Music, vibrations, and frequency analysis: Procedure

ME 240: Fundamentals of Instrumentation & Measurement • D. H. Kelley and I. Mohammad

Introduction

Considering the frequency content of a signal or measurement often brings powerful insights. Though spectral analysis (as it is called) can be used with any signal that varies over time or space, its use with sound is especially familiar. In this exercise, you will perform two tasks, both using spectral analysis.

Your first task is to tune a ukulele and demonstrate that you have tuned it correctly by plotting the spectrum produced by each of its strings. A musical instrument's pitch is synonymous with its fundamental frequency, and any instrument could be tuned using a procedure similar to what you will do. Stringed instruments, including ukuleles, have one-dimensional vibrational modes that can be visualized directly.

Your second task is to predict the frequency of the noise produced by a damaged ball bearing and experimentally verify your prediction. Bearings are essential for machinery and can be damaged by grit, corrosion, and impacts. Even in good conditions, bearings eventually wear out. Because bearings are often located deep inside a machine, visual inspection is not always possible, so determining a bearing's health by the sounds it makes can save a lot of time.

Learning goals

- Record and play sounds using Matlab.
- Calculate and analyze power spectra.
- Gain familiarity with Fourier transforms and methods for their calculation.
- Tune a ukulele using spectral analysis.
- Predict the frequency of the noise caused by a damaged bearing
- Experimentally verify the frequency of the noise caused by a damaged bearing

Materials

- **Tuning a ukulele:** ukulele, microphone
- **Diagnosing a damaged bearing:** motorized bearing assembly, power supply, microphone, tachometer, extra pulleys, calipers

Safety

Closed-toed shoes are required. Long hair must be pulled back and kept away from moving machinery.

Tuning a ukulele

With the microphone connected to the computer, audio can be recorded and played back in Matlab with commands like these:

```
Fs = 44100; % sampling rate (samples per second)
Nbits = 16; % bits per sample
duration = 5; % record for 5 s
obj = audiorecorder(Fs,Nbits,1);
recordblocking(obj,duration);
y = getaudiodata(obj); % sound amplitude
sound(y,Fs) % play it back
```

Remember to use `doc` if you need more information about how to use a Matlab function. Audio is typically sampled with frequency 44,100 Hz. In the Deliverables document, explain why. Then, record the sound of the A string of your ukulele, as labeled in Fig. 1. Make two plots of the amplitude of the sound as it varies over time, one above the other, using commands like

```
figure; subplot(2,1,1); % create upper axes
plot(...); xlabel(...); ylabel(...); title(...);
subplot(2,1,2); % create lower axes
plot(...); xlabel(...); ylabel(...); title(...);
```

In the upper plot, show the entire duration of your recording. In the lower plot, reduce the horizontal axis limits to a duration of 0.2 s, so that individual oscillations are visible. Ensure that the time axes of both plots use seconds (not sample number) as the unit of measurement. Label axes, add titles, and include your figure, with a caption, in the Deliverables document. Also, consider saving your figure as a `.fig` file and saving your data in a `.mat` file.

Figure 1: The strings of a ukulele are typically tuned to the notes G, C, E, and A, as shown.

The spectral power of a digital signal can be calculated using the Fast Fourier Transform algorithm, which is implemented in Matlab as `fft.m`. Calculate and plot the power spectrum of the sound you recorded using commands like these:

```
Ny = numel(y); % number of samples
Y = abs(fft(y)).^2; % spectral power
Y = Y(1:floor(Ny/2)); % ignore power of negative frequencies
f = Fs/Ny*(0:Ny/2-1); % corresponding frequencies (Hz)
figure; semilogy(f,Y)
```

The Fourier transform of a real signal is typically complex, with its real and imaginary parts representing components 90° out of phase (recall $e^{j\theta} = \cos\theta + j\sin\theta$, where $j = \sqrt{-1}$). Spectral power is given by the square of its magnitude. The corresponding frequencies range from zero to half the sampling frequency, as dictated by the Nyquist-Shannon sampling theorem. Since a signal's Fourier transform necessarily contains exactly as much information as the signal itself, and since the Fourier transform has both a real and imaginary part, the number of frequencies resolved must be half the number of samples recorded.

