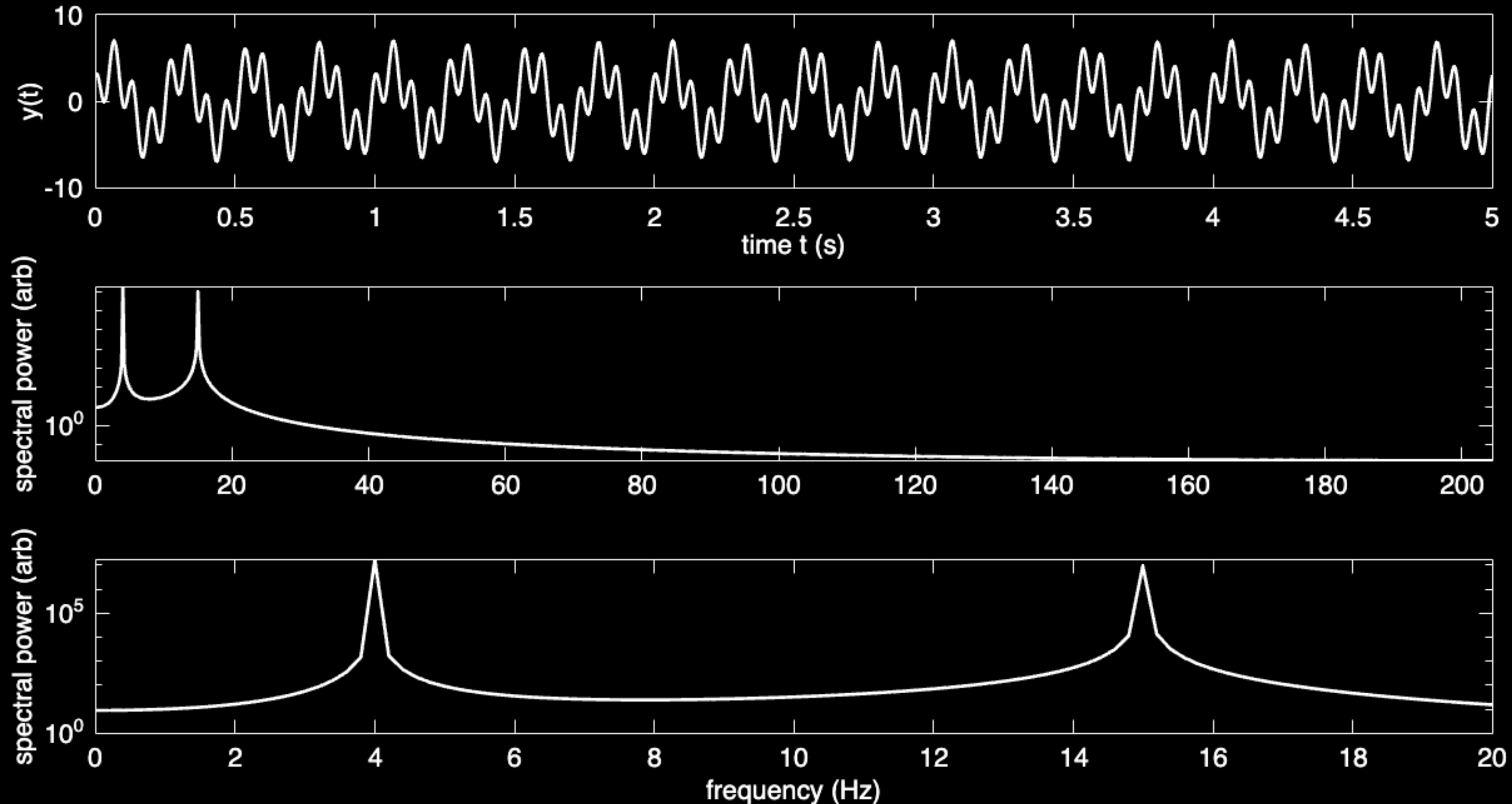


Ex: Using computational software, generate 2048 values of the function⁶⁹
 $y(t) = 3 \cos 30\pi t + 4 \sin 8\pi t$ (where t is in seconds) between $t = 0$ and
5 s. Then, perform an FFT and plot the spectral power.



Two-dimensional spectral analysis: membrane

- Strings are (roughly) one-dimensional, but many objects are (roughly) two-dimensional (e.g., drum head, airplane body panel) or three-dimensional (e.g., Baja car wheel mount, rotating table)
- Any object has a countably infinite set of natural vibrational modes, each with a characteristic frequency

