

Least-squares fitting

- Wheeler & Ganji 6.6
- We often want to fit measurements to a curve of analytic form
 - Can make empirical predictions from past trends
 - Can gain physical insight from parameter values (e.g., cooling rate of an object)
- Least-squares fitting lets us calculate the parameter values for the best fit to data
- Suppose you have n measurements (x_i, y_i) and want to fit the line $Y = ax + b$
 - Goal: Determine values of a and b that minimize error between $Y(x_i)$ and y_i
 - For a line, a and b are sometimes called “coefficients of linear regression”
 - Square the error so negative values don’t offset positive values

Least-squares fitting

- Total squared error is $E = \sum_{i=1}^n \underbrace{(Y(x_i) - y_i)}_{\text{model}}^2 = \sum_{i=1}^n \underbrace{(ax_i + b - y_i)}_{\text{model}}^2$

- When E is minimum, $\frac{\partial E}{\partial a} = \frac{\partial E}{\partial b} = 0$, so

- $$0 = \frac{\partial E}{\partial a} = \sum_{i=1}^n 2(ax_i + b - y_i)x_i$$

$$0 = \frac{\partial E}{\partial b} = \sum_{i=1}^n 2(\underbrace{ax_i}_{\text{two unknowns}} + \underbrace{b}_{\text{two unknowns}} - y_i)$$

$\left. \begin{array}{l} \text{two} \\ \text{equations} \end{array} \right\}$

- Can solve algebraically!
- For a parabola or exponential instead of a line, different model but same procedure

How accurate is the fit?

- Quantify accuracy of a fit with coefficient of determination r^2 :

$$r^2 = 1 - \frac{\sum_{i=1}^n \overbrace{(ax_i + b - y_i)^2}^{\text{model}}}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

— deviations from model
— deviations about mean
|
mean of
measurements

- Units of r^2 ? **None.**
- Value of r^2 if the fit is perfect? **$r^2 = 1$**
- Matlab's fit.m determines parameter values, coefficient of determination, more
 - polynomials, exponentials, power laws, logarithmics, Gaussians, surfaces, splines, sigmoids, ... and custom-written models

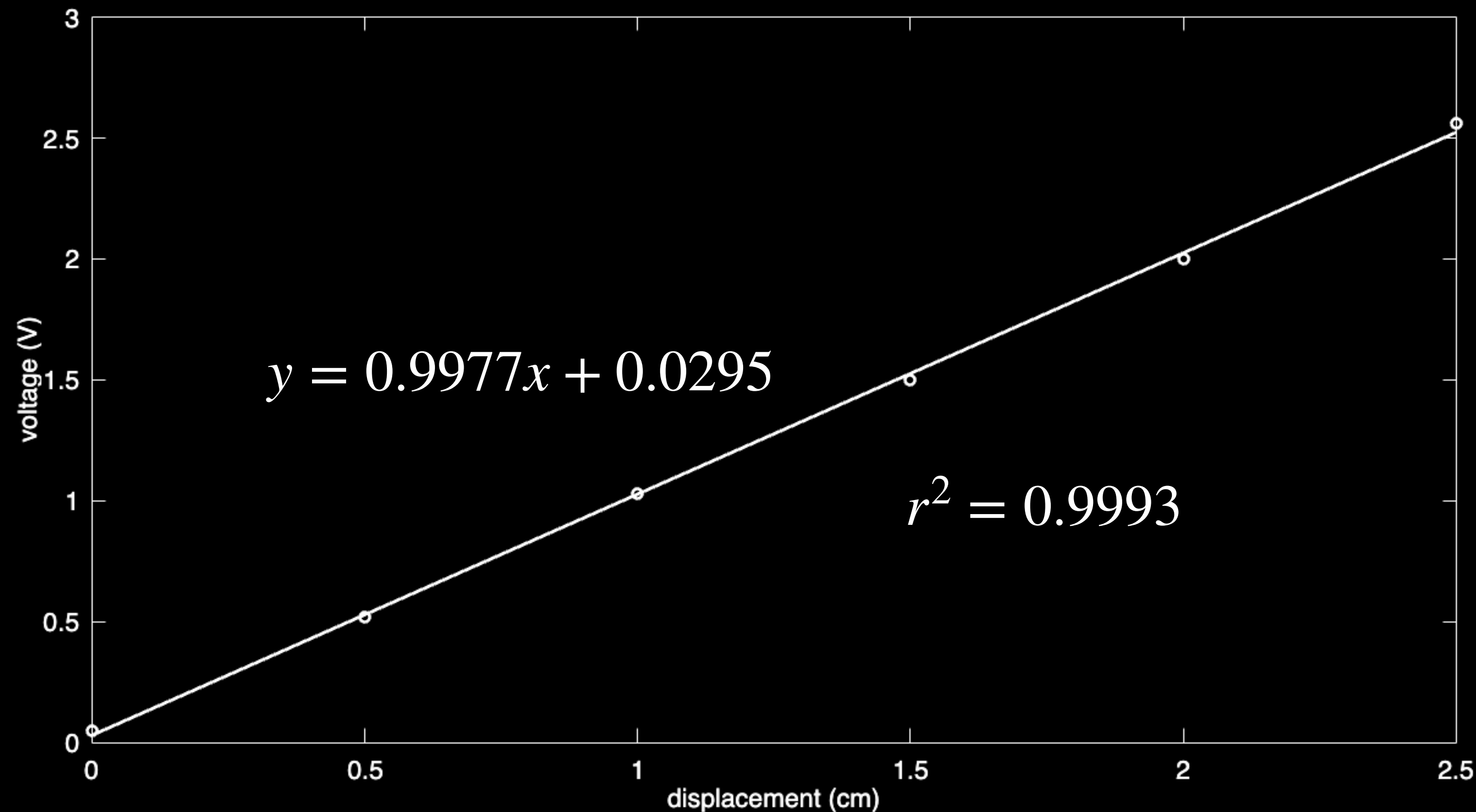
Ex. 6.20: A linear variable differential transformer (LVDT) is a transducer for measuring displacements, which outputs a voltage. Six displacements L_i and corresponding voltages V_i are listed below. Calculate the best linear fit and coefficient of determination.