Identify the fundamental vibrational frequency of the A string — the frequency with greatest spectral power — and write it in the Deliverables document. Label the plot axes, add a title to indicate which string was recorded, and include your plot, with a caption, in the Deliverables document. Then, record the C, E, and G strings, plot their spectra, include those plots, with captions, in the Deliverables document, and write the fundamental frequency of each string in the Deliverables document.

In music, the A note above middle C is defined by having 440 Hz as its fundamental frequency. Adjust the tuning peg of your ukulele until its A string does indeed produce a tone with fundamental frequency 440 Hz±1%. Once you have tuned the A string, include a plot of its power spectrum in the Deliverables document, with a caption. Write down the measured fundamental frequency.

A musical octave spans frequencies ranging over a factor of two, and notes separated by an octave share the same name, so the next lower A note has frequency 220 Hz. Western music names 12 notes per octave, for example like this:

A, A#, B, C, C#, D, D#, E, F, F#, G, G#, A.

With so-called even tuning, the ratio of the frequencies of one note to the next is the same for all pairs, that is, the spacing is even on a logarithmic scale. Use that fact to calculate the frequencies of the C, E, and G notes which lie between 220 Hz and 440 Hz. Record your answers in the Deliverables. Then, adjust the tuning pegs of your ukulele until the G, C, and E strings produce tones with fundamental frequencies matching the ones you calculated within 1%. Include plots of all three power spectra, with captions, and write down the measured fundamental frequencies in the Deliverables document. Congratulations, you have now tuned a ukulele!

Diagnosing a damaged bearing

Figure 2 shows a ball bearing on a shaft driven by a motor. The ball bearing is damaged, and your goal in this task is to predict and experimentally verify the vibrational signature of the damage.

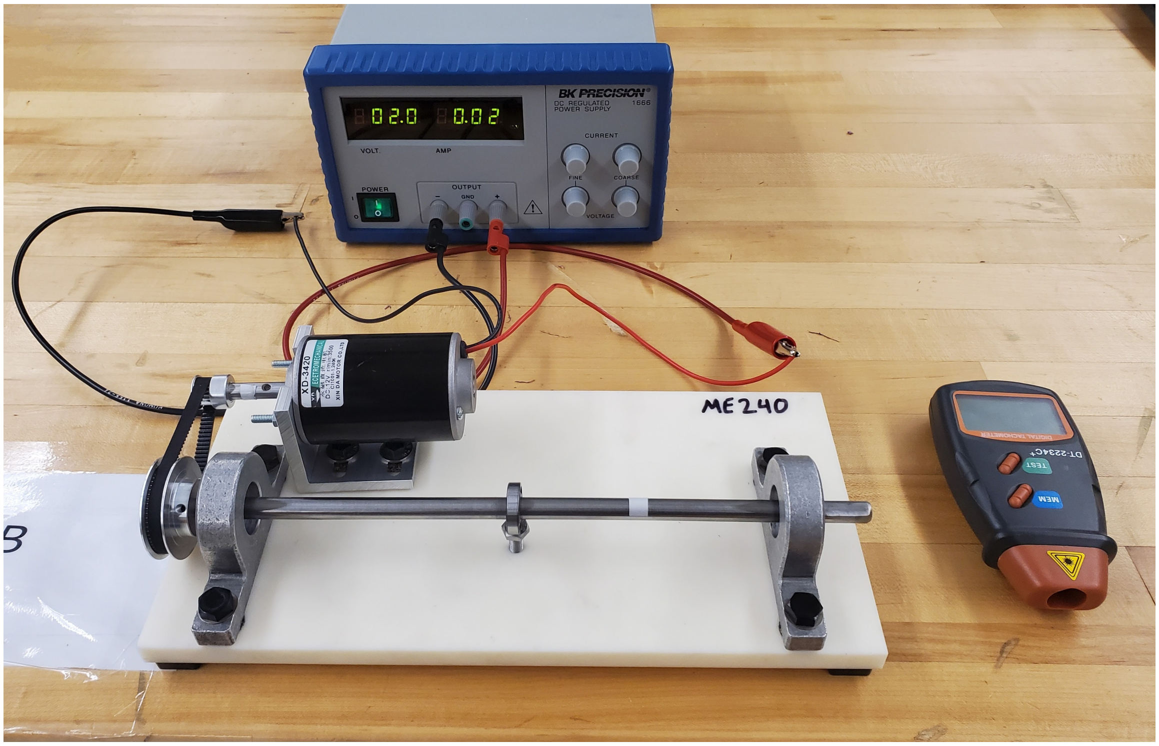

Figure 2: A damaged ball bearing (at center) on a shaft driven by a motor. To the right sits a tachometer, useful for measuring rotational frequencies.

