Particle tracking velocimetry: Procedure

ME 240: Fundamentals of Instrumentation & Measurement • D. H. Kelley and I. Mohammad

Introduction

Optical methods are powerful, convenient, and common for measuring the velocity of transparent fluids. To make motion visible, small tracer particles are typically added to the fluid and assumed to follow fluid motion faithfully. Each particle can then be tracked with an automated image processing algorithm, such that particle position is recorded in each frame of a movie, and velocity and acceleration can be calculated via numerical differentiation. This approach is known as particle tracking velocimetry (PTV).

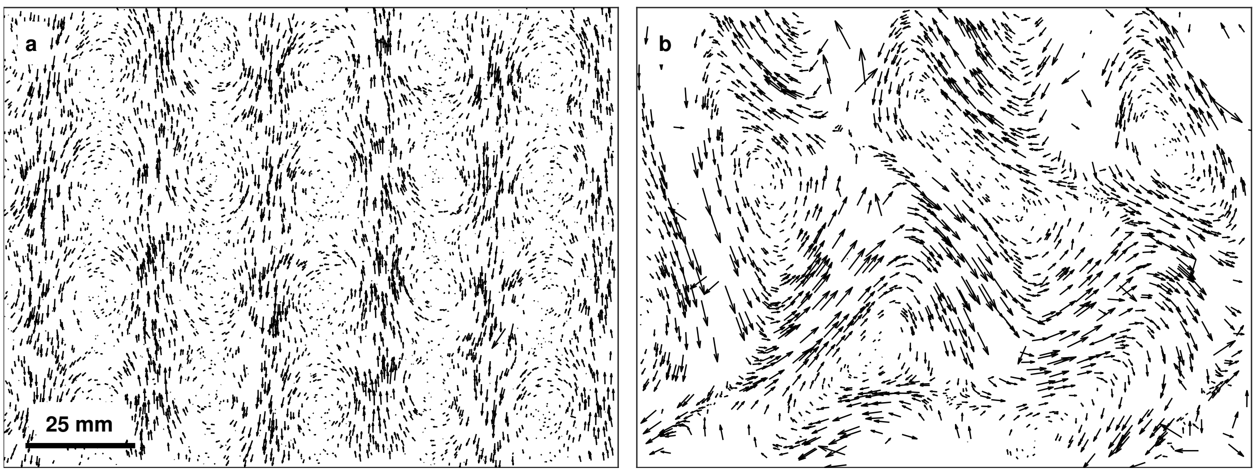

Figure 1: Fluid driven by uniform electrical current and magnets whose polarity alternates in stripes can move in parallel shear bands (a) or in vortices (b), depending on the Reynolds number. Here, each arrow indicates the position and velocity of a tracer particle.

In this exercise, you will use PTV to measure flows in the laboratory. The flows will be produced by passing electrical current through a thin layer of conductive fluid above an array of magnets, with their polarities arranged in stripes. Currents and magnetic fields combine to produce forces that can drive fluid flow. In this arrangement, small currents produce slow flows comprised of parallel shear bands aligned with the stripes, as shown in Fig. 1a. Large currents, however, produce flows that are not only faster but also more complicated in their spatial and temporal structure, as shown in Fig. 1b. Changes occur because the laws of fluid dynamics admit multiple solutions, and a different solution is stable (and therefore manifested) for slow flow than for fast flow. The transition from parallel shear bands to a more complicated flow structure (initially, an array of vortices) is known as an instability, and it can be predicted by the Reynolds number $\mathrm{Re} = UL\rho/\mu$, where $U = \langle \boldsymbol{u}^2 \rangle^{1/2}$ is the root-mean-square velocity ($\boldsymbol{u}$ is the velocity and brackets $\langle \cdot \rangle$ denote averaging over both space and time), $L/2$ is the magnet stripe width (Refer to Fig. 2), ρ is the density, and μ is the dynamic viscosity.

To calculate Re, you will measure U and separately determine the viscosity μ using the Stokes' law, which states that the viscous drag force on a sphere is $F_D = 6\pi\mu RV$, where R

is the radius of the sphere and V is its velocity. For a free-falling sphere at terminal velocity, the drag force and buoyant force are balanced by the weight, so μ can be calculated if V is measured.