L_i (cm): 0.00, 0.50, 1.00, 1.50, 2.00, 2.50

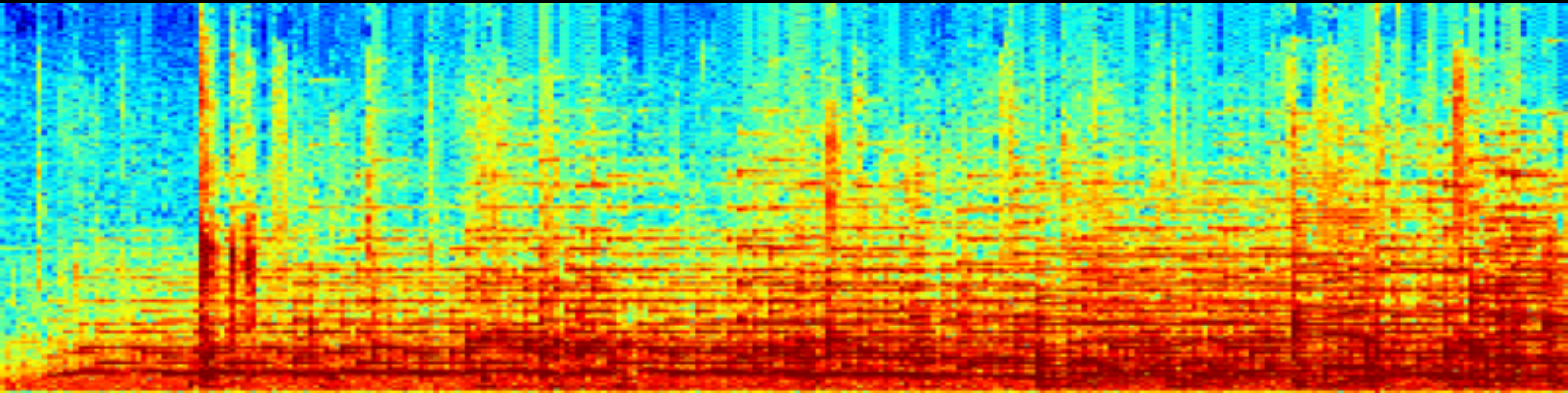
V_i (V): 0.05, 0.52, 1.03, 1.50, 2.00, 2.56