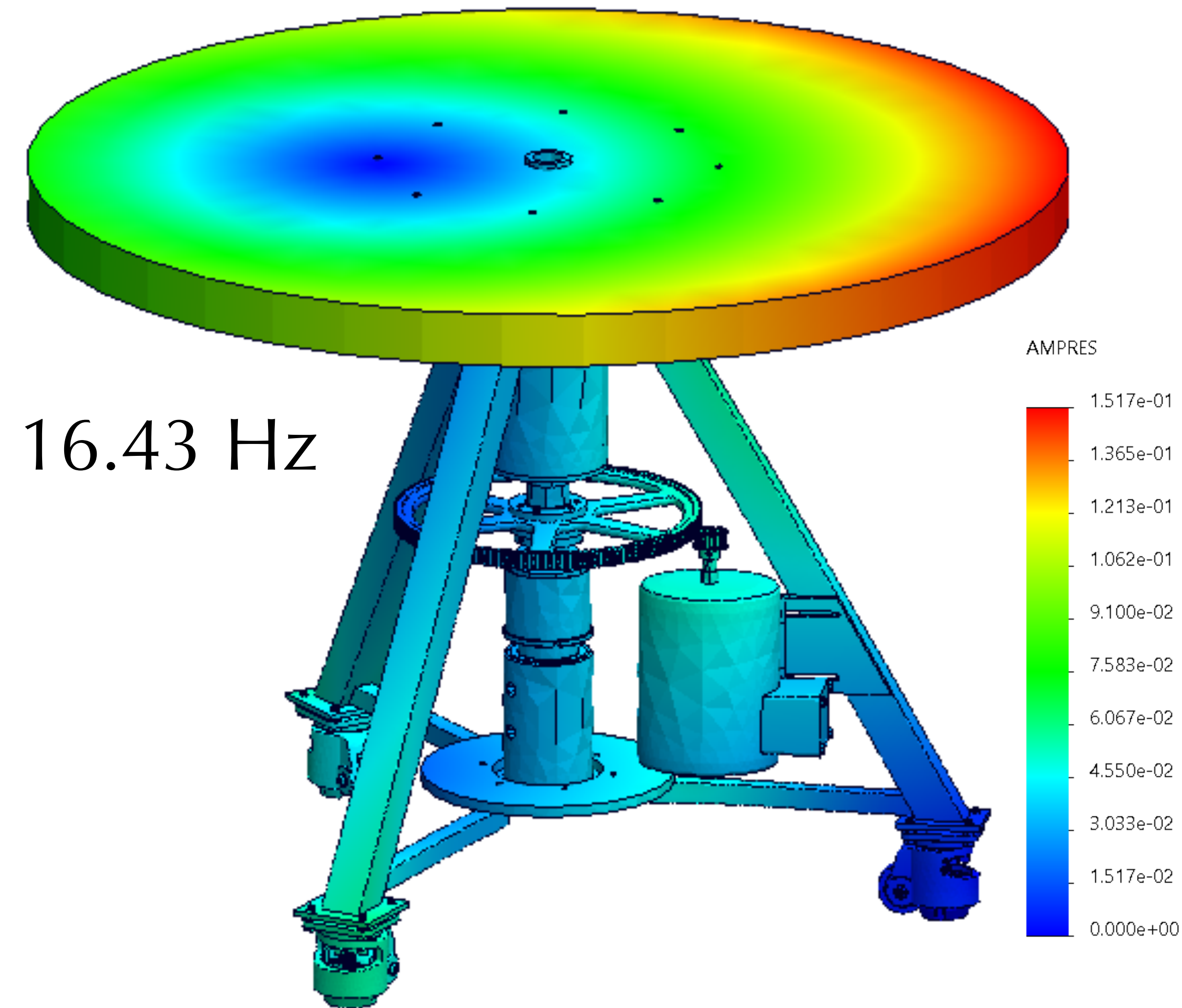
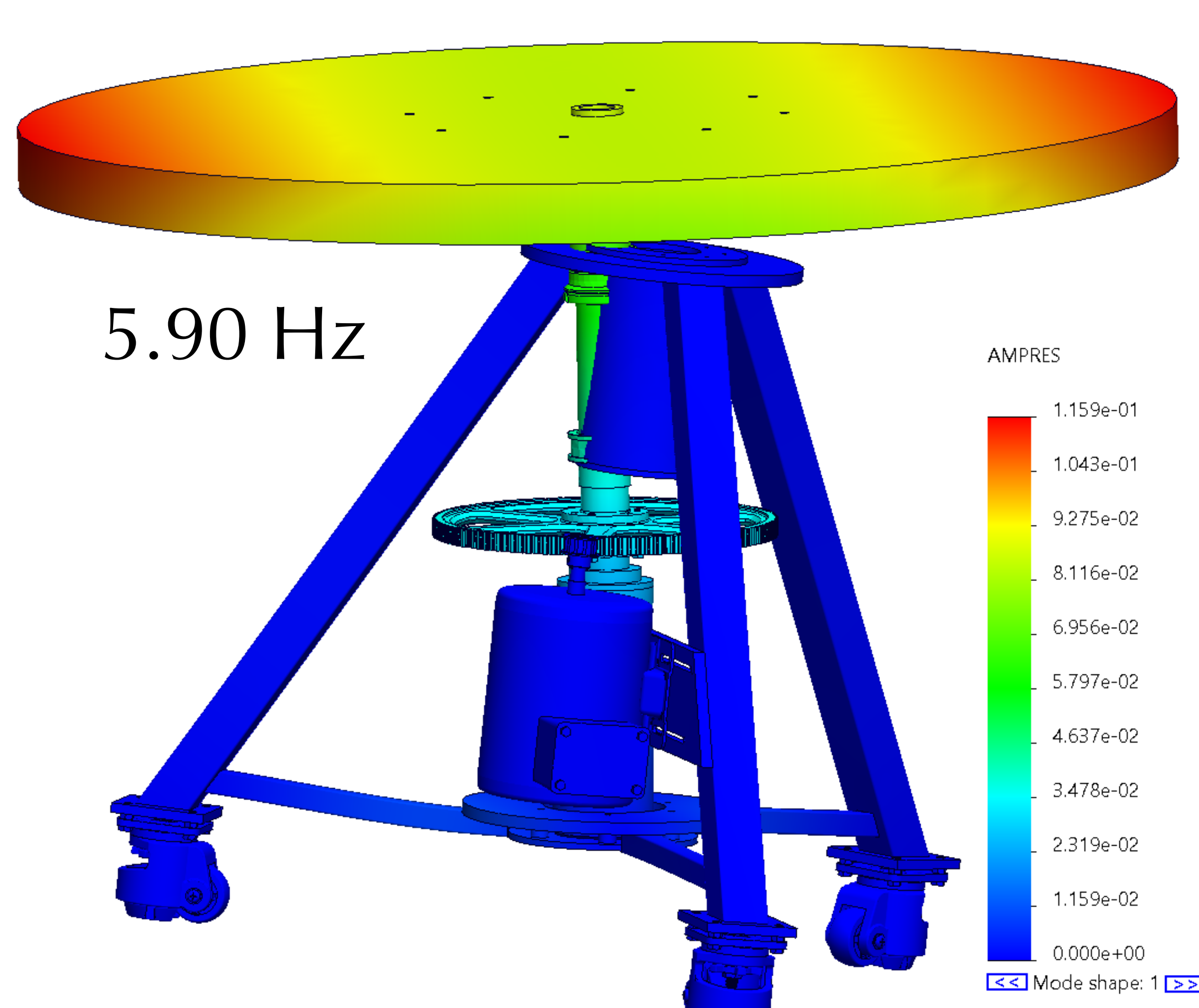
1 antinode in x ,
1 antinode in y

