Repeatability and statistics: Procedure

ME 240: Fundamentals of Instrumentation & Measurement • D. H. Kelley and I. Mohammad

Introduction

No measurement is perfect, so engineers commonly measure repeatedly, even for a single object or operation. Moreover, we often make measurements of many objects or operations, with the idea that they share characteristics which could be used to predict future results, for example in a factory producing parts meant to be as nearly identical as possible. Statistical tools allow such characterizations and predictions. In this exercise, you will apply statistical tools to characterize and predict the behavior of a spring-loaded launcher of metal balls (see Fig. 1).

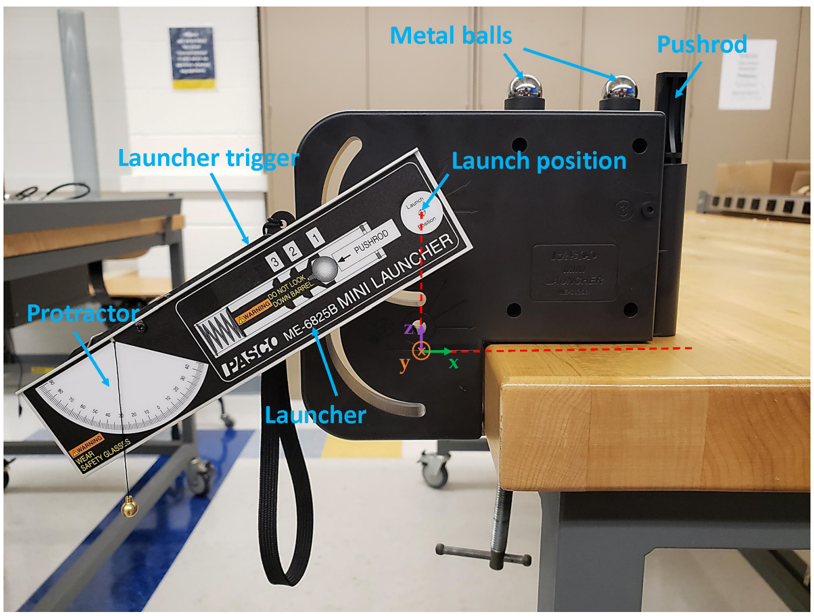

Figure 1: A ball launcher, with (x, y, z) directions labeled. The metal balls are spring-loaded into the launcher with the help of a pushrod. The launcher trigger releases the spring thus lifting the ball upto the launch position and projecting it at a particular velocity. The projection angle can be read by the built-in protractor.

For consistency, use a Cartesian coordinate system (x, y, z) whose origin is at the cross-point of the vertical line that goes through the center of the launch position, and the horizontal table top plane as shown in Fig. 1. The x direction increases horizontally along the length of the launcher, the y direction increases horizontally across the width of the launcher and the z direction increases vertically along the height of the launcher. Thus, if a ball fired by the launcher lands at location (x, y), x is the range of the shot and y is the lateral displacement.

Learning goals

- Calculate sample mean, sample standard deviation, and percent error.
- Construct histograms from measurements.
- Predict confidence intervals for future measurements.
- Use a χ^2 test to evaluate the assumption of a normal distribution.

Materials

- **Predicting the launcher's range:** Metal balls, launcher, mass balance, tape measure, protractor
- **Measuring launches:** Metal balls, launcher, carbon-less transfer graph paper, ruler or tape measure, clamps, tape

Safety

Closed-toed shoes and eye protection are required.

Predicting the launcher's range

This part of the exercise can be done either before or after measuring launches. Predict the launcher's range for the smallest speed (1 click) using your knowledge of projectile dynamics. One way to make the prediction is to use the fact that energy must be conserved, but other approaches are also possible. Assume the ball lands on the same flat, level surface on which the launcher rests. You may neglect the mass of the spring and friction within the spring. Your prediction should depend on the mass of the ball, the stiffness (spring constant: 200 N/m) of the spring, the angle of the launcher (30^o), and the initial position of the ball inside the launcher and with the spring compressed, in three dimensions, and perhaps other quantities as well. Measure as necessary. Does your prediction seem reasonable, based on your intuition? Check your prediction with your instructor.

Measuring launches

This part of the exercise can be done either before or after predicting the launcher's range. Set the launcher near the edge of a lab table such that the ball falls onto the table when launched, and use one or two clamps to hold the launcher in place. Consider using a bubble level to check that the table is leveled and adjusting it if necessary. Adjusting the screws on the back of the launcher, set the projection angle to 30^o. Place a piece of carbon-less transfer graph paper on the table at the place where you expect the ball to land after being launched, allowing for lateral variation of the landing point. Align the graph paper with the coordinate system, then tape it to the table and measure the three-dimensional position of its corner. Write "Set 1" on the paper. Consider placing a backstop beyond the paper. Launch the ball 20 times using the smallest speed (1 click), being as consistent as possible.

Each time it lands on the carbon-less paper, it will make a mark on the underlying sheet. Circle each with a pen or pencil and record the position of each. Secondary marks produced after bouncing should be ignored. Photograph the underlying sheet. Replace the paper with a new piece, labeled “Set 2”, aligning and locating it. Launch the ball 20 more times, recording the three-dimensional position of each landing. Photograph the underlying sheet and include both photos in the Deliverables.

Analysis, statistics, and predictions

Calculate the sample mean and sample standard deviation of the range and lateral displacement for each set of data (Set 1 and Set 2). Calculate the percent error for each mean value, given by

$$\text{percent error} = \frac{\text{actual} - \text{predicted}}{\text{predicted}} \times 100\%.$$

For each data set, construct a histogram of the range and a histogram of the lateral displacement. On the same axes, plot normal distributions with the same mean and standard deviation as the measurements. From each normal distribution, what is the interval in which you would expect future measurements to fall, with 95% confidence? Apply the χ^2 goodness-of-fit test to each data set to test the assumption of a normal distribution.