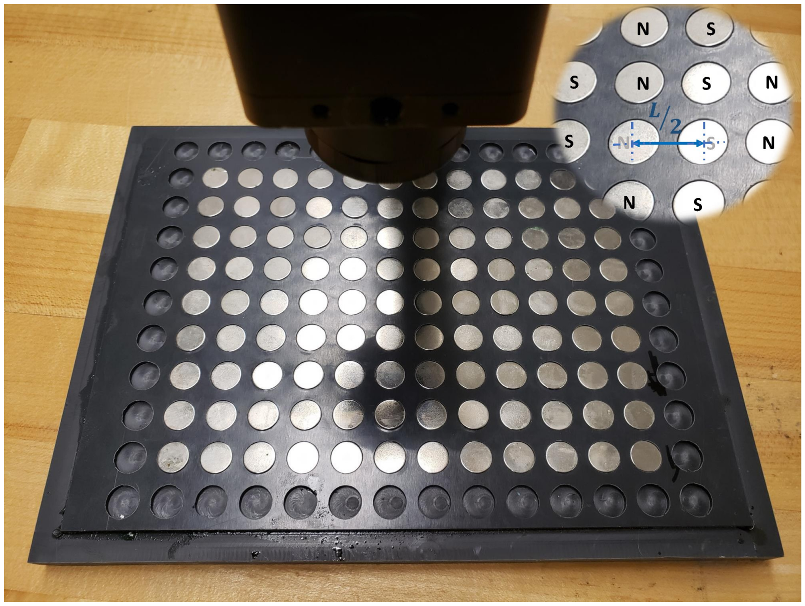

Figure 2: The array of magnets with their polarities arranged in stripes. $L/2$ is the center-to-center distance between two adjacent magnets.

Learning goals

- Measure the viscosity of a fluid.
- Measure the velocity of a fluid flow as it varies over space and time.
- Gain familiarity with optical and image processing techniques.
- Gain familiarity with fluid dynamical instabilities and their dependence on the Reynolds number (Re).
- Gain familiarity with viscous drag.
- Determine the value of Re at which a flow in the laboratory experiences an instability, transitioning from parallel shear bands to vortices.

Materials

- **Mixing the solution:** Water, glycerol, copper sulfate salt, mass balance, graduated cylinder, beakers

- **Determining fluid density:** Mass balance, graduated cylinder, beakers, stopwatch, phone holder
- **Determining the viscosity:** Nylon spheres, tall beaker, ruler, webcam
- **Imaging flows:** Plastic tray with electrodes, magnet board, cover with lights and camera, fluorescent tracer particles, wooden coffee stir, dish soap, power supply, multimeter, cables, plastic/wooden ruler

Safety

Closed-toed shoes, eye protection, and chemical gloves are required.

Mixing the solution

First, produce 175 g of a solution that is 10% copper sulfate (by mass) in water. (That's 10% of the *total* mass, not 10% of the water mass.) Note the mass of water and the mass of copper sulfate in the Deliverables document. Then, combine your copper sulfate solution with glycerol (also known as glycerin) to produce a new solution that is 20% glycerol by volume (20% of the *total* volume). Note the required glycerol volume in the Deliverables document. Re-using solution that is more than a few hours old tends to produce poor results.

Determining the fluid density

Use a graduated cylinder, a mass balance, and perhaps a beaker to determine the density of your solution. Note the volume, mass, and density in the Deliverables document.

Determining the viscosity

The viscosity can be determined using Stokes' law. A sphere falling through a fluid initially accelerates but eventually reaches a terminal velocity, fast enough that the drag force (which increases with speed) prevents further acceleration, and an equilibrium is achieved. In the Deliverables document, draw a free body diagram of a sphere falling at terminal velocity. You should include the drag force $F_D = 6\pi\mu RV$, the buoyant force $F_B = 4\pi R^3\rho g/3$ (which equals the weight of the water displaced by the sphere), and the weight of the sphere. Write the weight of the sphere in terms of the sphere density ρ_s, which is not the same as the fluid density ρ. Use your free body diagram and Newton's second law to write a formula for the viscosity μ in the Deliverables document. Verify your formula with your instructor before continuing.

Fill a tall beaker or graduated cylinder with your solution and place a stopwatch next to it. Position a phone such that its camera sees the beaker and stopwatch, being sure to keep the phone straight, perhaps using a holder. Start recording a movie. Place a nylon sphere into the fluid and measure the terminal velocity V. Find three moments in the movie at which the sphere is aligned with three different marks on the vessel or on a ruler held next to the vessel. Knowing the change in the sphere's position and the change in time, you can calculate V for the interval between the first pair of marks and for the interval between the second pair of marks. If the two velocities indicate that the sphere is accelerating, and not at equilibrium, try again, using later times. Once you know the terminal velocity V, use your formula to calculate the viscosity μ. The nylon spheres have radius $R = 794$ μm and density $\rho_s = 1150$ kg/m^3. Note your results in the Deliverables document.

