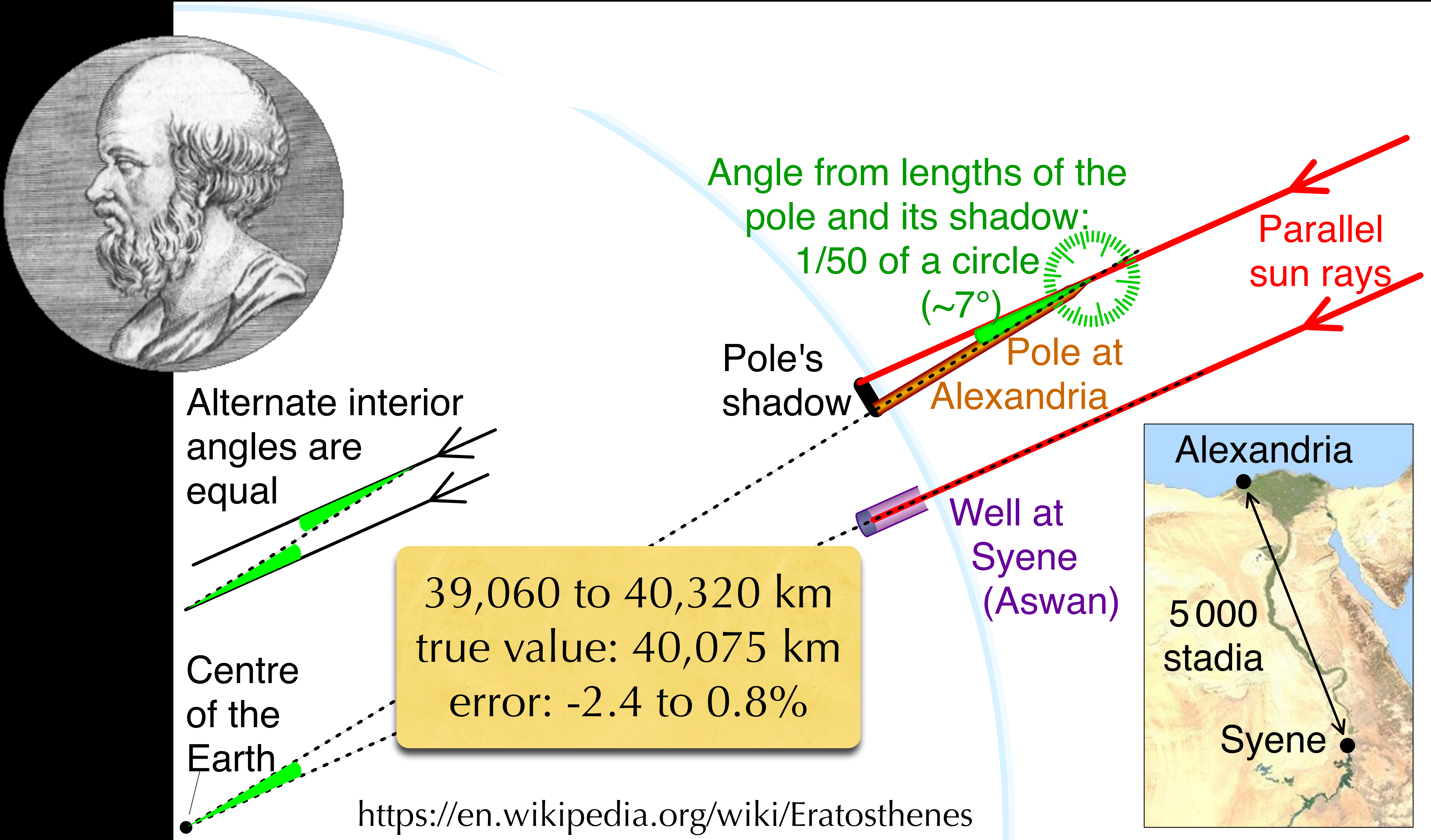
ME 240: Fundamentals of Instrumentation & Measurement