$$0 = \sum_{i=1}^n 2(ax_i + b - y_i)x_i$$

$$0 = \sum_{i=1}^n 2(ax_i + b - y_i)$$



Discrete Sampling and Analysis of Time-Varying Signals Wheeler & Ganji Chapter 5

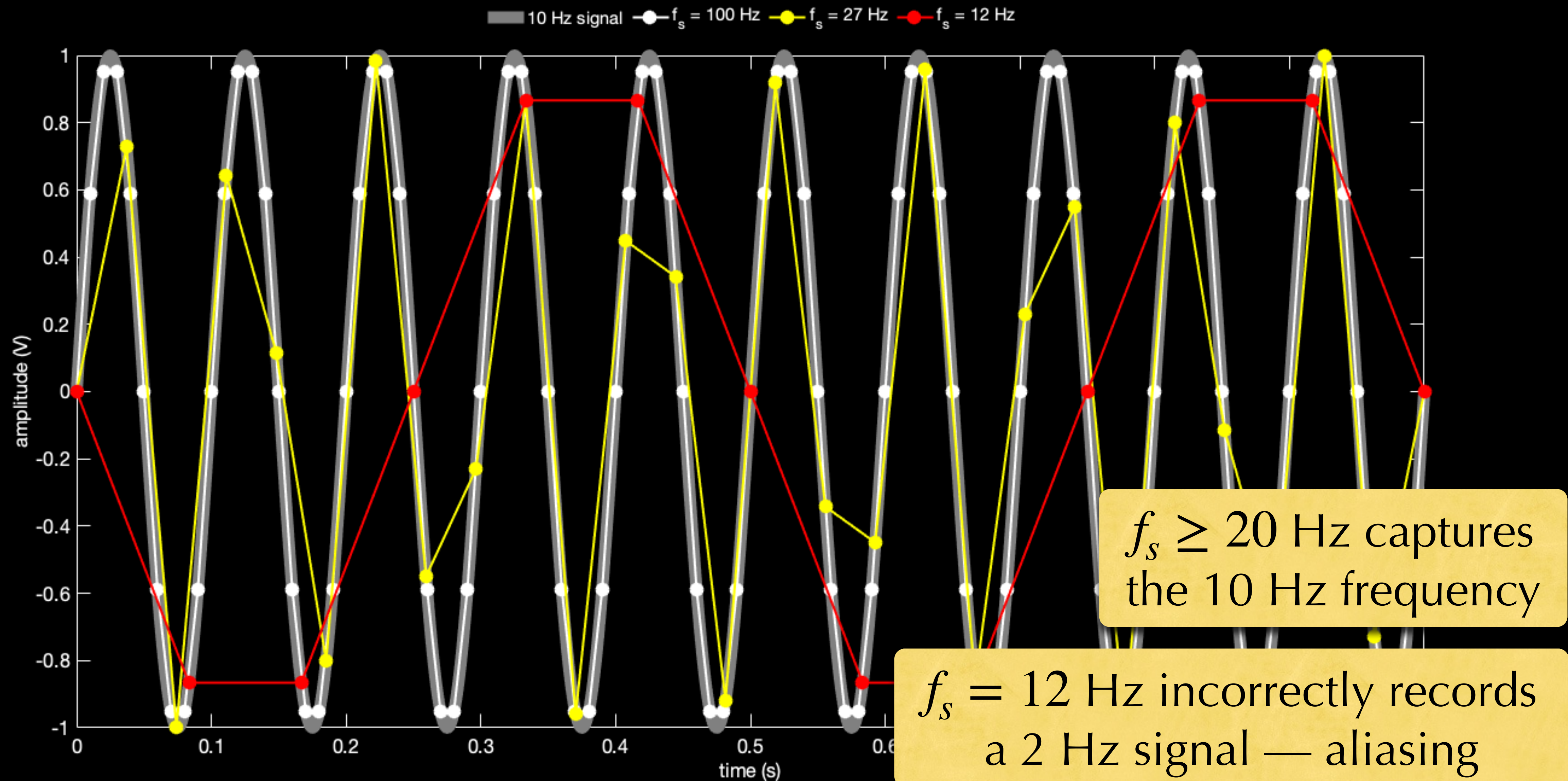