1 antinode in x ,
2 antinodes in y

2 antinodes in x ,
1 antinode in y

2 antinodes in x ,
3 antinodes in y

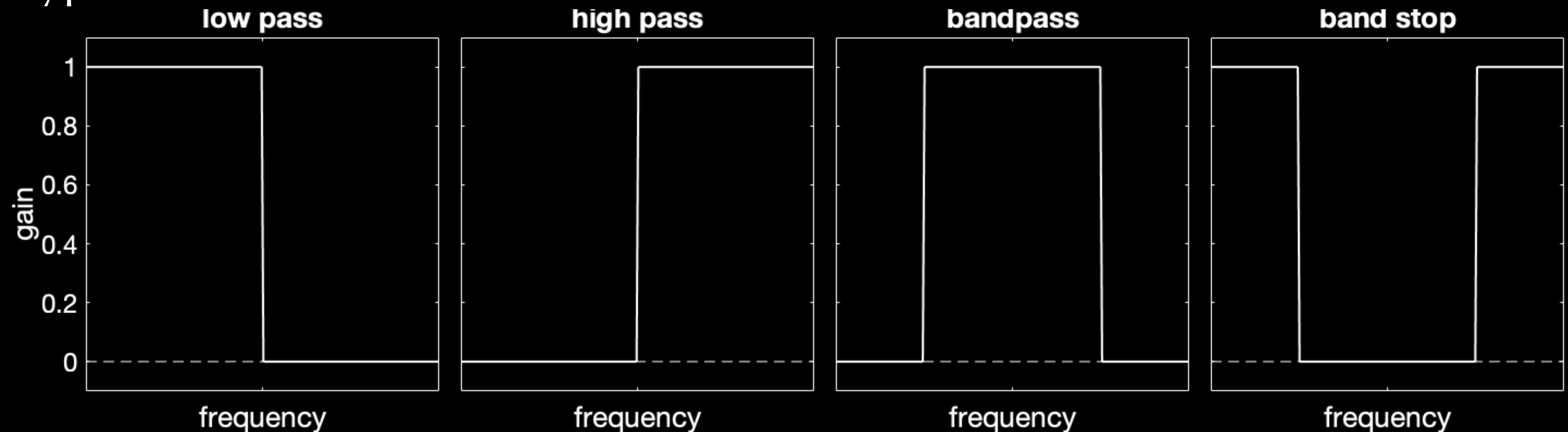
First two vibrational modes of rotating table



- These frequencies are much higher than any planned drive frequency (rotation 2 Hz or less) so resonance is unlikely

Filters

- Any time-varying voltage can be expressed as a superposition of sinusoids of different frequencies
- A *filter* removes undesired frequencies
 - Noise (e.g. 60 Hz, high-frequency hiss)
 - Irrelevant frequencies (e.g. < 20 Hz or > 20 kHz for audio) to prevent aliasing
- Ideal types:

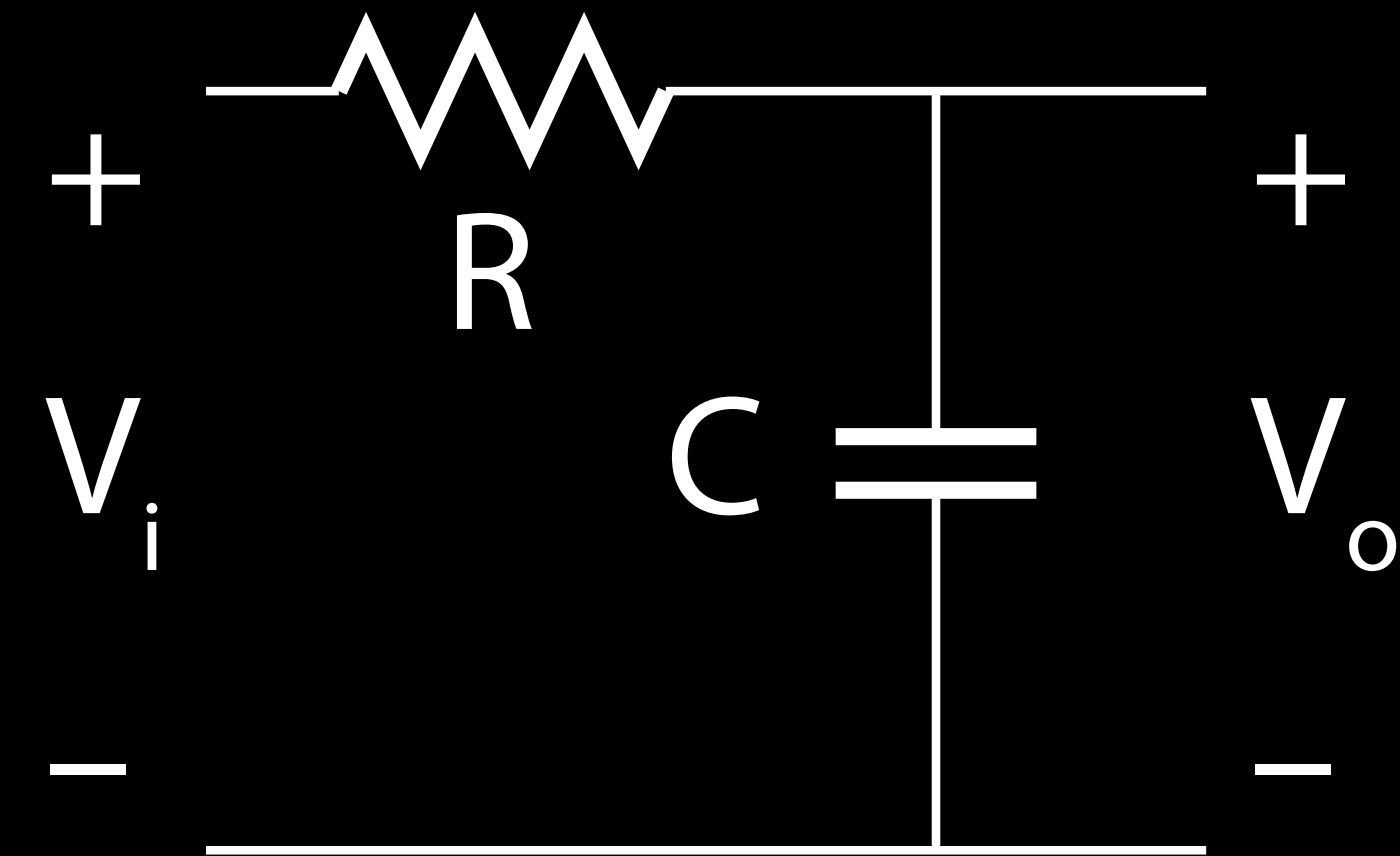


The simplest low-pass filter

- First-order Butterworth, aka RC
- Voltage law: $RC \frac{\partial V_o}{\partial t} + V_o = V_i$
- If $V_i = A \sin \omega t$, the long-term output is