UNIVERSITY *of* ROCHESTER

Eratosthenes of Cyrene

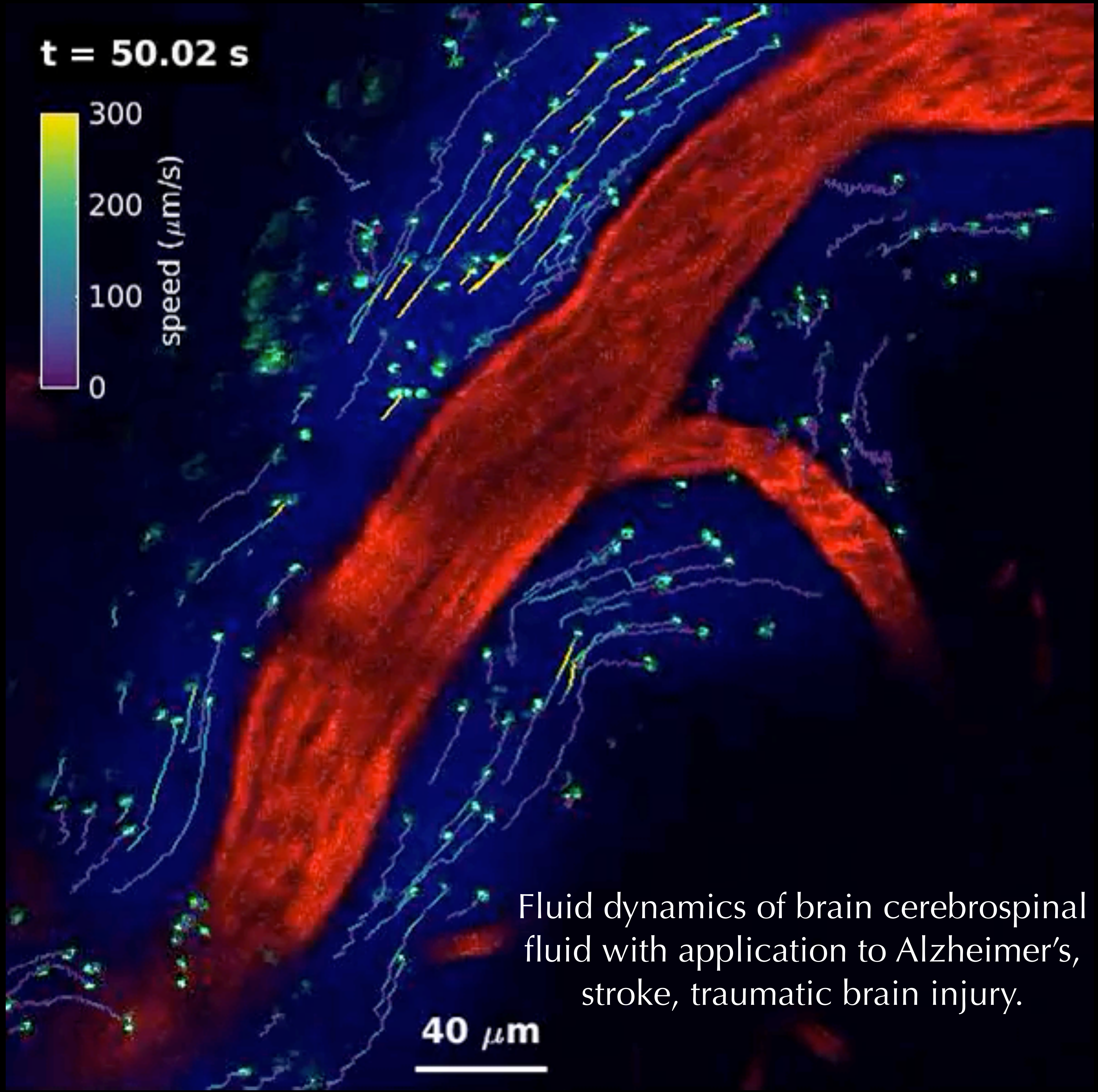


39,060 to 40,320 km
true value: 40,075 km
error: -2.4 to 0.8%

https://en.wikipedia.org/wiki/Eratosthenes

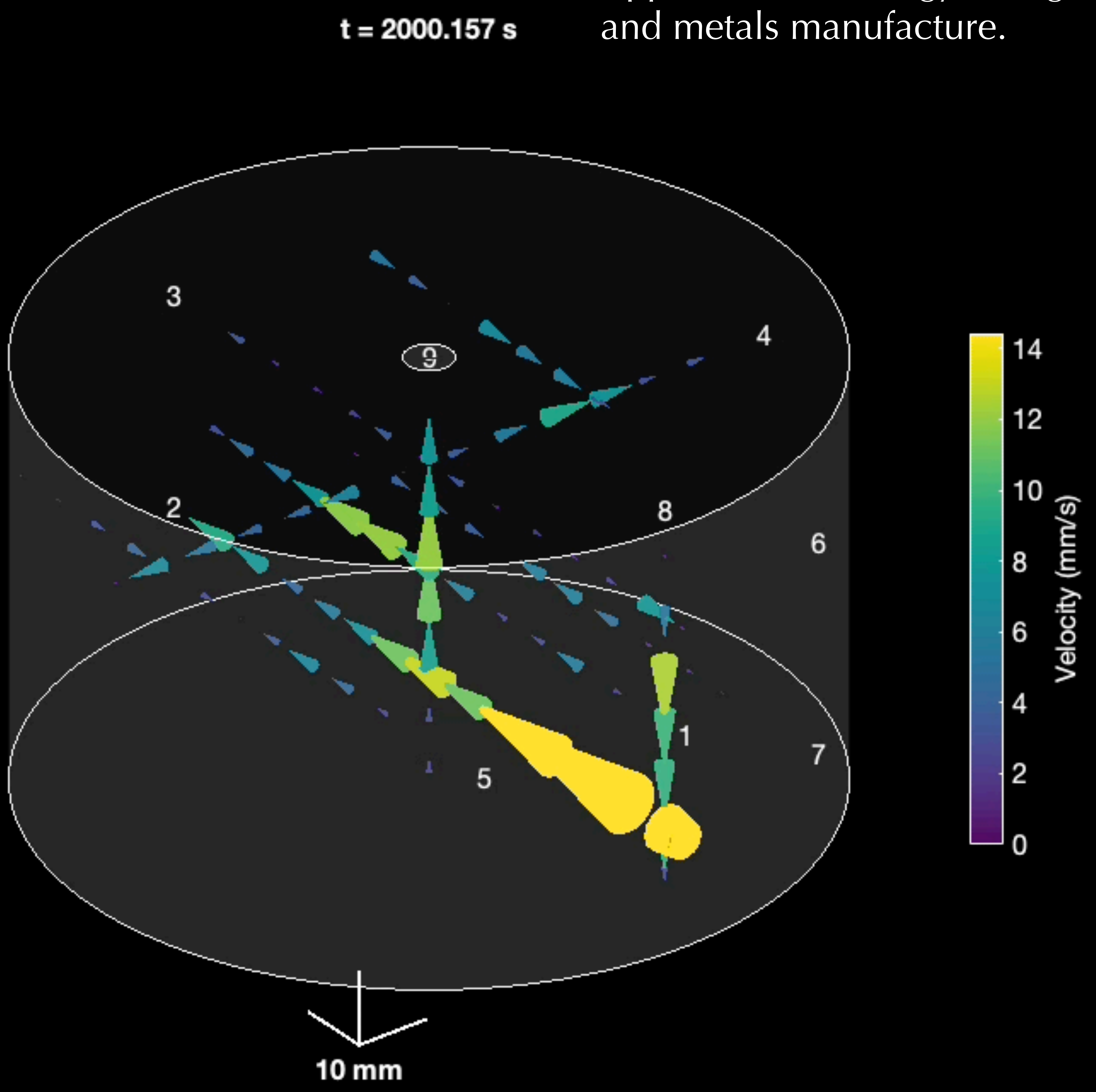
MixingLab

Douglas H. Kelley
hajim.rochester.edu/me/sites/kelley/
UNIVERSITY of ROCHESTER

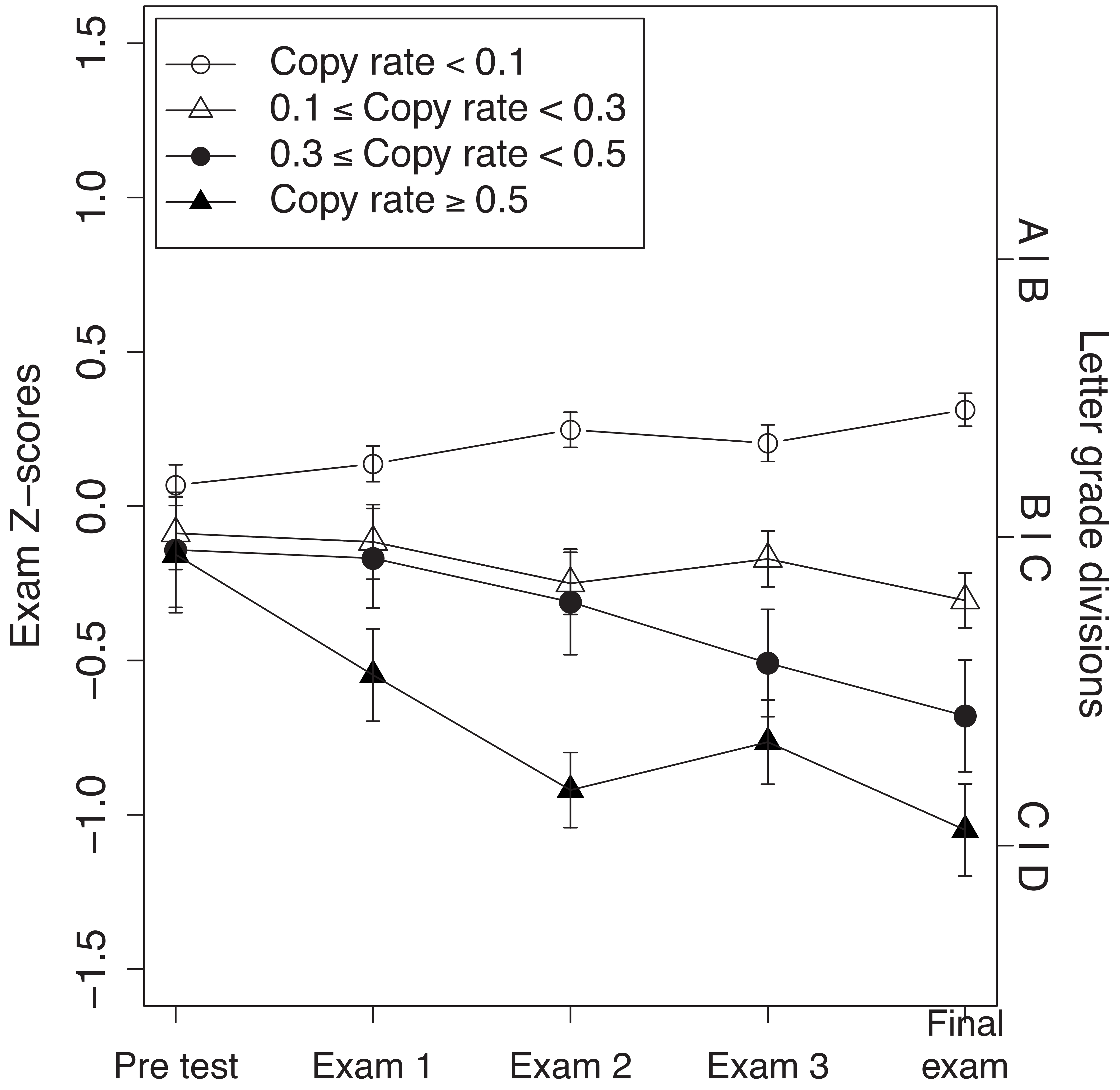


Fluid dynamics of brain cerebrospinal fluid with application to Alzheimer's, stroke, traumatic brain injury.

Liquid metal fluid dynamics with application to energy storage and metals manufacture.