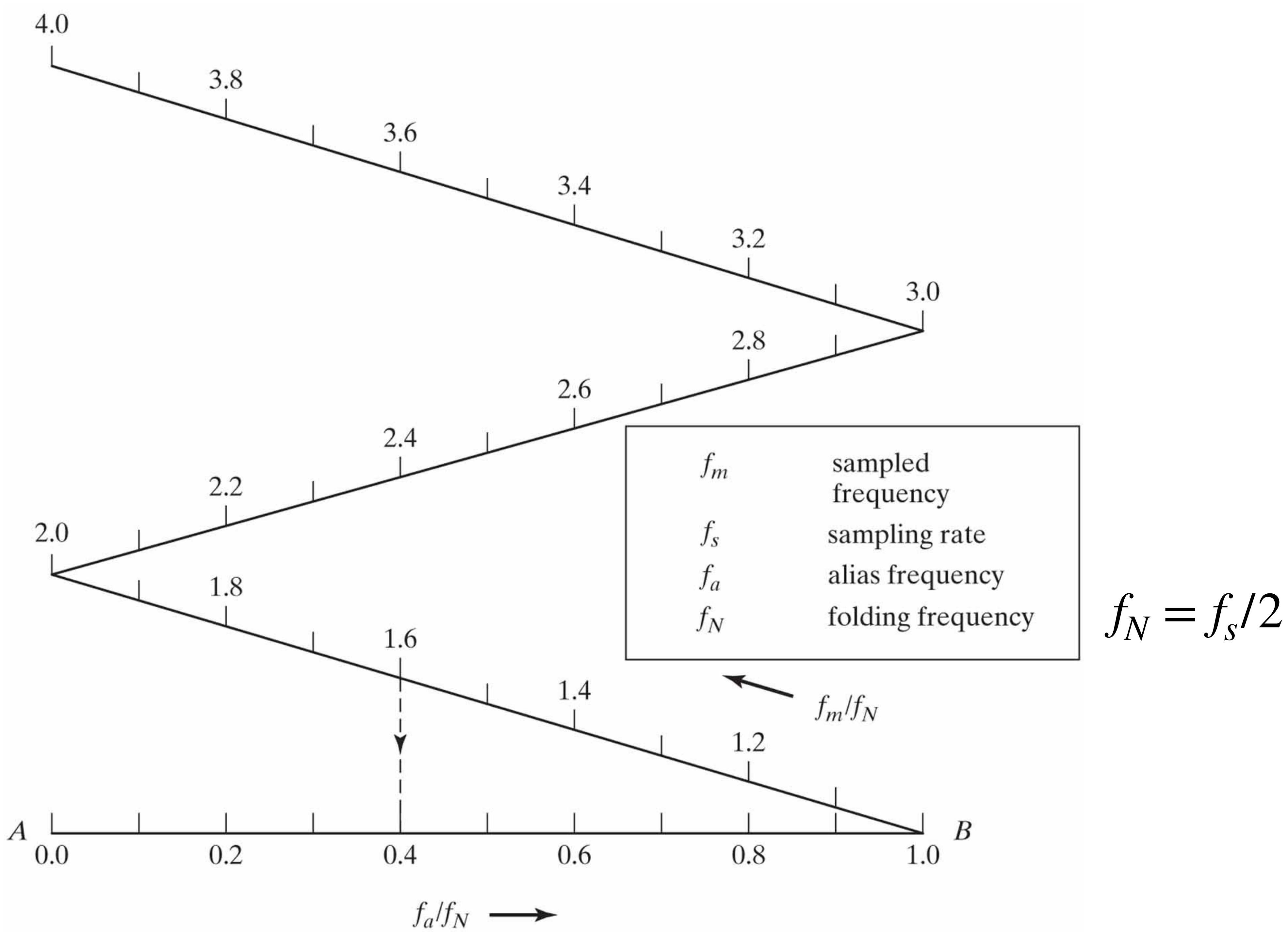
Nyquist sampling theorem

If a signal is sampled at frequency f_s and contains no frequencies exceeding $f_s/2$, then the signal can be exactly reconstructed from the samples.

