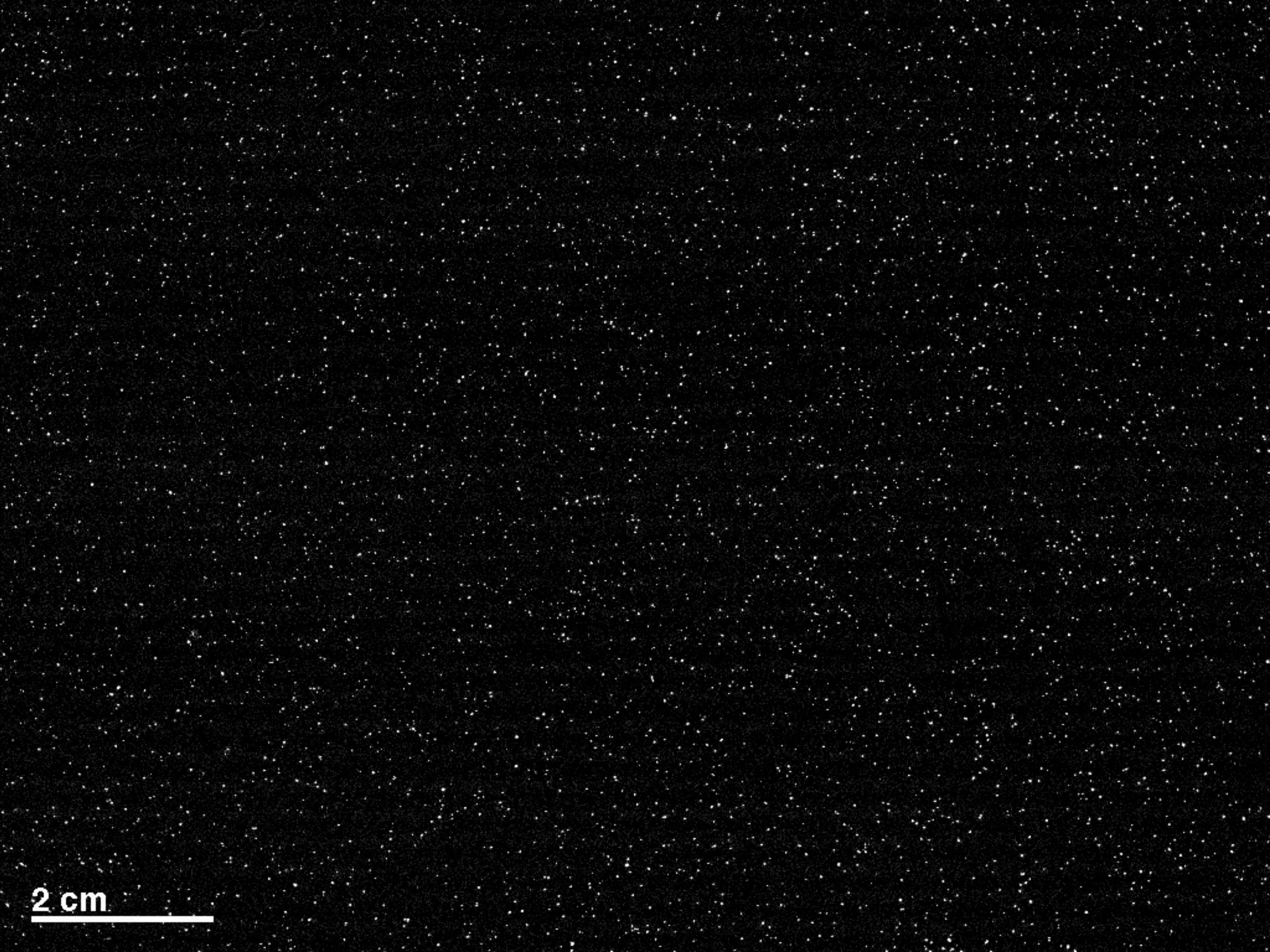


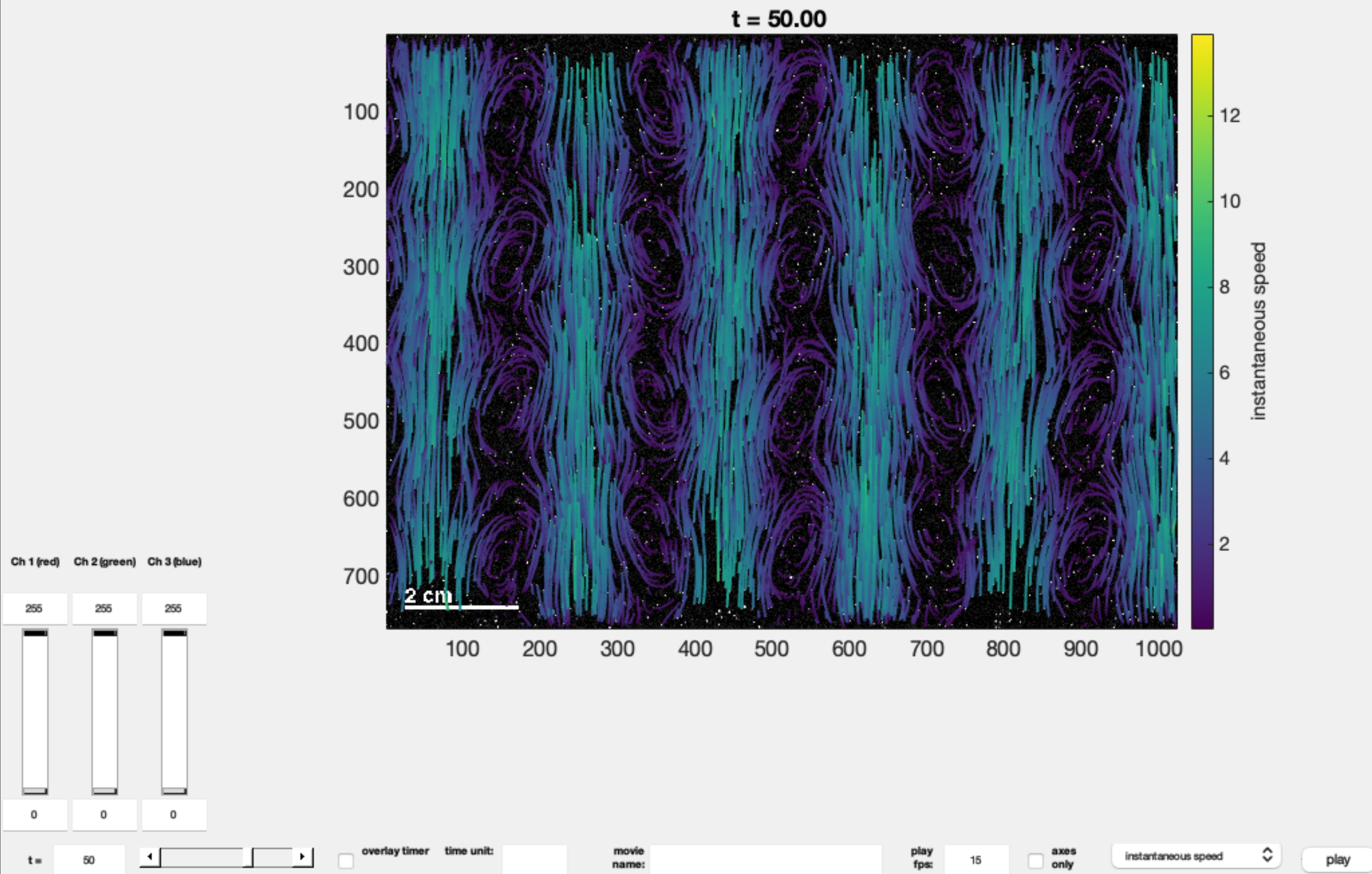
# Writing a Methods section

Explain how the lab exercise was conducted with enough detail that an engineer could repeat your work.

- Begin by posing the question being addressed, and then state what data you used to answer this question, explaining why this data was appropriate to answer it.
- Afterwards, explain the process you used for data collection, listing tools used (including make and model for tools that aren't trivially common), materials, etc.
- Finally, explain how you analyzed the data.
- Optionally, you may use figures, tables, and equations.
- Feel free to paraphrase the given procedure and refer to its figures as needed.
- Anywhere you augmented or deviated from the given procedure, make a note.