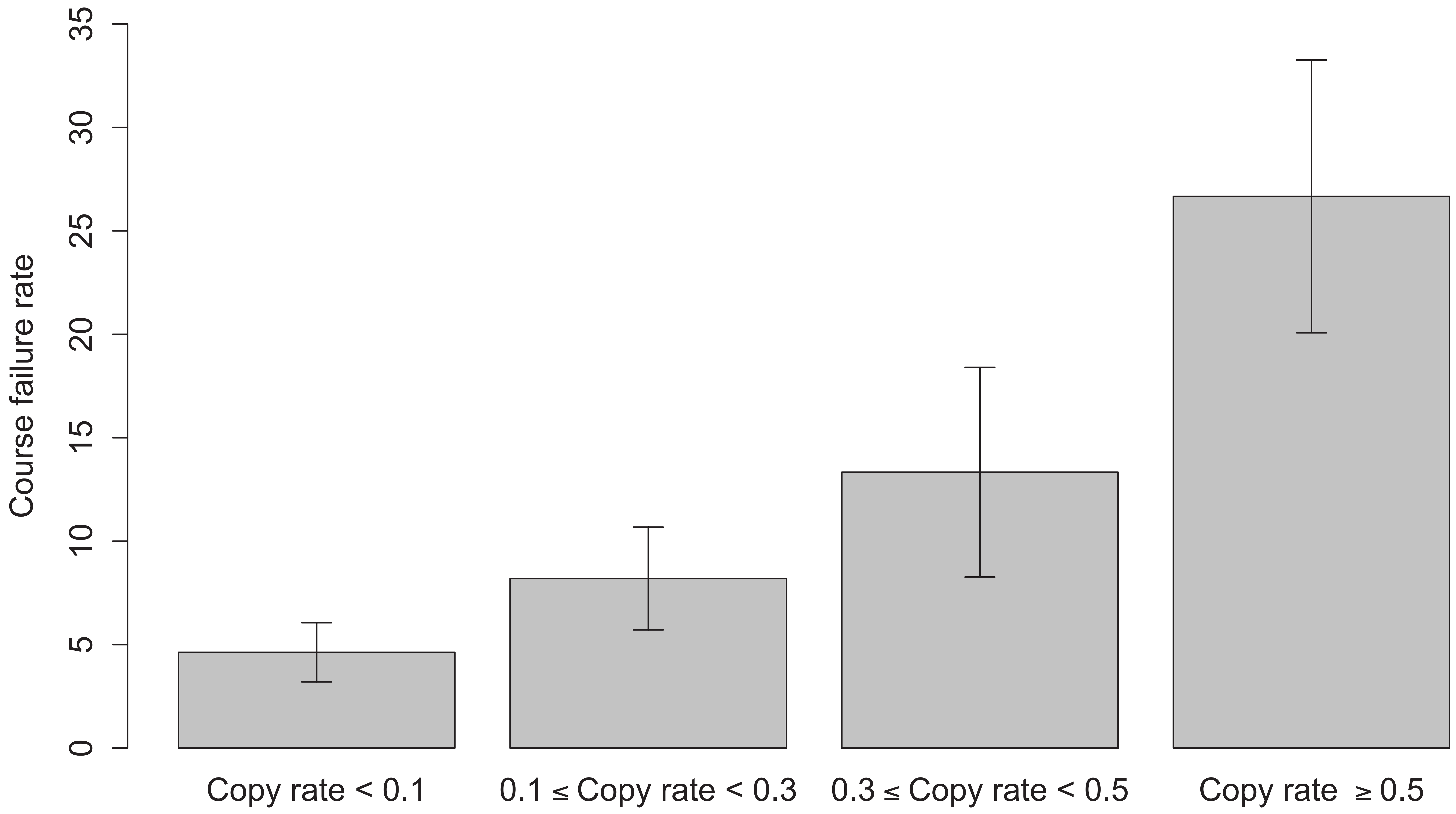


Copying homework and your grade



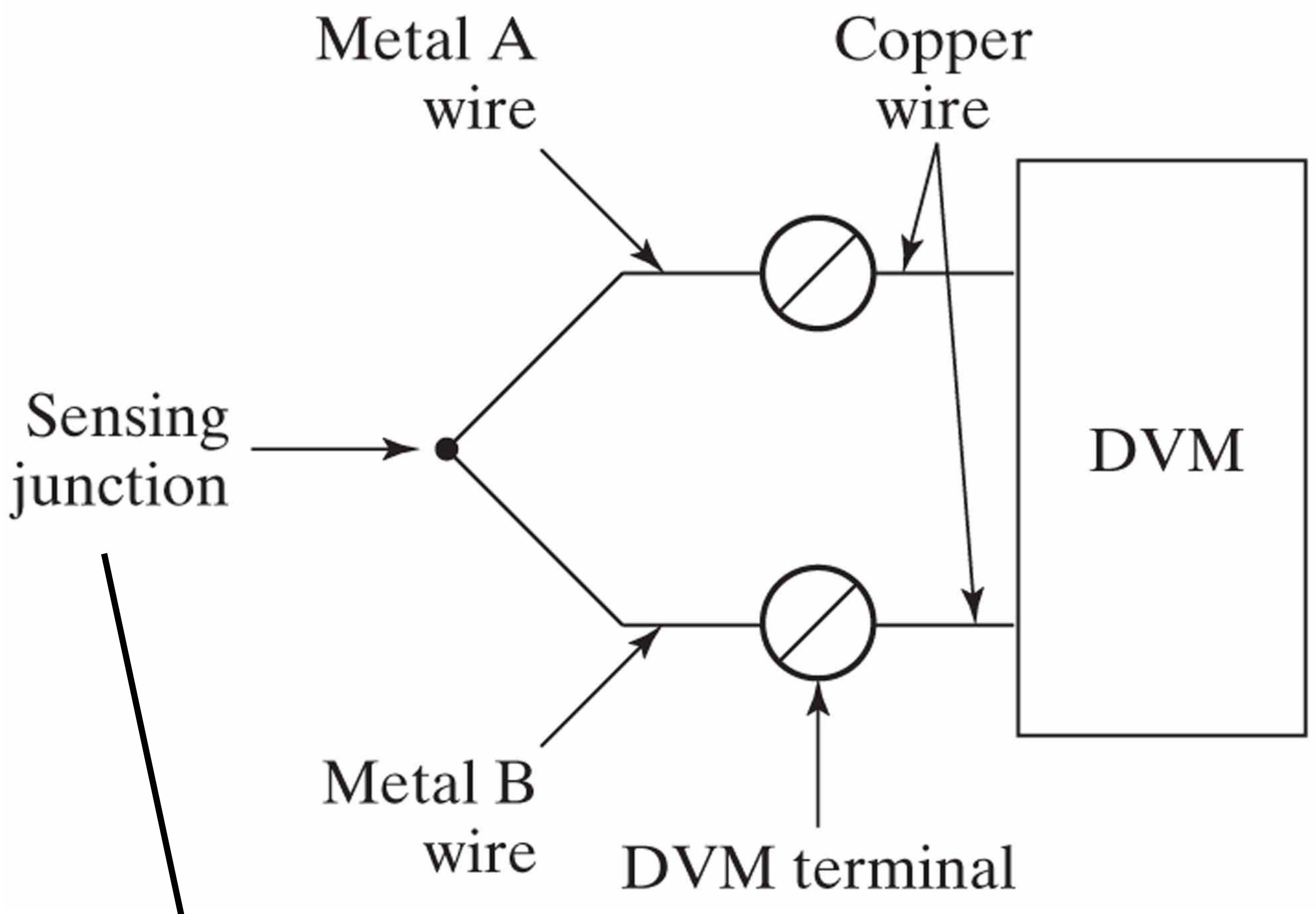
Palazzo *et al. Phys. Rev. Spec. Topics: Phys. Ed. Res.* 2010