- ... assuming there's no quantization error.
- $f_s/2$ is the “Nyquist frequency”, aka f_N
- If the signal does contain frequencies exceeding $f_s/2$, sampling causes them to appear incorrectly as frequencies below $f_s/2$; this is called “aliasing”.

Aliasing when sampling rate fails the Nyquist criterion ⁵²



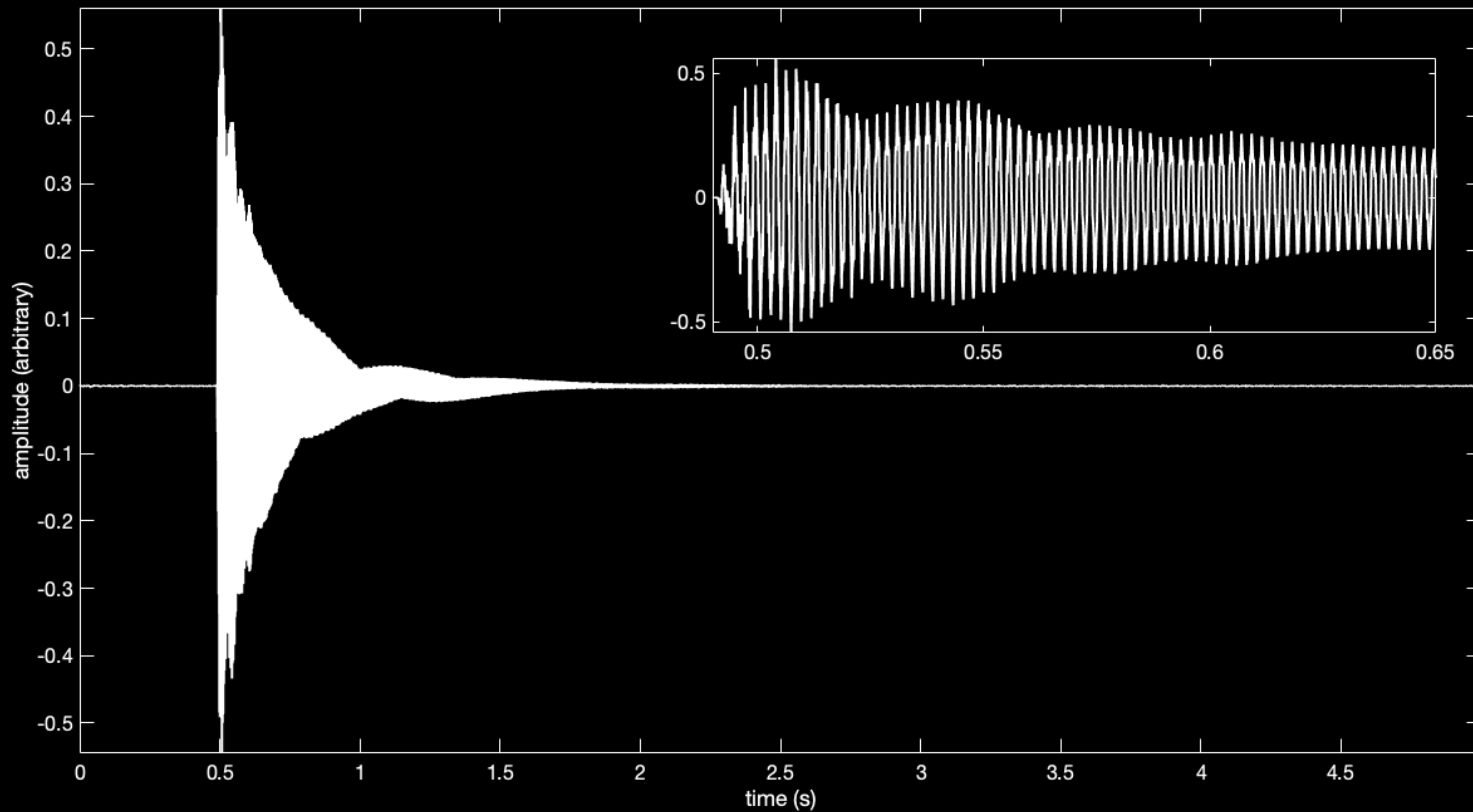


Spectral analysis