Imaging flows

First, use the ruler to measure the magnet stripe width $L/2$, recording the value in the Deliverables document. Add your fluid to the tray until the electrodes are well connected via fluid. Scoop a tiny pinch of fluorescent tracer particles (results are typically better with fewer particles) from the bottle using a wooden coffee stir and add them to the solution. Add one tiny drop of dish soap, then mix to distribute the particles. Sweep away any soap bubbles. Turn on the lights, then cover the apparatus with a box to exclude ambient light. Connect the power supply, the multimeter and (the electrodes of) the fluid apparatus in series so you can use the multimeter to measure current, since power supplies do not provide high-accuracy current measurements. Apply a small current I. Though flow forces are proportional to current, because the resistance of the fluid layer is large, adjusting the voltage may give you better control. Watch and verify that the particles move in parallel shear bands. Increase the current gradually, over a minute or two, and verify that the shear flow first accelerates, then gives way to vortices. In the Deliverables document, note the critical current I_c and critical voltage V_c at which vortices arise.

Observe the flow using the webcam and the Windows Camera App, remembering to remove the lens cap if necessary. Use manual focus to get a sharp image. Spending a few extra minutes to produce good images will save much more time later! Consider asking your instructor's opinion. Then, photograph a plastic or wooden ruler near your fluid layer and use the image to determine the size of each pixel in mm, noting that value in the Deliverables document.

Apply a small current to produce a slow shear flow. Once the flow has stabilized, record a movie 5-10 s long. Watch the movie to verify that particles are bright, lighting is uniform, and images are not blurry, recording a new movie if necessary. Note the current, frame rate (look in camera settings or properties of the movie file), and movie file name in the Deliverables document. Then, record at least eight more movies, each 5-10 s long, each with a larger current than the one before, always allowing the flow to stabilize before recording. Your main objective is to determine the value of the Reynolds number Re at which shear flow gives way to vortices, so record movies at currents above and below the critical value. Dispose of your solution in an appropriate waste container.

Measuring and analyzing flow velocities

All data analysis will be done in Matlab, leveraging a collection of custom-written functions, which are installed on the computers in the lab and can also be downloaded from the course website if you want to work at home[1].

[1] Put the files in your Matlab home folder (typically `C:\Users\your_username\Documents\MATLAB` or similar) so the Matlab app knows where to find them.

Start by calculating a background image from one of your movies (often, the fastest flow works best).

```
bg = BackgroundImage('your_movie','background.png',2) % e.g., IMG0001.mp4
```

For more information about any of the Matlab functions, use the `doc` command, e.g., `doc BackgroundImage`. Include the image in the Deliverables document. Then, track particles in your slowest movie first. Start by entering these commands at the Matlab command line:

```
obj = PredictiveTrackerClass;
obj.Fnames = 'your_movie'; % e.g., IMG0007.mp4
obj.Bg = 'background.png';
obj.Arealim = 1; % any bright pixel is a particle
obj.Chan = 2; % green particles (not red or blue)
obj.Framerange = [1 75] % first 75 frames, for starters
```

Next, you must choose two important tracking parameters. The first is the threshold `obj.Thr`. A bright pixel will be identified as a particle if and only if its brightness exceeds the background image by this value. Brightness in digital images often ranges from 0 (e.g., for no brightness in the green channel) to $2^8 - 1 = 255$ (for brightest green). The second is the search radius `obj.Max_disp`. The algorithm forms particle tracks by connecting bright spots from frame to frame. Once a particle has been located in one frame, its likely position in the next frame is estimated from its current position and velocity. A connection is made to whatever bright spot appears closest to the predicted location — but only if it falls within the search radius `obj.Max_disp` (measured in pixels). Choosing values for `obj.Thr` and `obj.Max_disp` will require iteration, so start by making guesses and tracking particles:

```
obj.Thr = your_threshold; % between 0 and 255
obj.Max_disp = your_Max_disp; % pixels
myTracks = PredictiveTracks(obj,1) % '1' enables visualization
```

If the tracking is inconveniently slow, stop the calculation (`CTRL-C`) and increase `obj.Thr`. Otherwise, in the resulting interactive window (see Fig. 3), click "play" at bottom right to see the first 75 frames played with particle tracks overlaid in color. Tracks should faithfully follow visible particles throughout the field of view. If fast flow regions are devoid of tracks, increase `obj.Max_disp`. If particles connect in non-physical ways causing tracks to cross like spiderwebs, decrease `obj.Max_disp`. If many particles go untracked, reduce `obj.Thr`. If image noise is tracked like particles, increase `obj.Thr`. Repeat the tracking as necessary, varying `obj.Thr` and `obj.Max_disp`, until you are satisfied with the tracks. Then, note your values of `obj.Thr` and `obj.Max_disp` in the Deliverables document. Finally, extend the frame range via

```
obj.Framerange = [1 inf];
```

and do the tracking one more time.

