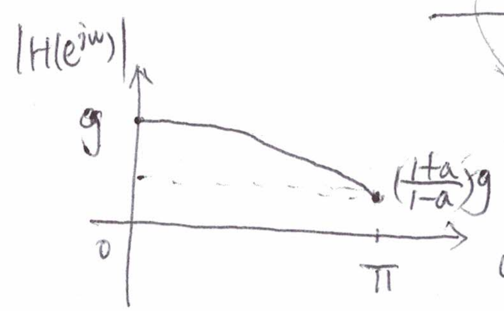
$$H_{Loss}(z) = g\frac{1+a}{1+az^{-1}},$$

pole: $z=-a$



$0<g<1, \quad -1<a\leq 0$

$$|H(e^{j\omega})| = \frac{g(1+a)}{|e^{j\omega}+a|}$$

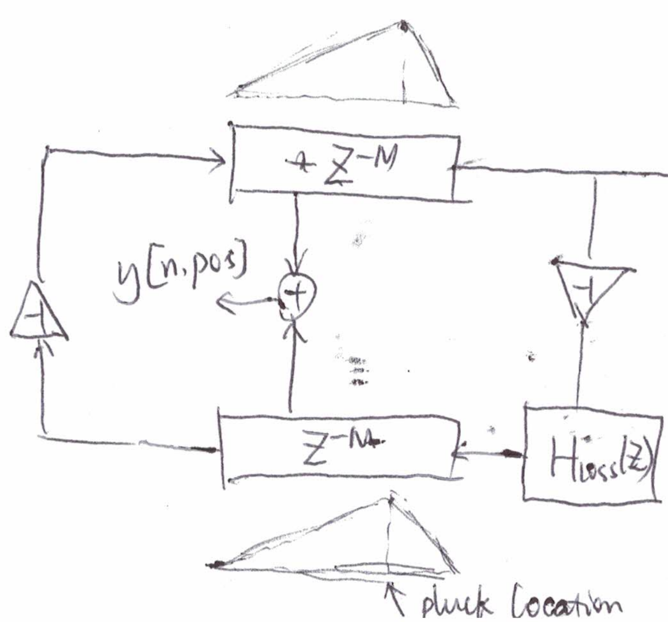


LPF: high frequencies decay faster.

A simple <u>plucked</u> String Model: How to simulate this pluck?

A pluck is simulated as the <u>initial displacement</u> of the string.

triangular shape.

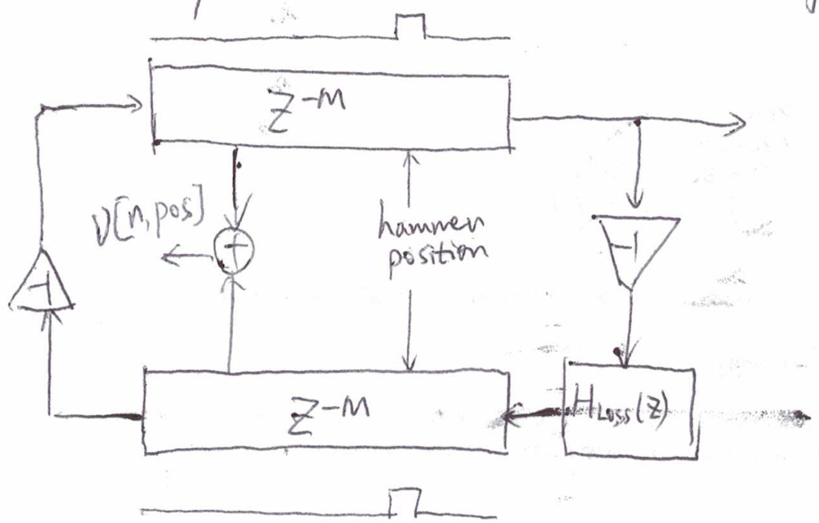


output to bridge

An Ideal Struck String: How to simulate a struck? (4)

A struck is simulated as the initial velocity of the string.