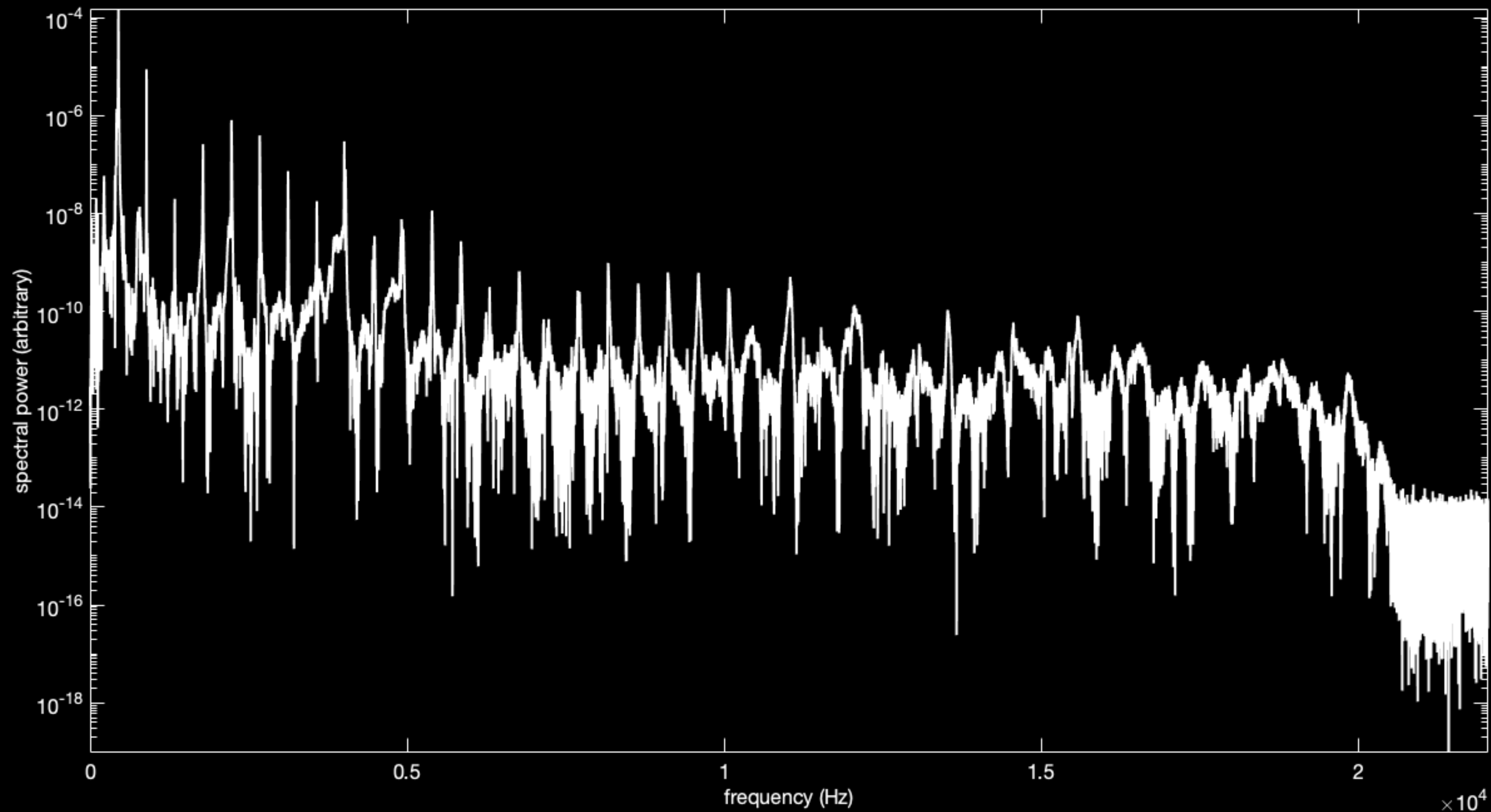
- Key idea: Think about what frequencies are in a signal
- Useful for vibrations, machinery, more
- Intuitive for sound (20 Hz to 20 kHz for human hearing)
 - 18 kHz
 - Fundamental frequency corresponds to pitch
 - 440 Hz (A above middle C)
- Electromagnetic waves
 - 400 to 790 THz visible to humans (750 to 380 nm)
- Frequencies can be as high or low as you like!