The shaft rotation frequency f_s depends on the motor rotation frequency f_m because the shaft is linked to the motor through a timing belt:

$$f_s = \gamma f_m,$$

where γ is the gear ratio. Do you expect the shaft to spin faster or slower than the motor? Record your answer in the Deliverables document. Examine and/or measure the pulleys on the motor and shaft, or the duplicate pulleys, to determine the value of γ, recording it in the Deliverables document.

Set the power supply's voltage knob to zero, then connect it to the motor and switch it on. Raise the voltage to increase the rotation rate, being sure that you never exceed the rated voltage (written on the motor) so that you do not destroy the motor. Use the tachometer to measure f_m and f_s, recording their values. Using those measured values, calculate f_s/f_m and compare the value to γ as predicted.

The bearing was damaged by making a hole in its outer race (ring). As each ball rolls, it makes a click when falling into the hole and another when climbing out. The clicking frequency f_c depends on the shaft rotation frequency f_s and can be predicted because a ball bearing is geometrically equivalent to a planetary gear train:

$$f_c = 2 n_b f_s \frac{R_i}{(R_i + R_o)}, \tag{1}$$

where R_i and R_o are the inner and outer radii, respectively (see Fig. 3), and n_b is the number of balls in the bearing. The factor of two arises because there are two clicks as each ball passes the hole.

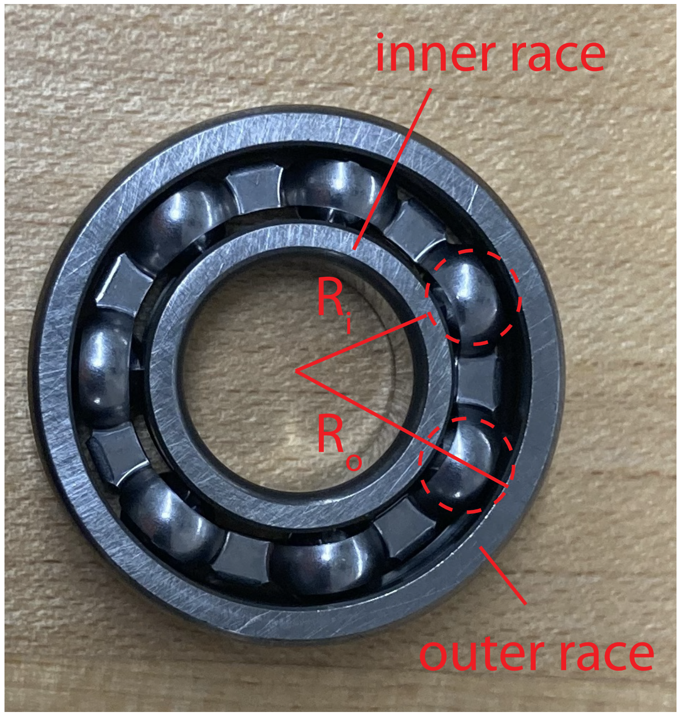

Figure 3: A ball bearing.

Measure the radii R_i and R_o. Determine the ball count n_b. Then, turn on the motor and use the tachometer to measure the shaft rotation frequency f_s. Use Eq. 1 to predict f_c. Record your results in the Deliverables document.

Use the microphone to record the sound of the spinning apparatus for a few seconds. Plot your recording, along with an enlargement 0.2 s long, as you did for the previous recordings. Label axes, add titles, and include your figure, with a caption, in the Deliverables document. Calculate and plot the power spectrum, which should have peaks at the shaft frequency and click frequency. Mark those two peaks and determine their frequencies from the data. Label axes, add titles, and include your figure, with a caption, in the Deliverables document. Compare the measured value of f_c to the predicted value, calculating the percent difference between them and recording it in the Deliverables document.