$$V_o = \frac{A}{\sqrt{1 + (RC\omega)^2}} \sin(\omega t - \arctan RC\omega)$$

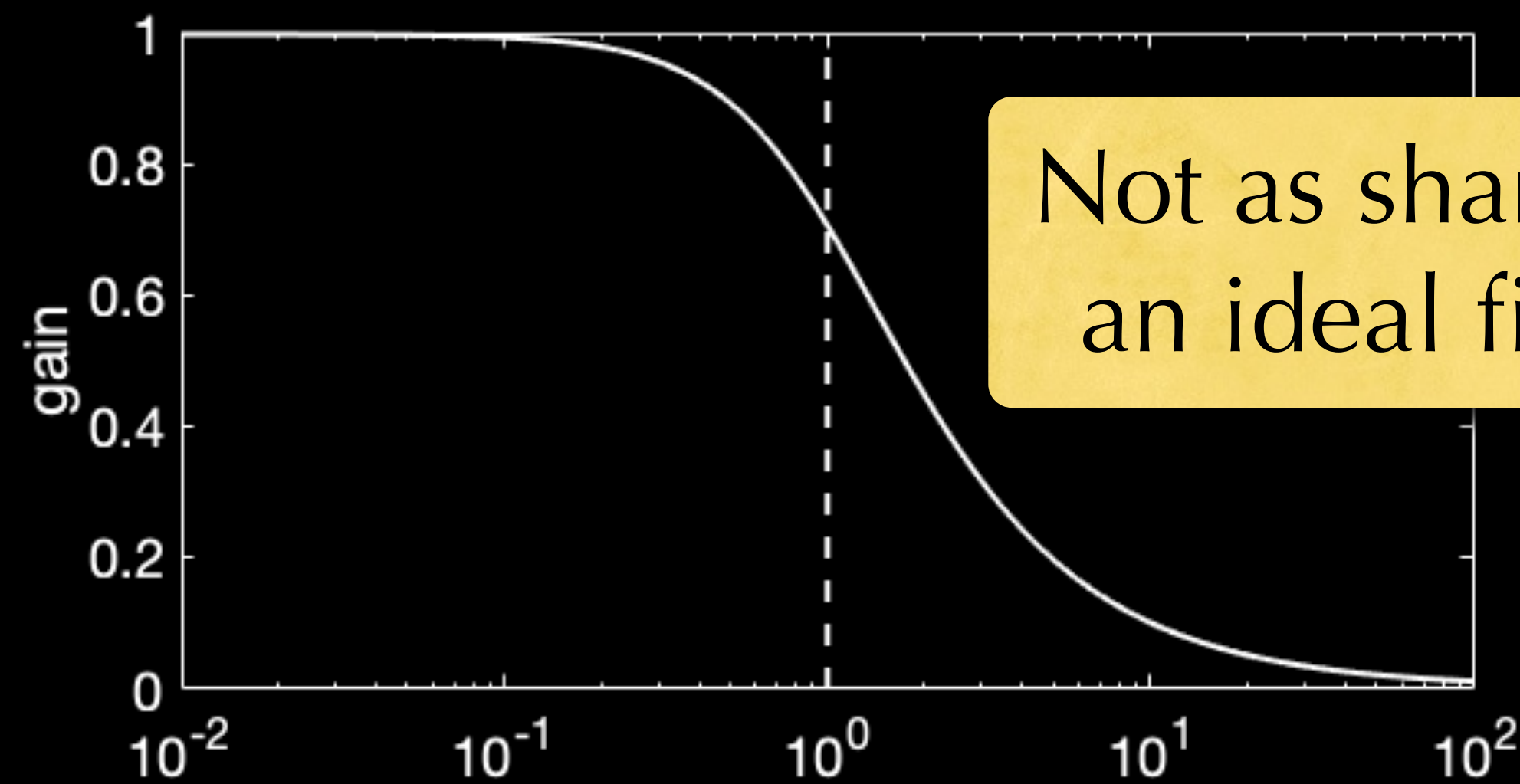
reduced amplitude
still a sin!
phase shift
- Amplitude of V_o at frequency $\omega = 0$? A
- Amplitude of V_o at frequency $\omega = \infty$? 0
- Cutoff frequency? $\frac{1}{RC}$