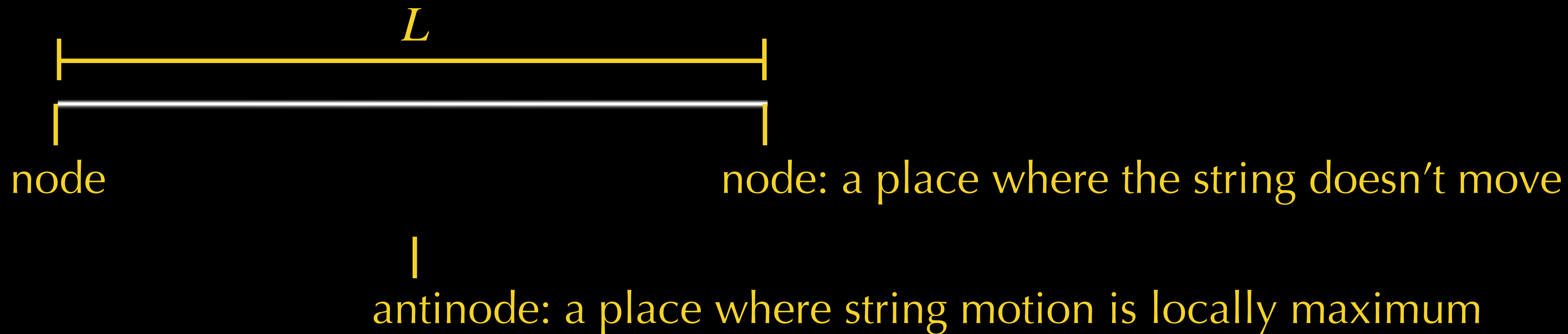
2 cm

A black and white photograph showing a dark, granular surface, likely a rock or mineral specimen. The surface is covered with numerous small, light-colored grains or crystals. A white scale bar is located in the bottom left corner, with the text "2 cm" above it.



# One-dimensional spectral analysis: ukulele string

- What is the boundary condition at the ends of a ukulele string? **Fixed.**
- Simplest possible string vibration: “fundamental mode”