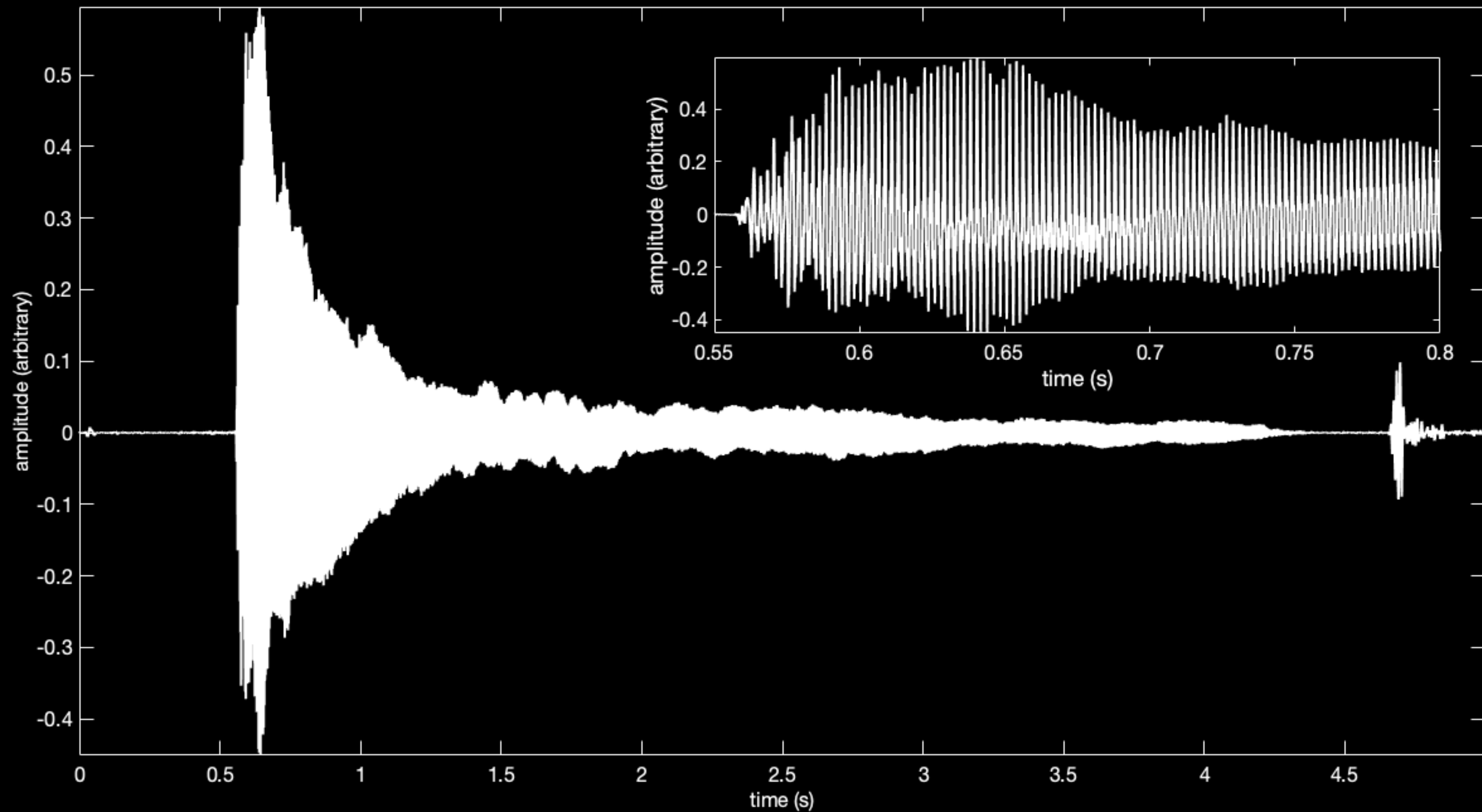
"A" note on ukelele



"A" note on ukelele



"A" note on piano



"A" note on piano