Consider velocity wave: Then take integration to obtain displacement.



Note: If we take the 2nd order derivative of the triangular displacement of a plucked string, we also get an impulse input.
This suggests that the struck string model can also be used to simulate plucked string vibration, after two integrations.

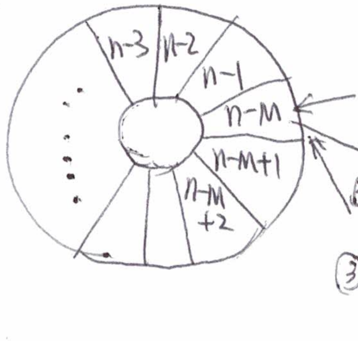
Delay Line with a fixed delay: Z^{-M}

$$Y(Z) = Z^{-M} \cdot X(Z)$$

$y[n] = x[n-M]$: At time n, the model needs to access the M-th previous sample of x.

We need to store the M previous values of x using a buffer (circular)

At time n:



C/C++: $0 \le curpos < M$
curpos = (curpos + 1) % M;

Matlab: curpos = curpos % M + 1;
$0 < curpos \le M$

Fractional Delay line: Interpolation using integer delay values ⑤

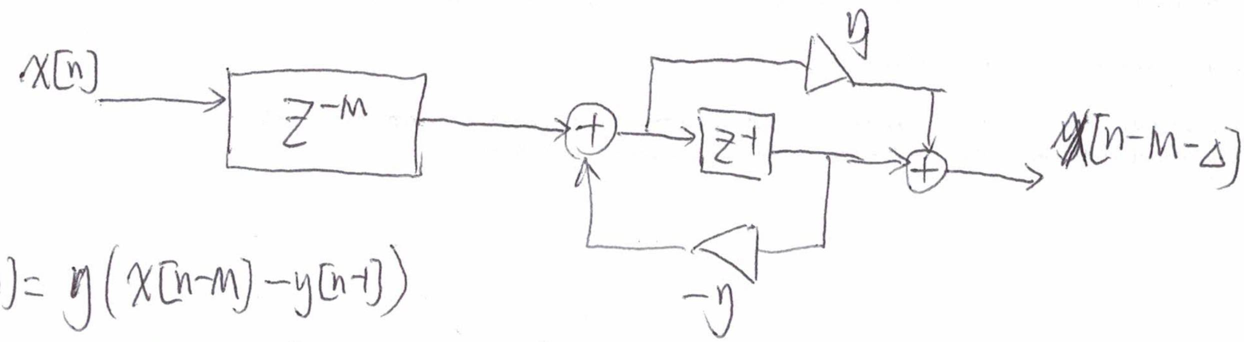
① Linear interpolation. $z^{-(M+\Delta)}$ Let ~~$M = \lfloor M \rfloor + \Delta$~~, $\Delta \in [0,1)$

~~$y[n] = (1-\Delta)\, x[M] + \Delta \cdot x[M-1]$~~

$$y[n] = (1-\Delta)\, x[n-M] + \Delta\, x[n-M-1]$$

LPF: amplitude error.

② First-order All-pass filter interpolation.



$y[n] = \eta\,(x[n-M] - y[n-1])$
$\quad + x[n-M-1]$

$$H(z) = \frac{\eta + z^{-1}}{1 + \eta z^{-1}}$$

At low frequencies $(z \to 1)$, phase delay $= \dfrac{1-\eta}{1+\eta}$

Let it equal to Δ, then $\eta = \dfrac{1-\Delta}{1+\Delta}$

In practice, split M and Δ, such that $\Delta \in [0.3,\ 1.3)$

This is because if Δ is close to zero, then $\eta \to 1$.

pole: $z = -\eta$,

Zero: $z = -\dfrac{1}{\eta}$

would possibly cancel due to numerical issues. there would be zero delay.