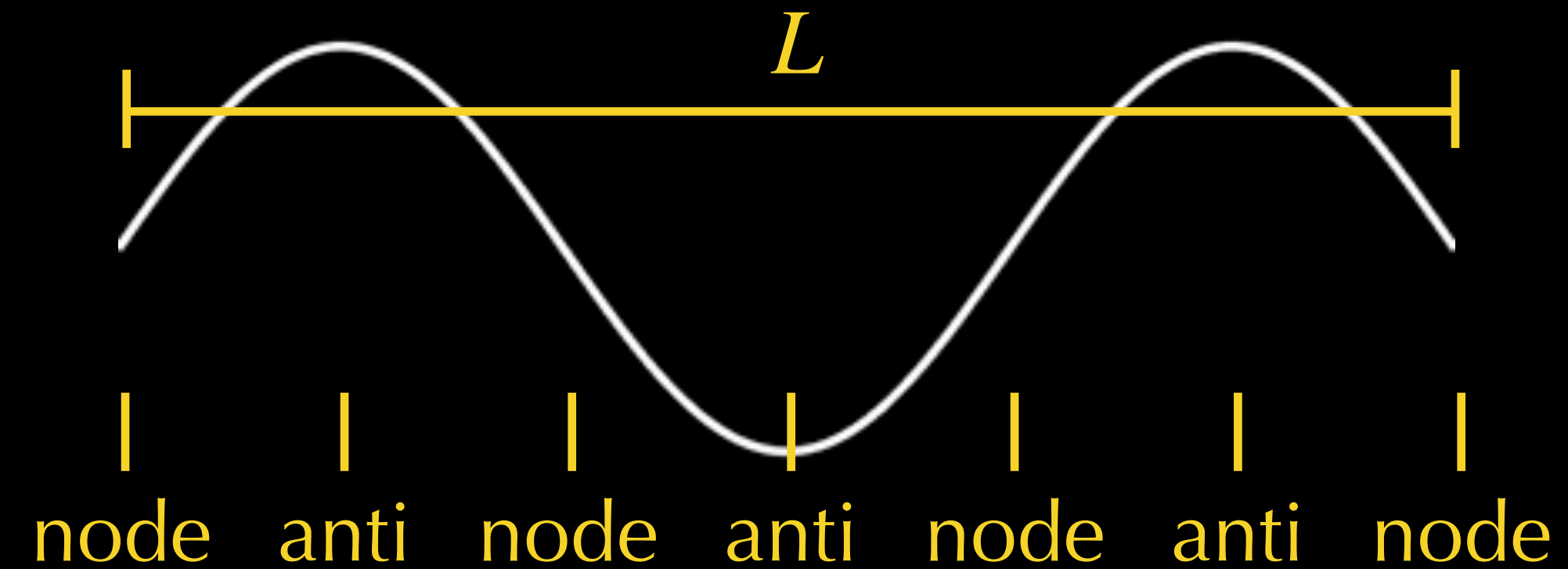
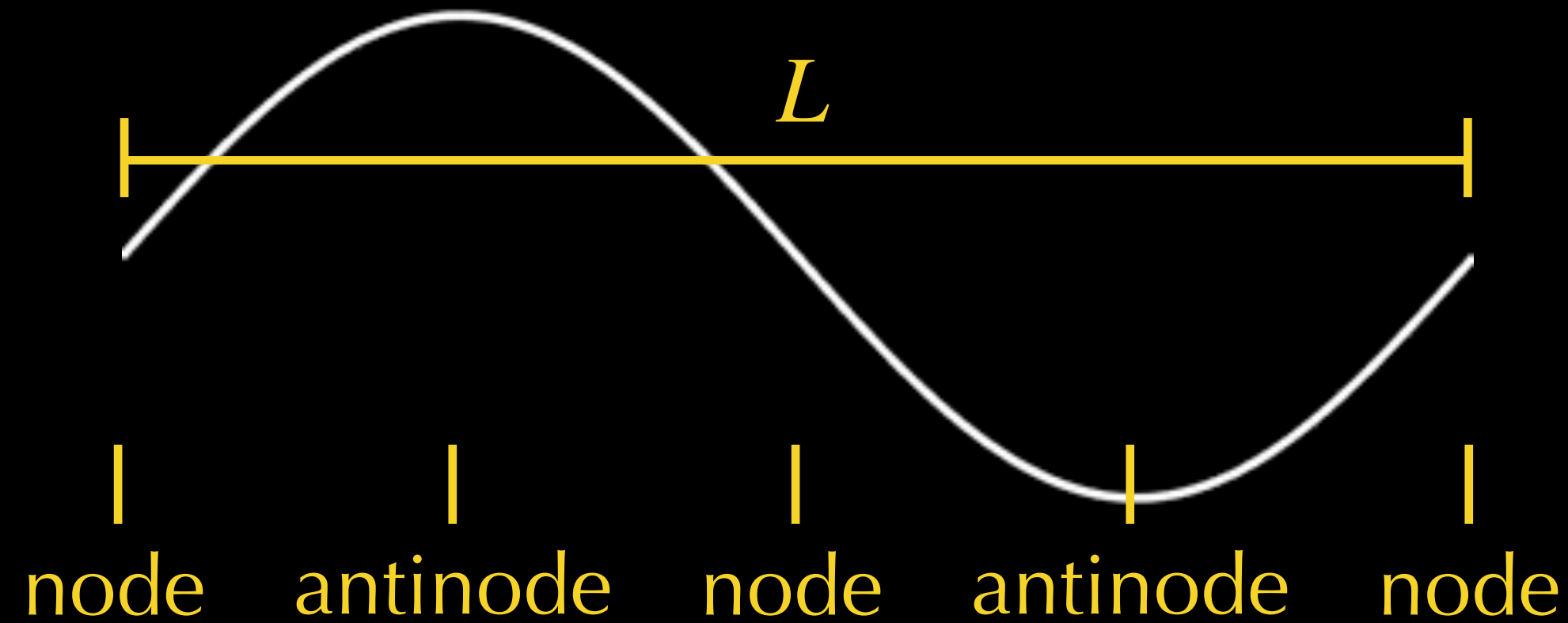