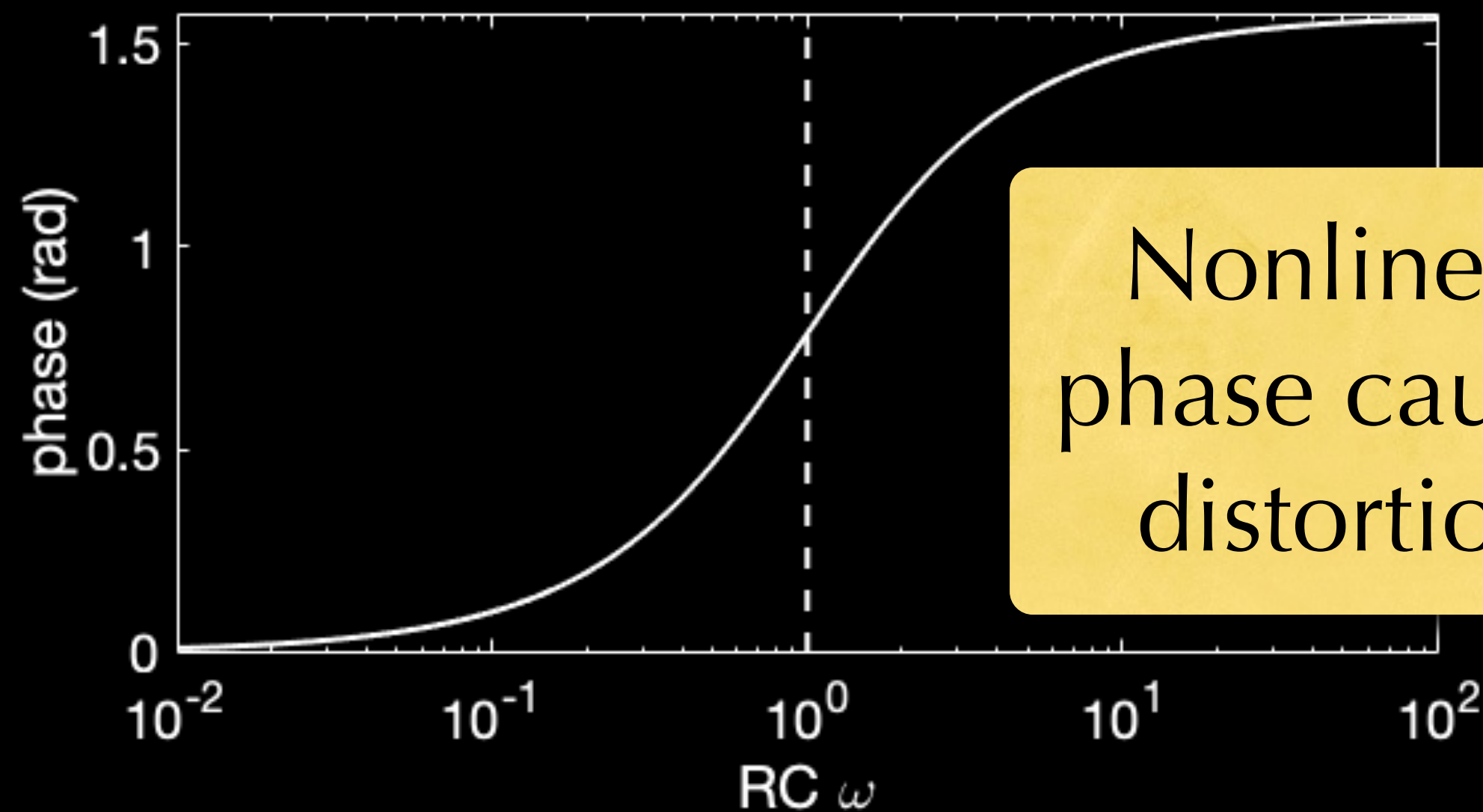
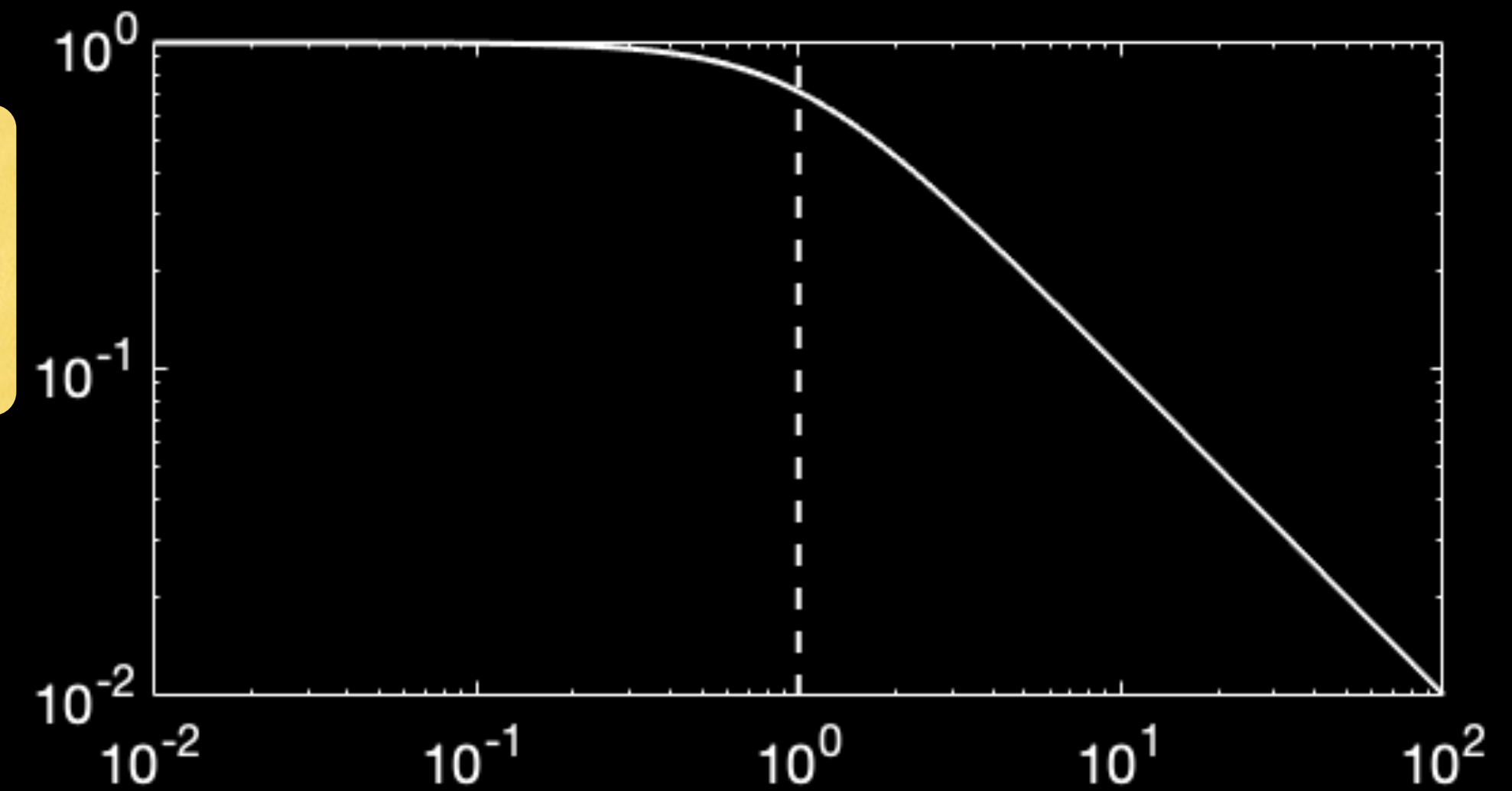
$$G = \frac{1}{\sqrt{1 + (RC\omega)^2}}$$