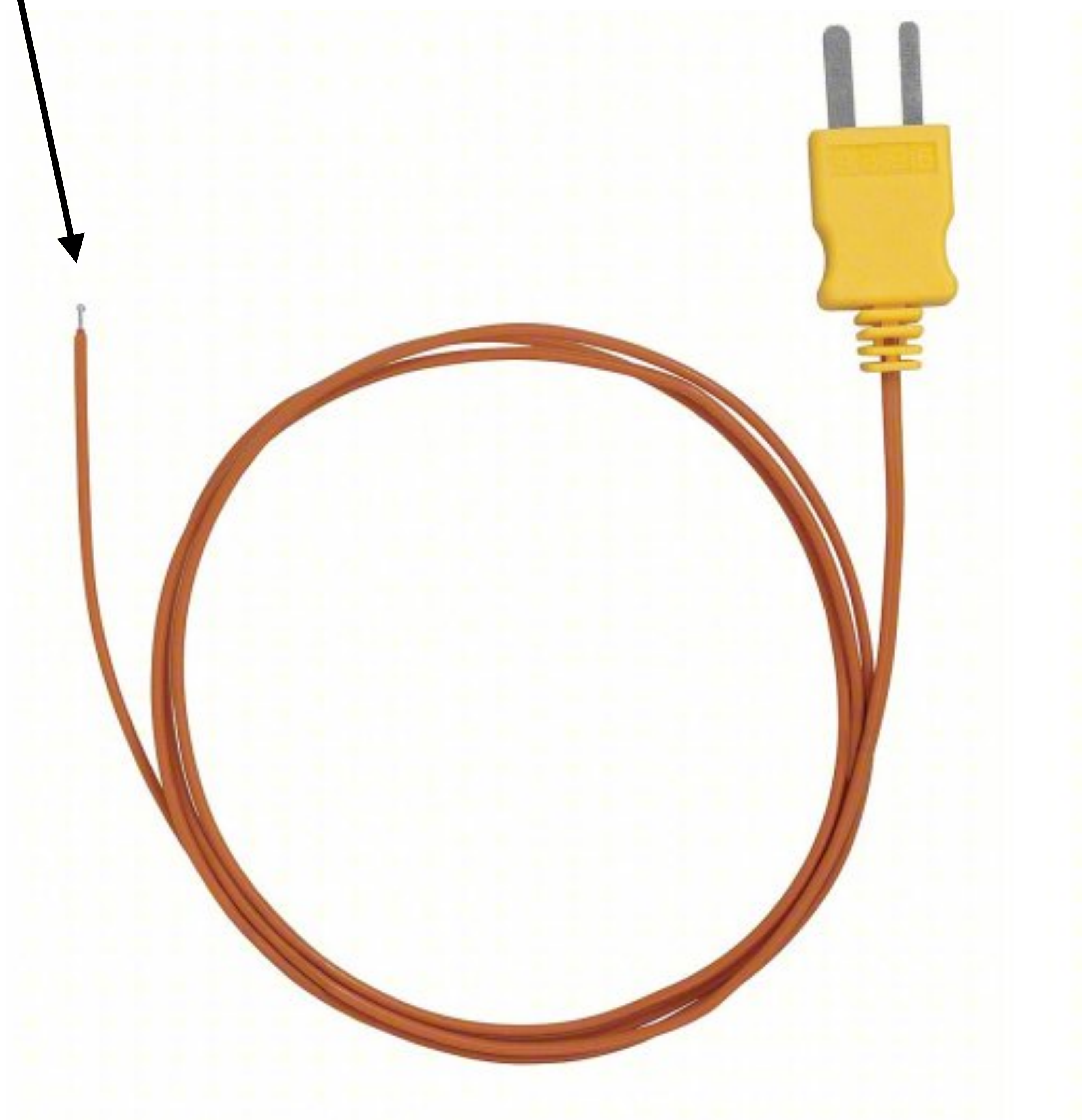
Copying homework and failure



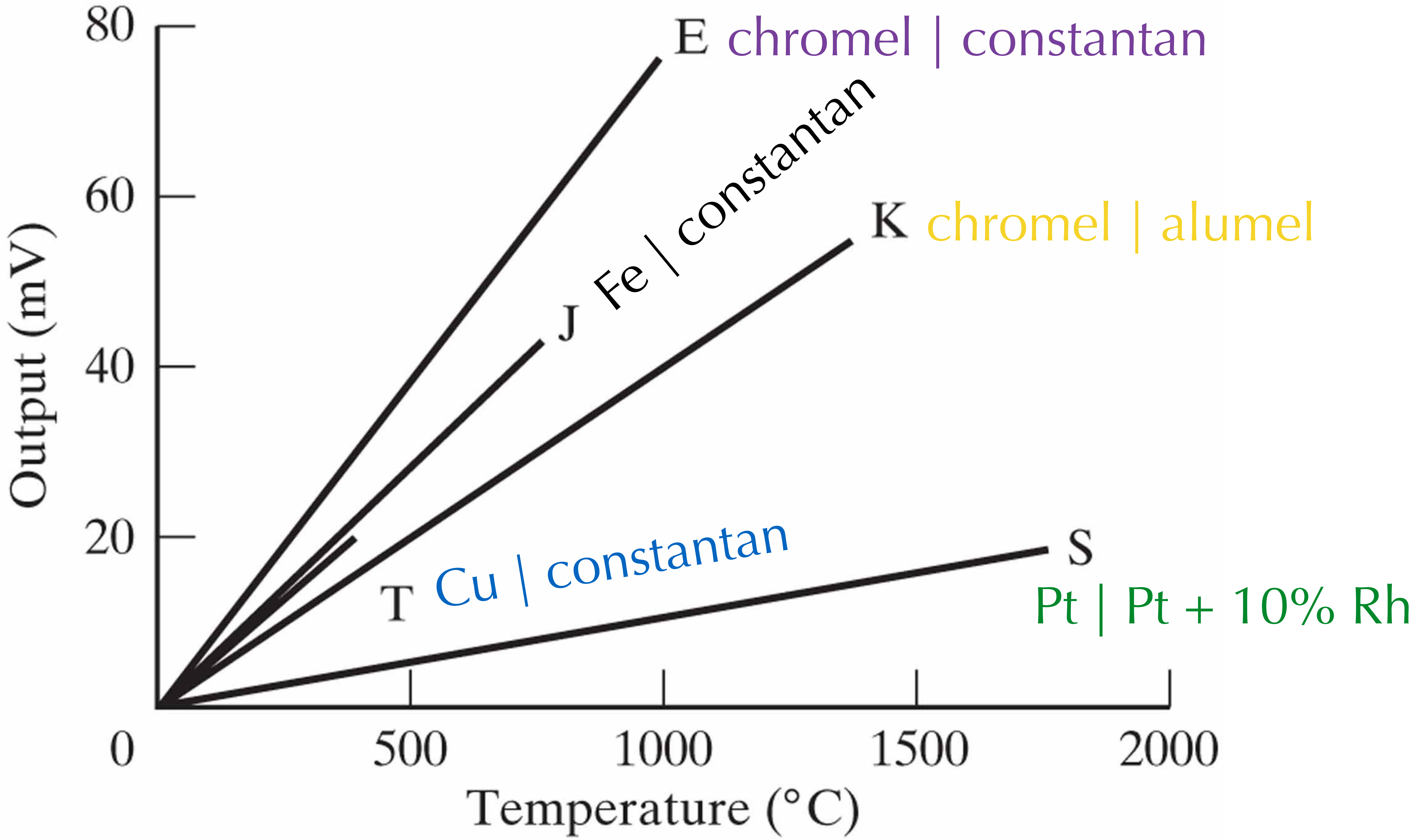
Palazzo *et al. Phys. Rev. Spec. Topics: Phys. Ed. Res.* 2010

Thermocouples





- Where two metals touch, a voltage is developed that is proportional to temperature (Peltier effect).
- Different proportionality for different metals.
- Lead wires of same (or compatible) metal



Radiation thermometers / thermal cameras

- All objects radiate always!

$$E = \epsilon\sigma T^4 = \epsilon \int_0^\infty \frac{C_1 \lambda^{-5}}{e^{\frac{C_2}{\lambda T}} - 1} d\lambda$$

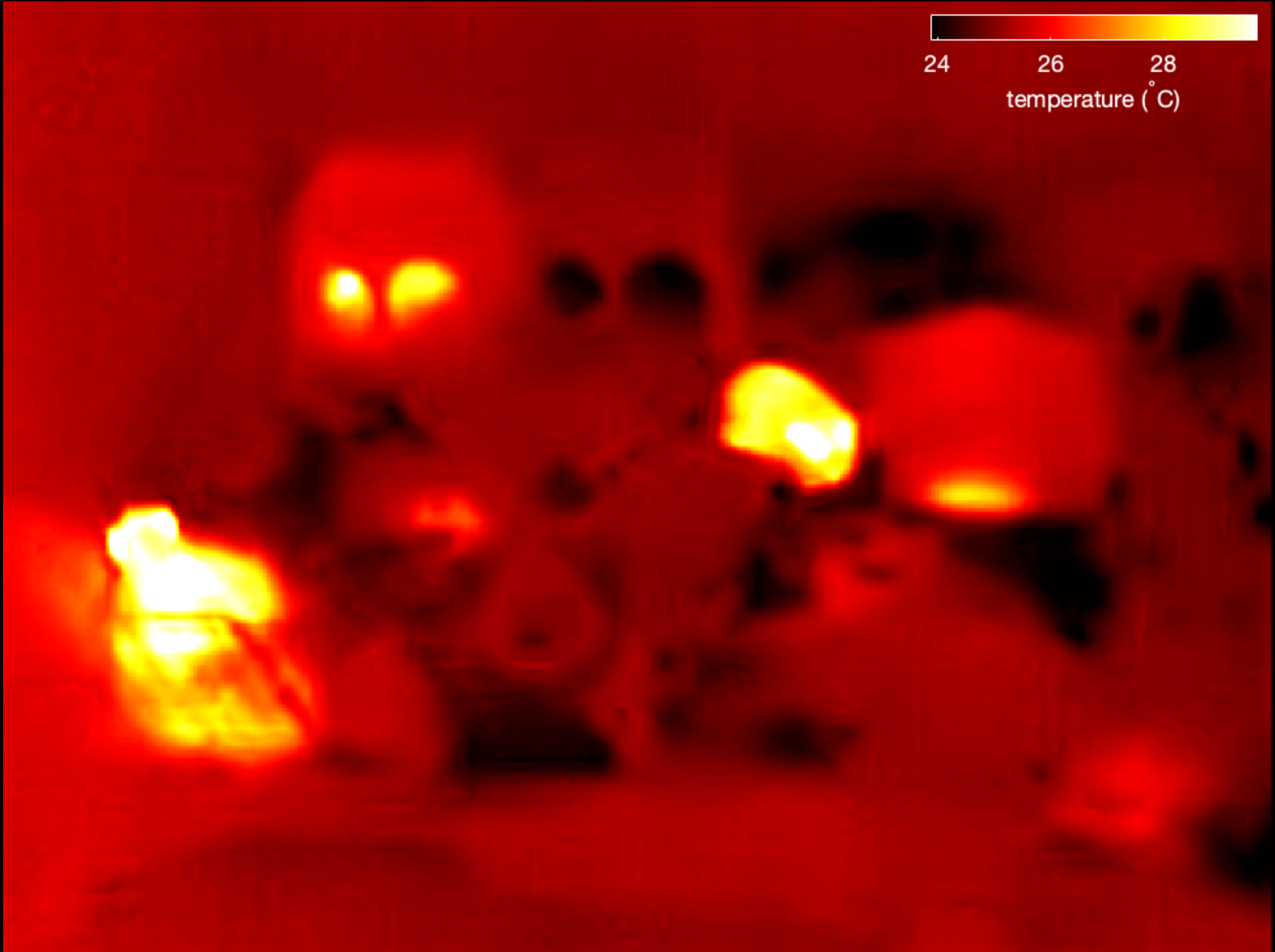
radiation power (E), emissivity (ϵ), temperature (T), Stefan-Boltzmann constant (σ), wavelength (λ)