- Wavelength  $\lambda$ ?  **$\lambda = 2L$**
- Frequency  $f$ ?  **$f = \frac{c}{\lambda} = \frac{c}{2L}$** 

wave speed
- The frequency of the fundamental mode is the “fundamental frequency”  $f_0$

# One-dimensional spectral analysis: ukulele string

- More possible modes ("harmonics"):



- Wavelength and frequency?

$$\lambda = L, f = \frac{c}{L} = 2f_0$$

$$\lambda = \frac{2}{3}L, f = \frac{3c}{2L} = 3f_0$$

- The  $n^{\text{th}}$  mode has  $n$  antinodes, wavelength  $\lambda = \frac{2L}{n}$ , and frequency  $nf_0$

# One-dimensional spectral analysis: ukulele string

- The set of modes  $n \in \{1, 2, \dots, \infty\}$  represents all the possible string modes ("basis set")
- Any vibration of a string can be represented as a weighted sum of these modes!

$$y(x) = b_1 \sin \frac{\pi}{L} x + b_2 \sin \frac{2\pi}{L} x + b_3 \sin \frac{3\pi}{L} x + \dots = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{L} x$$

Diagram illustrating the components of the equation:

- $y(x)$  is labeled "string displacement" (indicated by a yellow line).
- $b_1, b_2, b_3$  are collectively labeled "mode amplitudes" (indicated by a yellow line).
- $x$  is labeled "position" (indicated by a yellow line).

- $y(x = 0)$ ?  $y(x = 0) = 0$
- $y(x = L)$ ?  $y(x = L) = 0$
- $b_n$  is the amplitude of the frequency  $nf_0$

# Fourier series

- Math: similarly, any time-varying periodic signal with fundamental frequency  $f_0$  (period  $T = f_0^{-1}$ ) can be written as a weighted sum of sines and cosines:

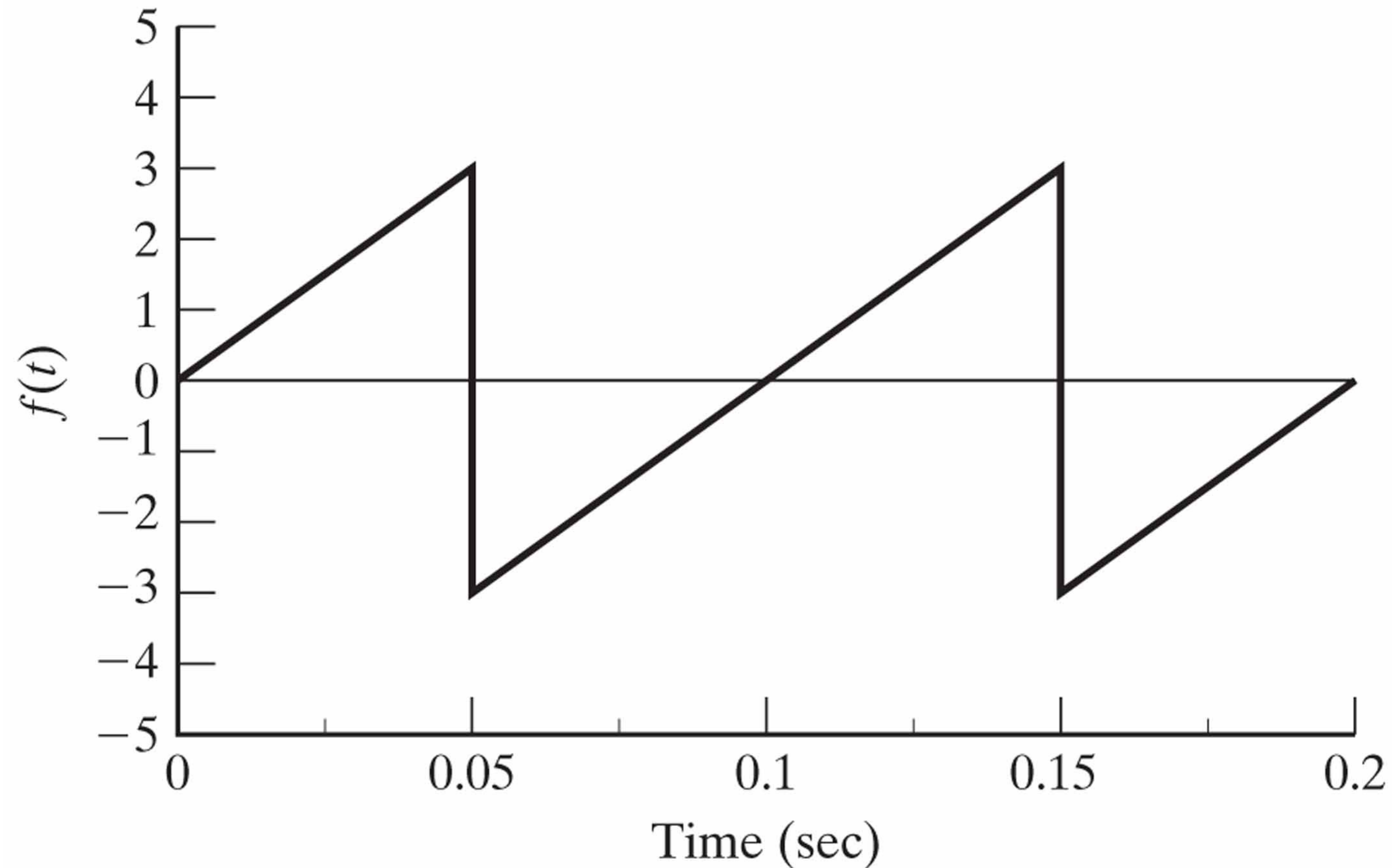
$$y(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos 2\pi n f_0 t + b_n \sin 2\pi n f_0 t),$$