$$\phi = \arctan RC\omega$$

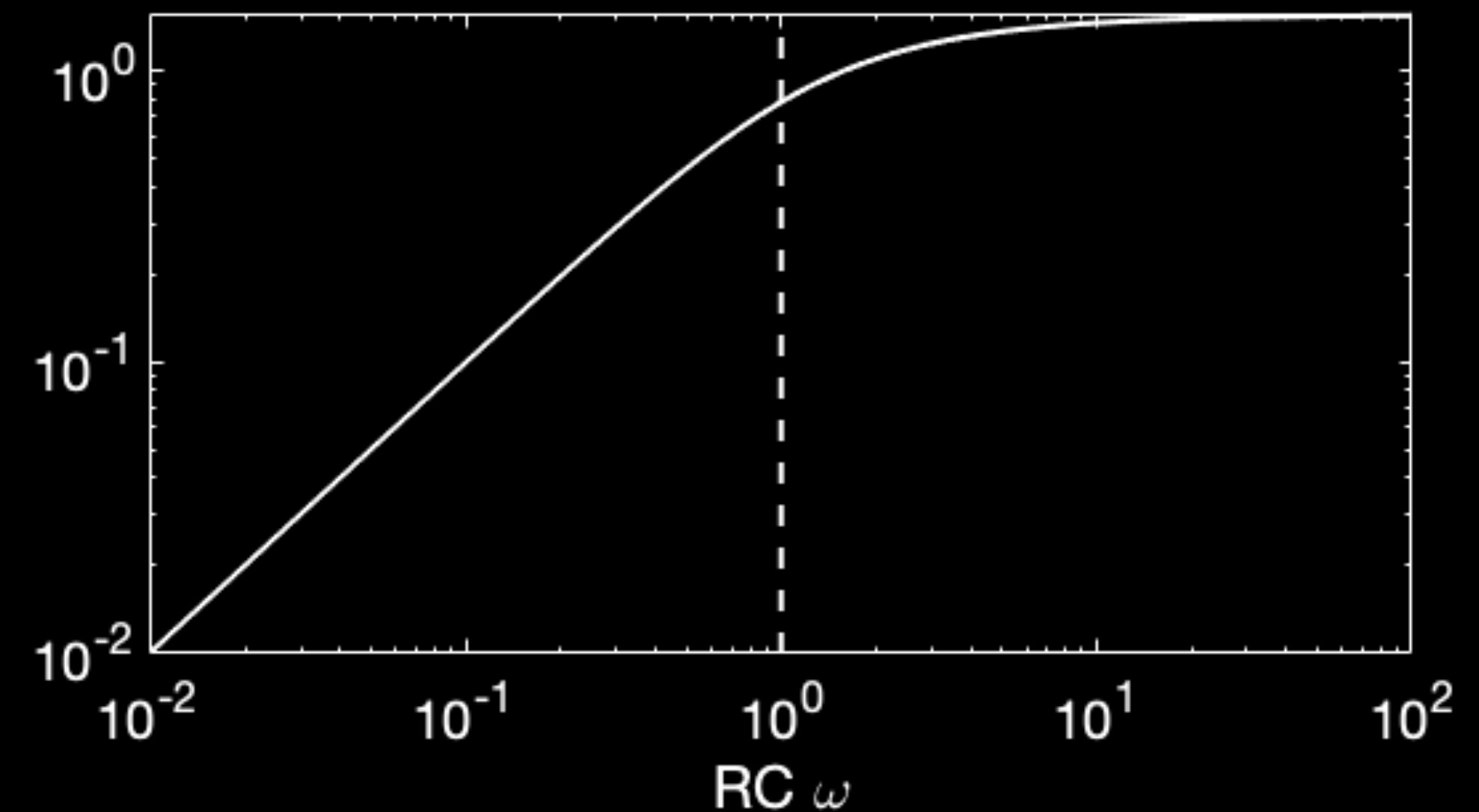
First-order low-pass Butterworth filter



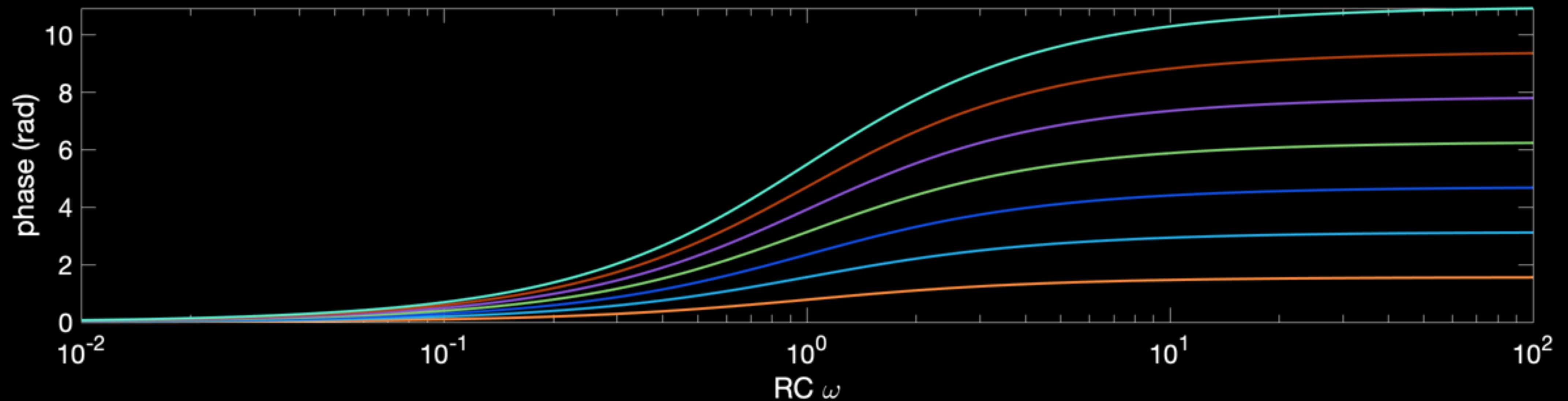
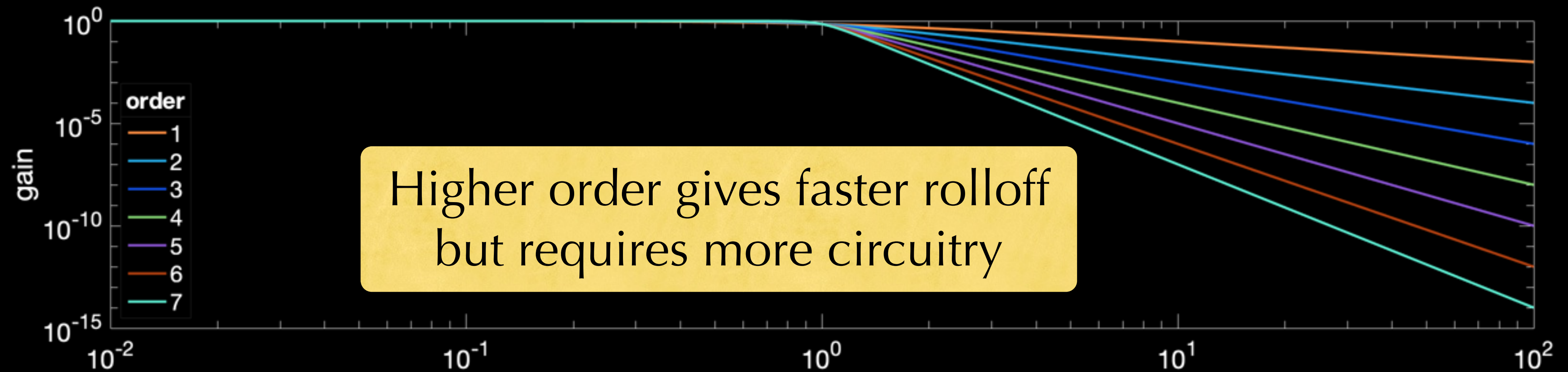
Not as sharp as
an ideal filter