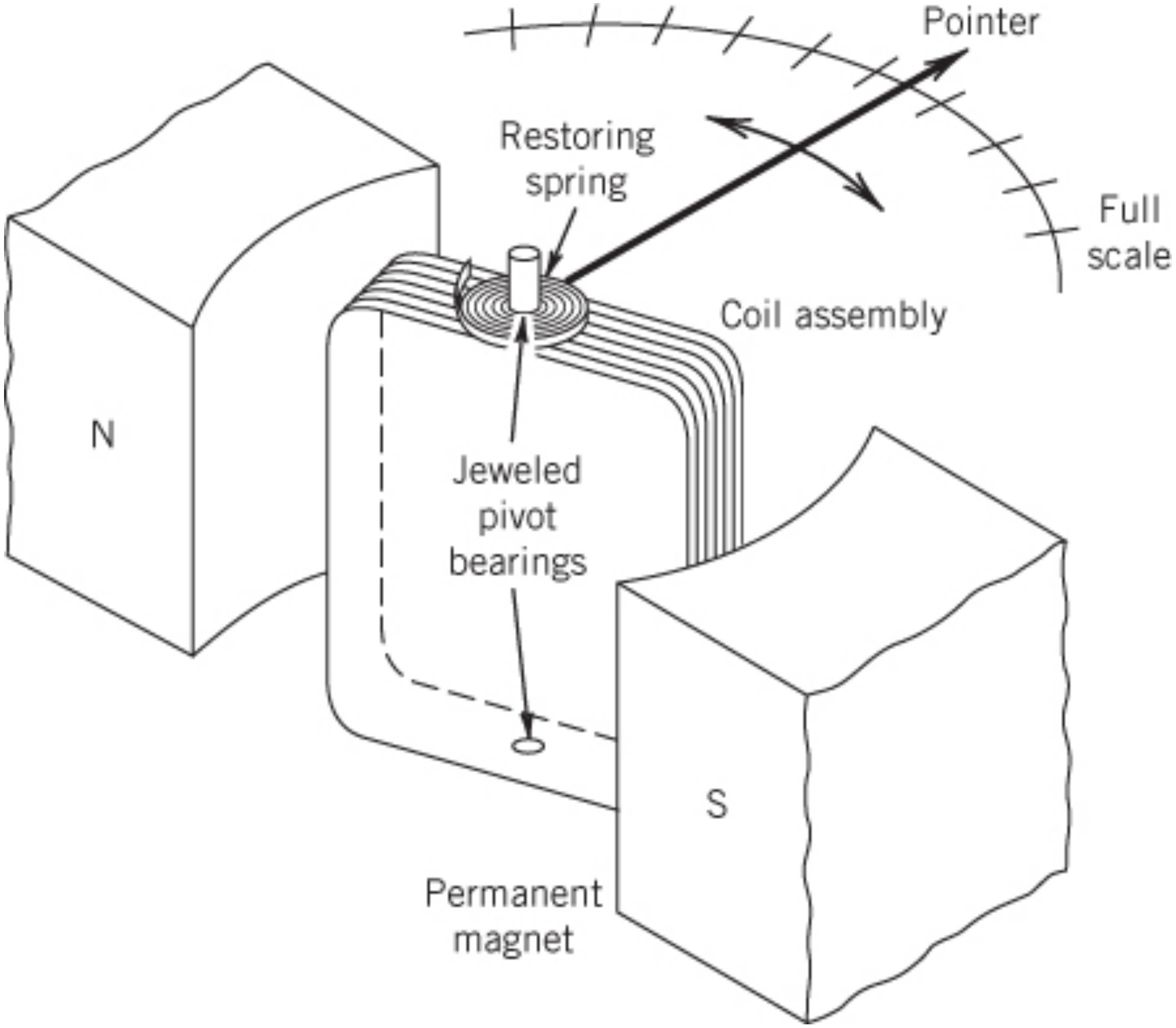
$\sigma = 5.669 \times 10^{-8}\,\mathrm{W m^{-2} K^{-4}}$

$C_1 = 3.743 \times 10^{8}\,\mathrm{W} \cdot \mu\mathrm{m}^4/\mathrm{m}^2$

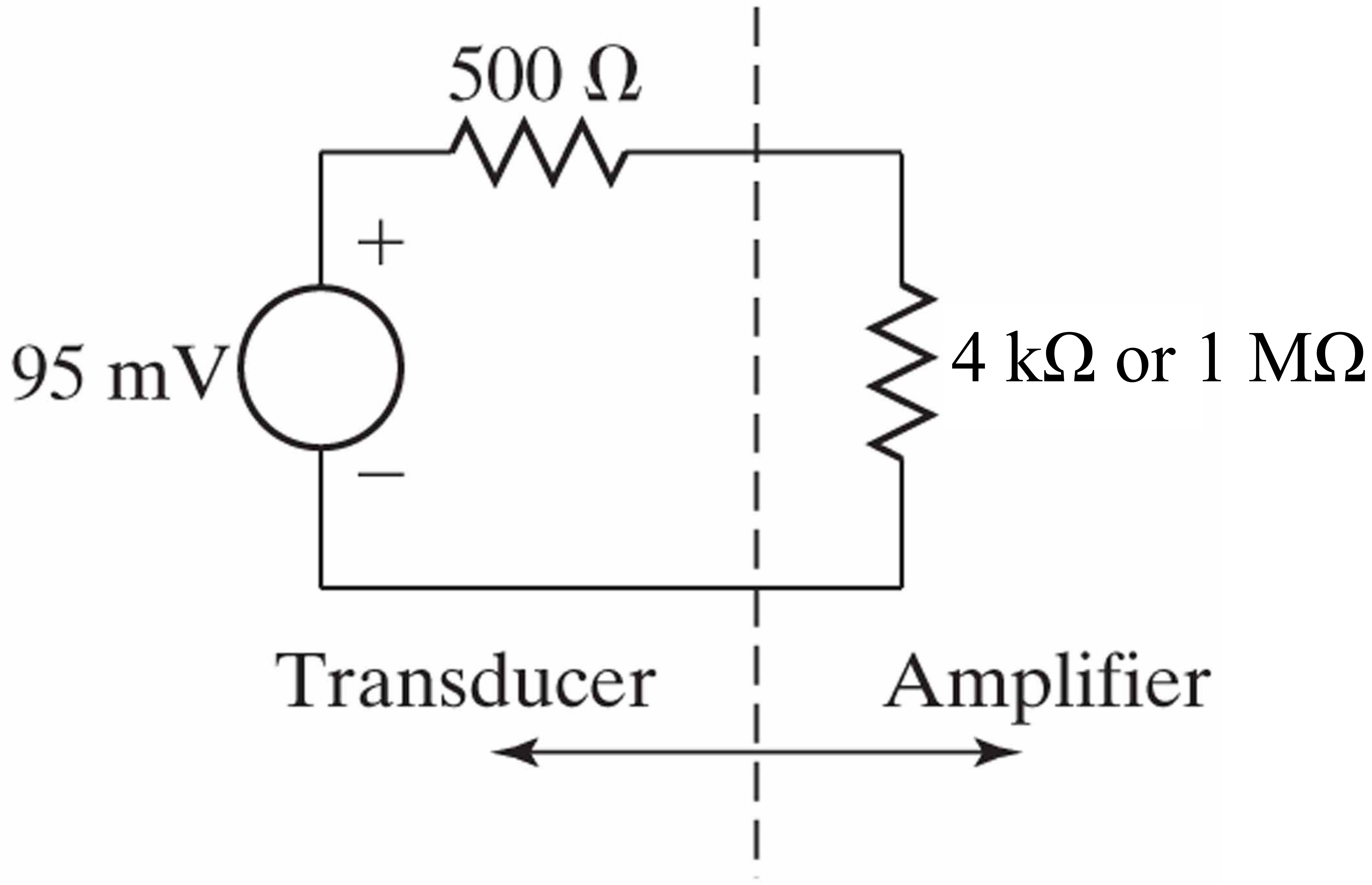
$C_2 = 1.4387 \times 10^{4} \mu\mathrm{m} \cdot K$

- $\epsilon = 1$ for ideal black body, $\epsilon = 0.018$ for shiny metals
- Common to measure E at two or more wavelengths, estimate T from ratio.