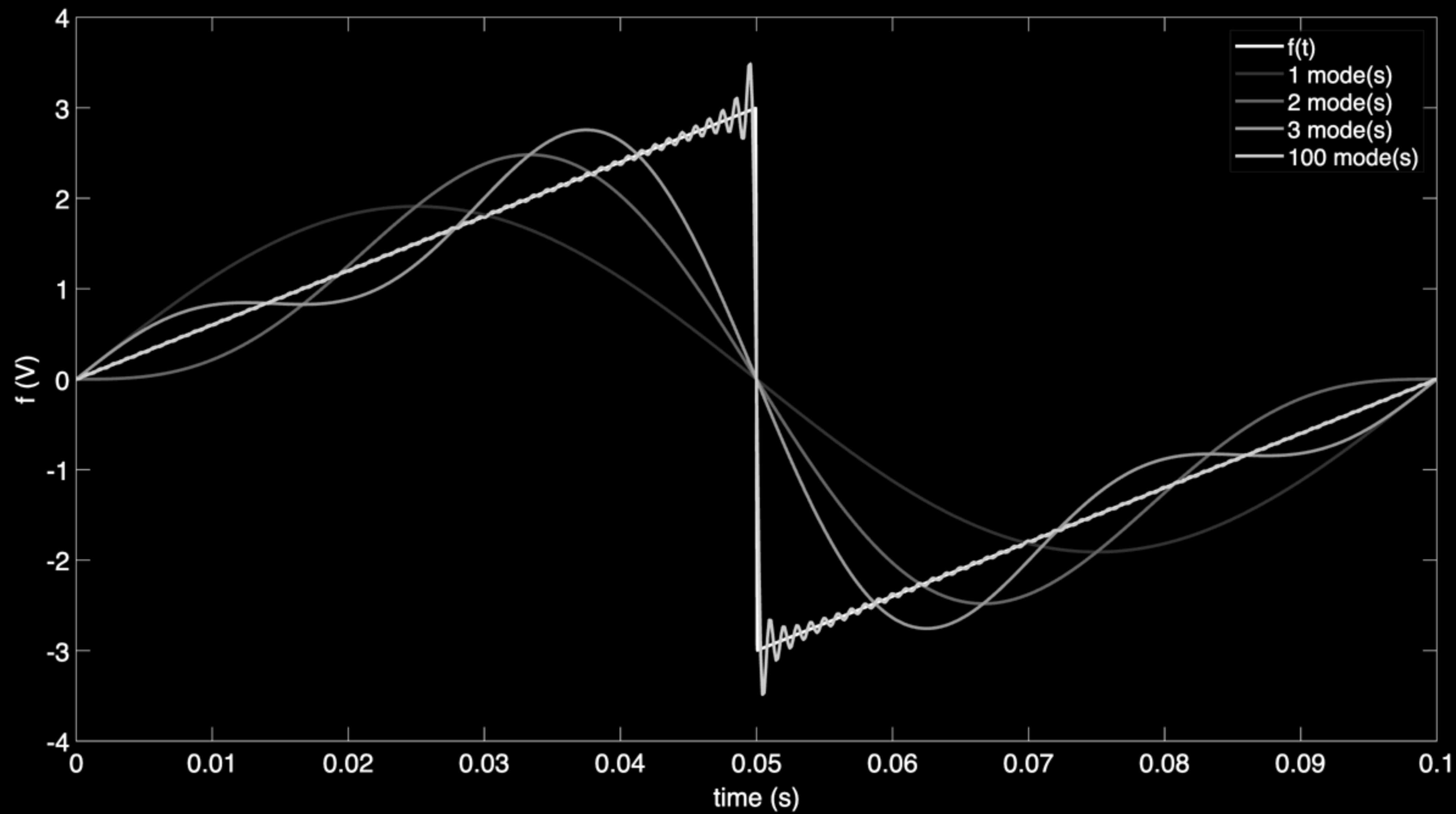
Diagram labels for the equation above:

- $y(t)$ : signal
- $a_0$ : mean of  $y(t)$
- $a_n$  and  $b_n$ : amplitudes
- $t$ : time

- where  $a_0 = \frac{1}{T} \int_0^T y(t) dt$ ,  $a_n = \frac{2}{T} \int_0^T y(t) \cos 2\pi f_0 n t dt$ ,  $b_n = \frac{2}{T} \int_0^T y(t) \sin 2\pi f_0 n t dt$
- $y(t)$  can be calculated from  $\{a_0, a_n, b_n\}$  (its “Fourier series”)
- $\{a_0, a_n, b_n\}$  can be calculated from  $y(t)$
- For brevity, can write  $2\pi f_0 = \omega_0$  (angular frequency)

Ex. 5.2: Determine the amplitudes of the first, second, and third harmonic components of the signal plotted below.





# Fourier transform

- Math: any time-varying signal – even an aperiodic one – can be written as an integral of sines and cosines
- Typically use complex exponentials for brevity:

$$\cos \theta = \frac{1}{2} (e^{j\theta} + e^{-j\theta}), \sin \theta = \frac{1}{2} (e^{j\theta} - e^{-j\theta}), \text{ and } e^{j\theta} = \cos \theta + \overset{j = \sqrt{-1}}{j} \sin \theta$$

$$\bullet \text{ So then } \underset{\text{signal}}{y(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y(\omega) e^{j\omega t} \underset{\text{angular frequency}}{\overset{\text{time}}{d\omega}},$$

$$\bullet \text{ where } Y(\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt$$

- $y(t)$  can be calculated from  $Y(\omega)$  (its “Fourier transform”)
- $Y(\omega)$  can be calculated from  $y(t)$  (its “inverse Fourier transform”)

# Discrete Fourier transform

- Math: A series of  $N$  measurements of a time-varying frequency, sampled at frequency  $f_s$ , can be written as a sum of sampled sines and cosines — a “discrete Fourier transform”
- Maximum accessible frequency? Nyquist frequency  $f_s/2$
- How to access higher frequencies? Sample faster
- Can discriminate frequencies as close as  $f_s/N$
- How to improve discrimination? Record longer
- $|Y(\omega)|^2$ , the “spectral power”, tells the strength of frequency  $\omega$
- Matlab: `fft.m` for discrete Fourier transform, `ifft.m` for inverse, `pwelch.m` for discrete Fourier transform with windowing to smooth out leakage