Visualize the first 100 frames of your tracks using

```
figure; plot_tracks(myTracks,bg,[0 100])
```

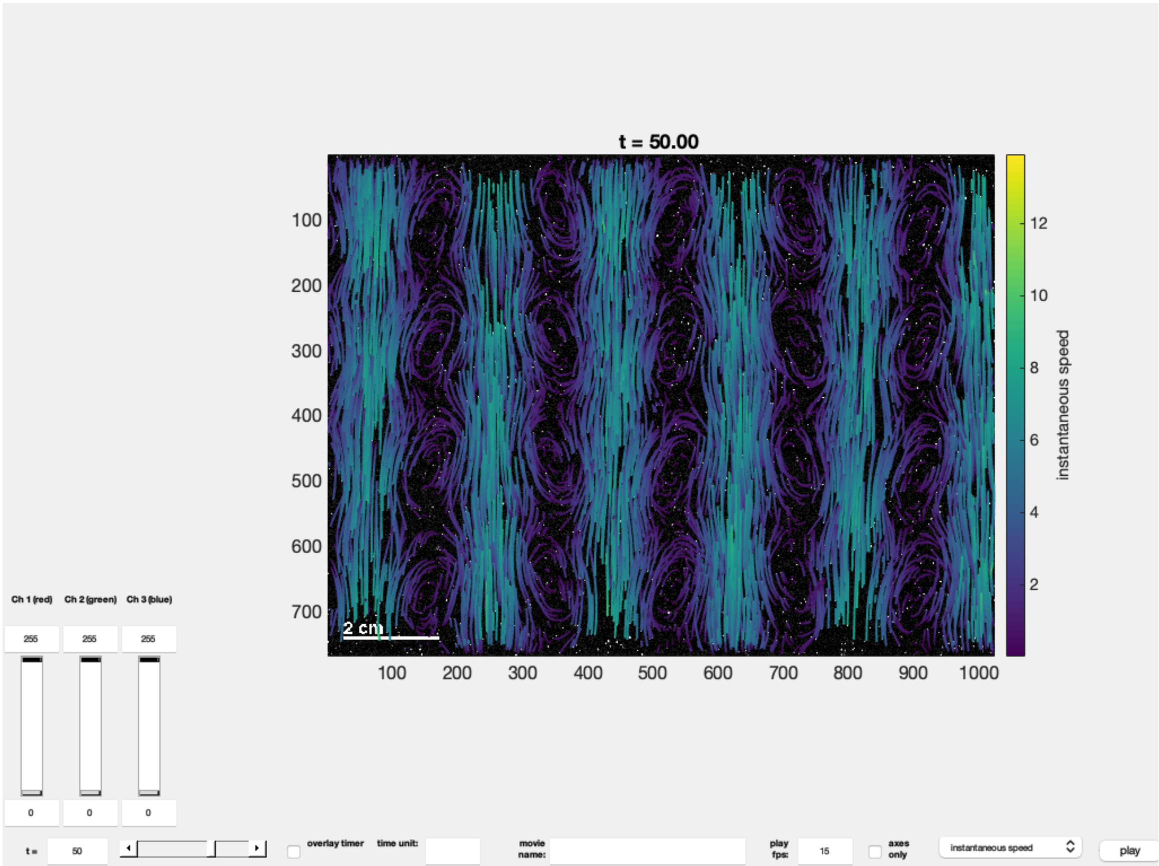

Figure 3: An interactive visualization window facilitates particle tracking.

Label the axes and add a labeled colorbar, then save the figure as an image to be included in the Deliverables document. (Consider saving a `.fig` file and/or a `.mat` file as well, for later.) The struct array `myTracks` contains information about your tracks, sorted particle-by-particle. Concatenate all the velocities using

```
[u,v] = Velocities(myTracks,[],0); % ‘0’ avoids visualization
```

Here `u` and `v` list measurements of all horizontal and vertical velocity components, respectively. Their units are pixels/frame. Calculate the root-mean-square *horizontal* velocity $\langle u^2 \rangle^{1/2}$, which measures horizontal flow — that is, deviation from the pattern of vertical shear bands. Calculate the Reynolds number Re using $U = \langle u^2 + v^2 \rangle^{1/2}$. Note $\langle u^2 \rangle^{1/2}$ and Re in the Deliverables document, converting units from pixels and frames to metric units using the pixel size and frame rate.

Repeat the tracking procedure for each of your other movies, noting $\langle u^2 \rangle^{1/2}$ and Re for each. Use `plot_tracks` to visualize the tracks in your fastest movie. Plot the variation of Re with current I. Plot the variation of $\langle u \rangle^{1/2}$ with Re. Label all axes and consider saving `.fig` files and `.mat` files, then include images of both plots in the Deliverables document. From your measurements, determine the value of the critical Reynolds number Re_c at which the instability occurs, causing shear bands to give way to vortices. Note its value in the Deliverables document.