24
26
28
temperature (°C)


Pointer
Restoring
spring
Full
scale
Coil assembly
N
Jeweled
pivot
bearings
S
Permanent
magnet

Ex. 3.1: A force-measuring transducer has an open-circuit output voltage of 95 mV and an output impedance of 500 Ω. To amplify the signal voltage, it is connected to an amplifier with a gain of 10. Estimate the input loading error if the amplifier has an input impedance of (a) 4 kΩ, or (b) 1 MΩ.



UA741CP

Texas Instruments

Images are for reference only
See Product Specifications

Share

Mouser #:	595-UA741CP
Mfr. #:	UA741CP
Mfr.:	Texas Instruments
Customer #:	Customer #
Description:	Operational Amplifiers - Op Amps GP Op Amp
Datasheet:	UA741CP Datasheet
ECAD Model:	PCB Symbol, Footprint & 3D Model

Download the free Library Loader to convert this file for your ECAD Tool. Learn more about ECAD Model.

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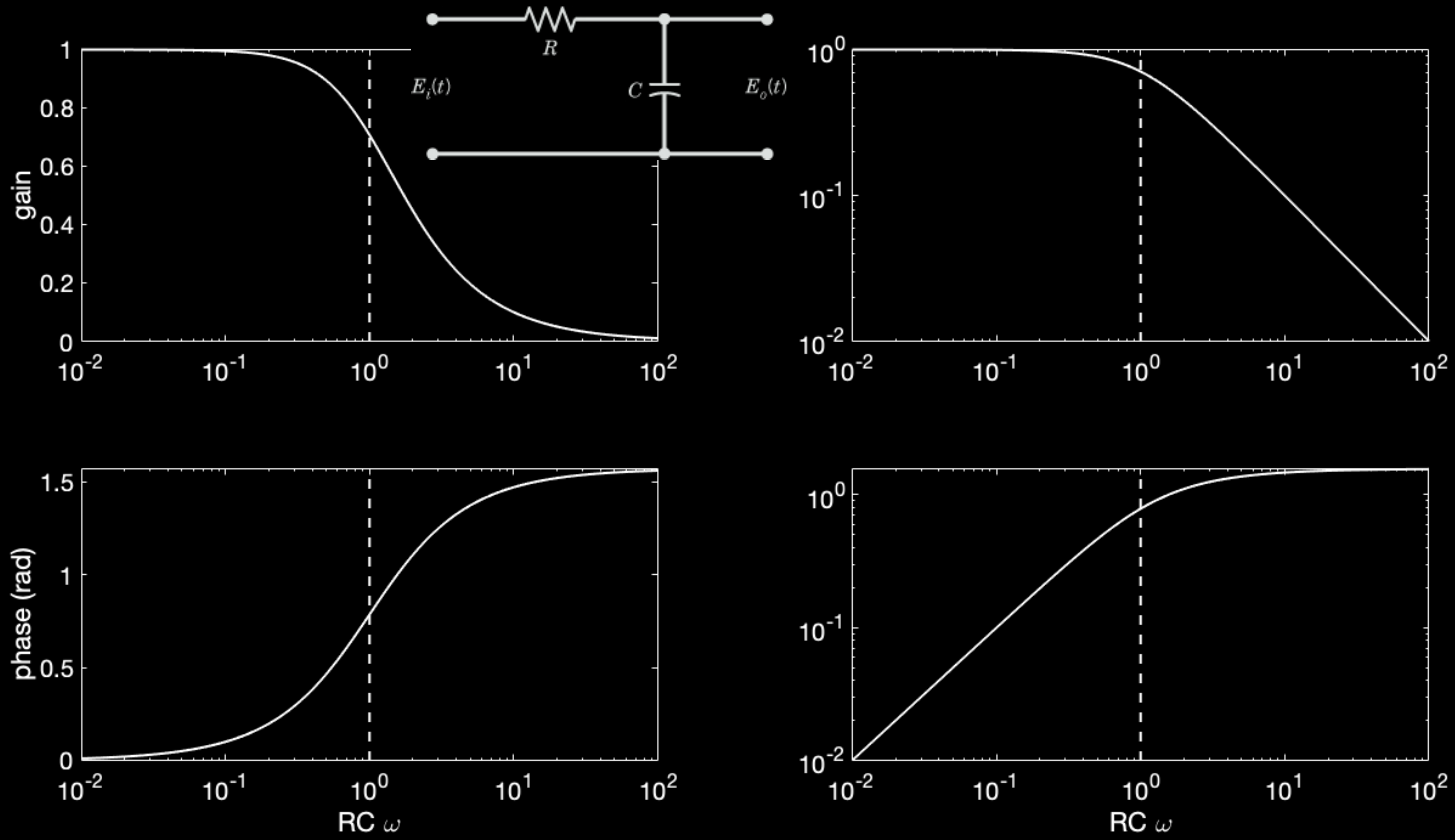
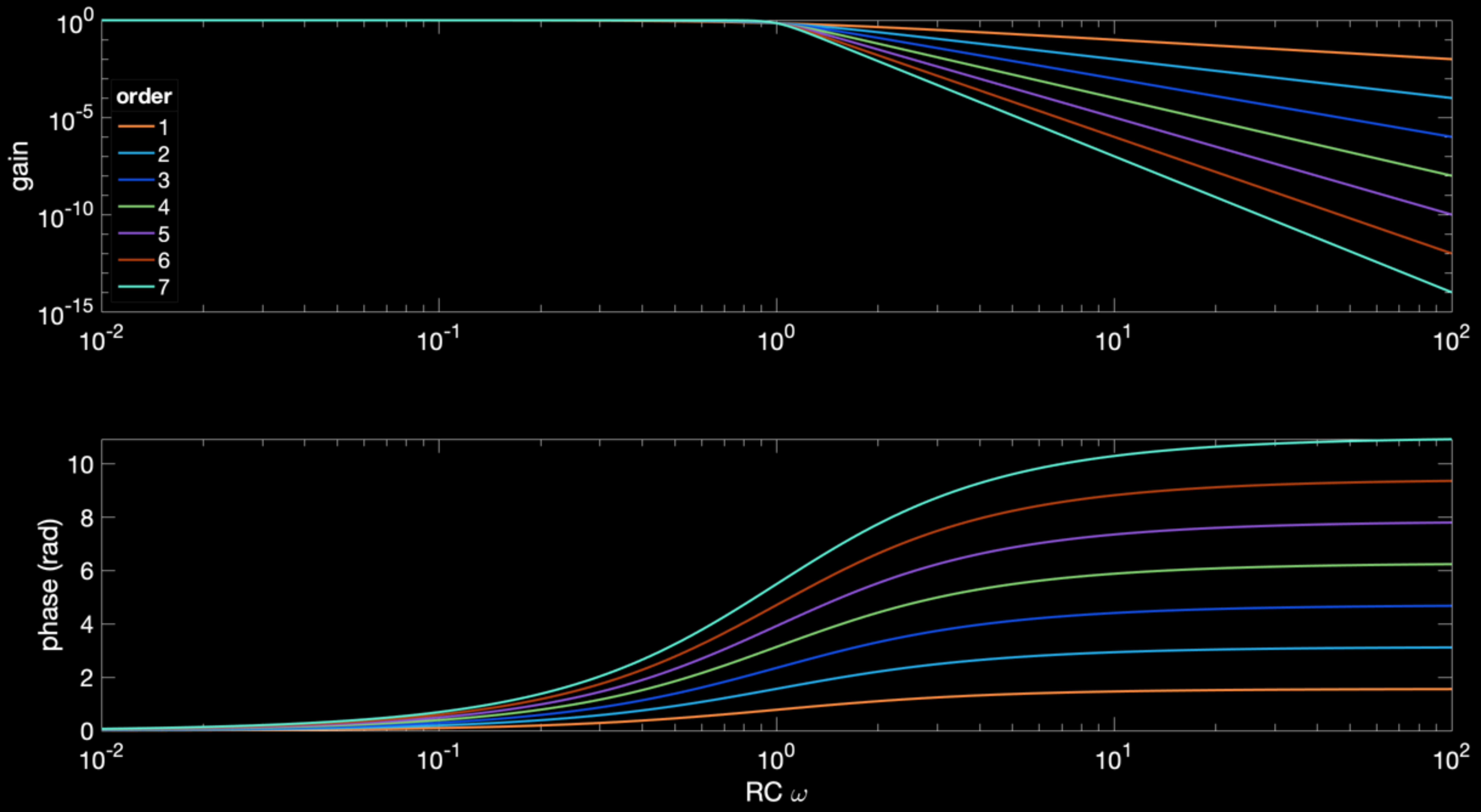
Single-stage low-pass Butterworth filter