Nonlinear
phase causes
distortion

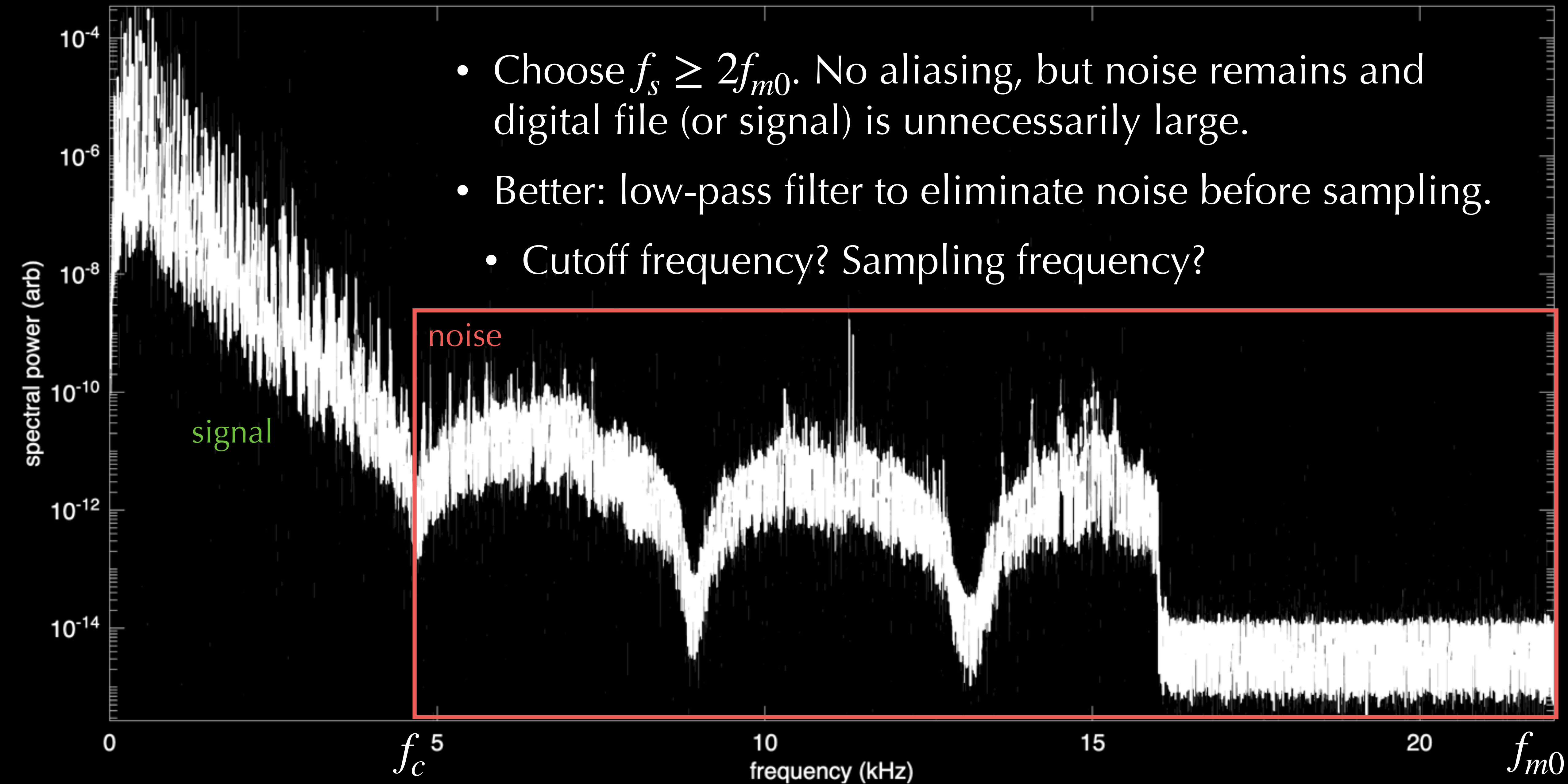


Low-pass Butterworth filters



How to avoid aliasing?

- Choose $f_s \geq 2f_{m0}$. No aliasing, but noise remains and digital file (or signal) is unnecessarily large.
- Better: low-pass filter to eliminate noise before sampling.
 - Cutoff frequency? Sampling frequency?



How to avoid aliasing?

- Choose filter cutoff at f_c and $f_s \geq 2f_{mf}$, where $f_{mf} = f_c 2^{N_{\text{oct}}}$ is the frequency at which signal power drops below the least significant bit of the analog-to-digital converter, $20 \log_{10} 2^N$ where $N_{\text{oct}} = \frac{20 \log_{10} 2^N}{\text{filter attenuation rate (dB/octave)}}$ and N is the bit depth.