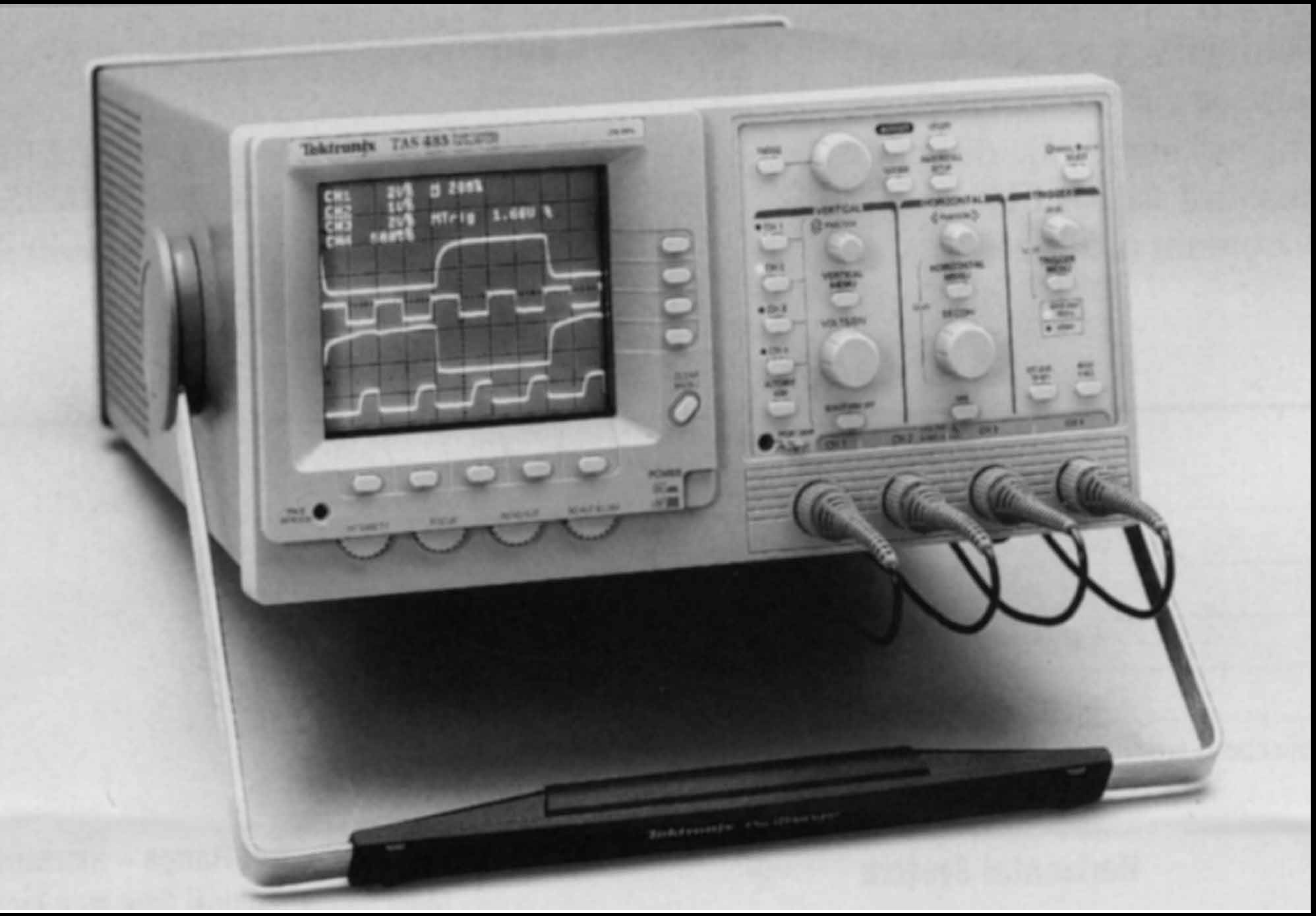
Low-pass Butterworth filters



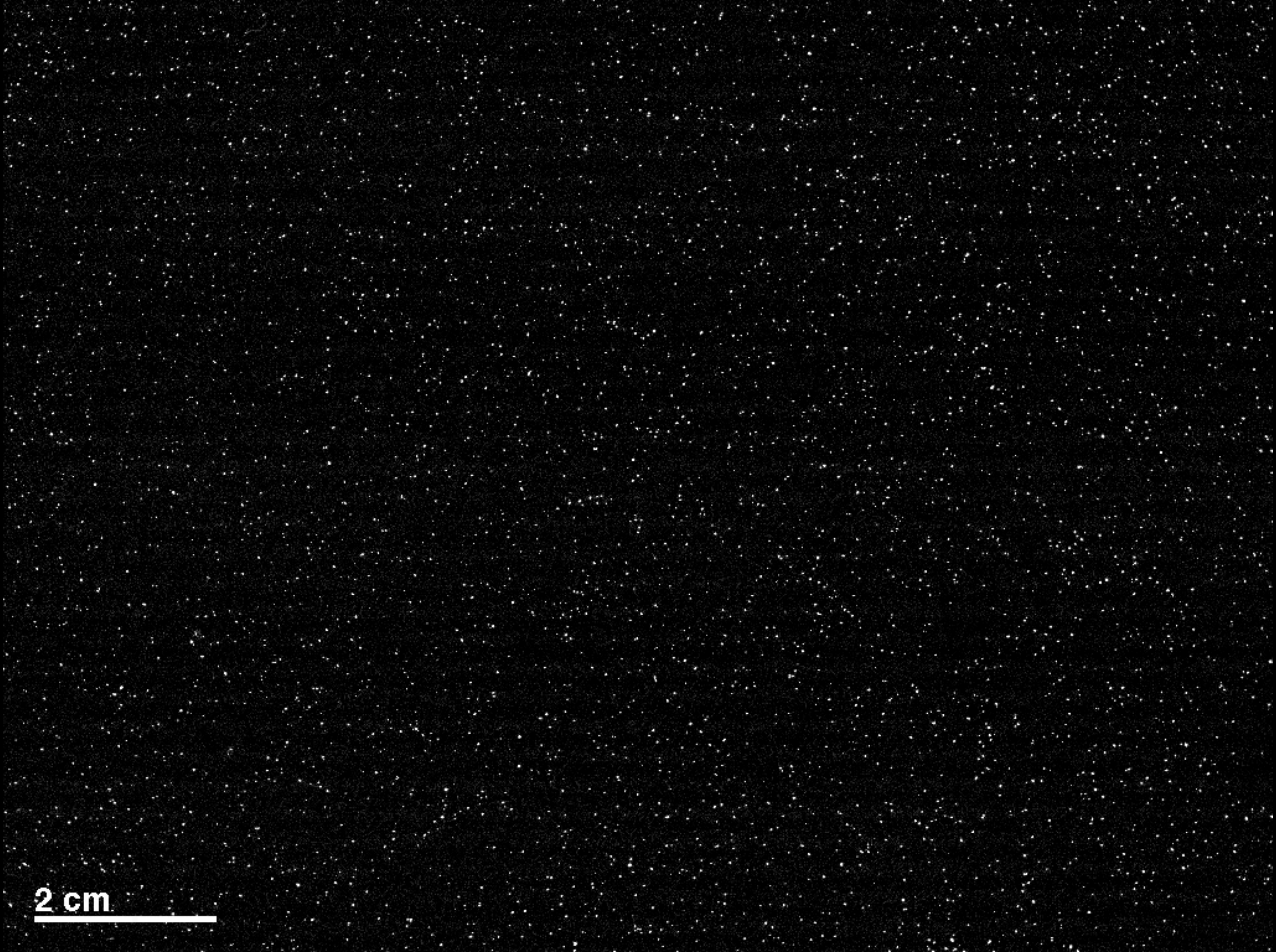
Digital multimeter

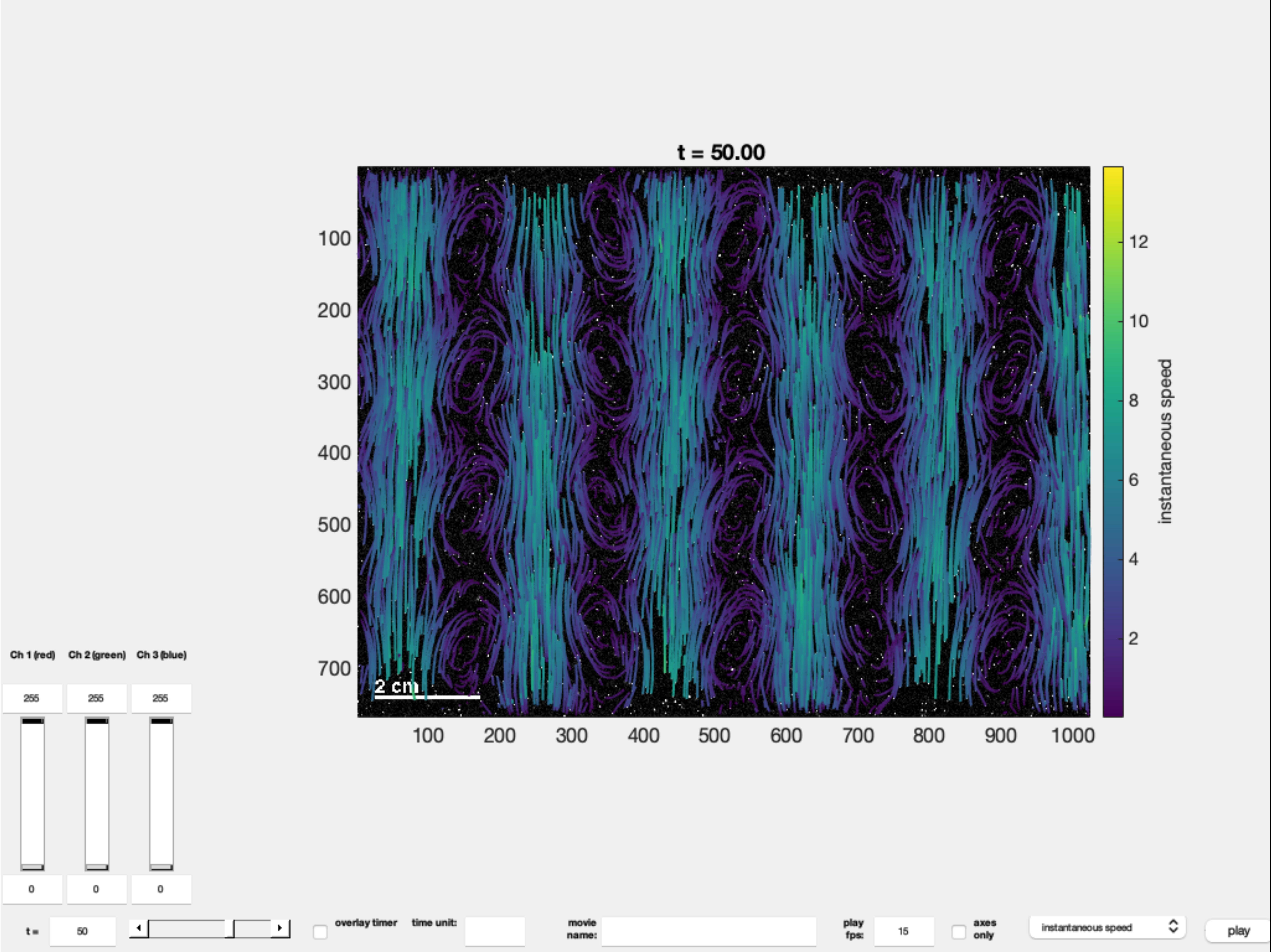
- High input impedance for voltage measurements
- Low input impedance for current measurements
- DC or AC
- Can measure resistance
- Some can measure capacitance, read thermocouples, …
- Accuracy <1% of reading
- Not working? Check leads, battery, and fuse!

Oscilloscope

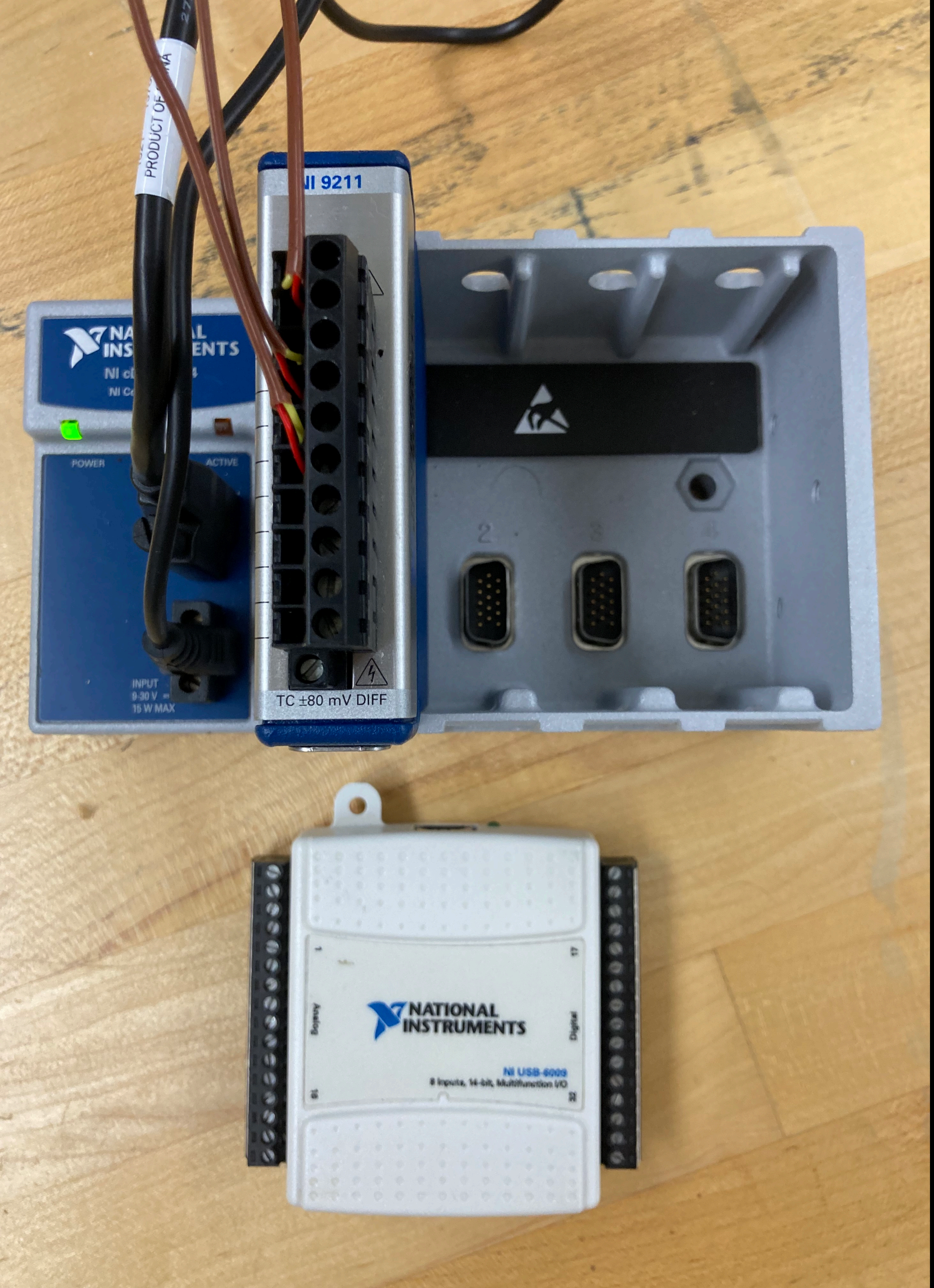


- For measuring time-varying voltages, typically periodic
- High input impedance
- Accuracy <1% of reading
- Multiple channels
- Up to 100 MHz
- Usually USB output


2 cm


t = 50.00
100
200
300
400
500
600
700
100
200
300
400
500
600
700
800
900
1000
2 cm
12
10
8
6
4
2
instantaneous speed
Ch 1 (red)
Ch 2 (green)
Ch 3 (blue)
255
255
255
0
0
0
t =
50
overlay timer
time unit:
movie name:
play fps:
15
axes only
instantaneous speed
play

Computerized data acquisition

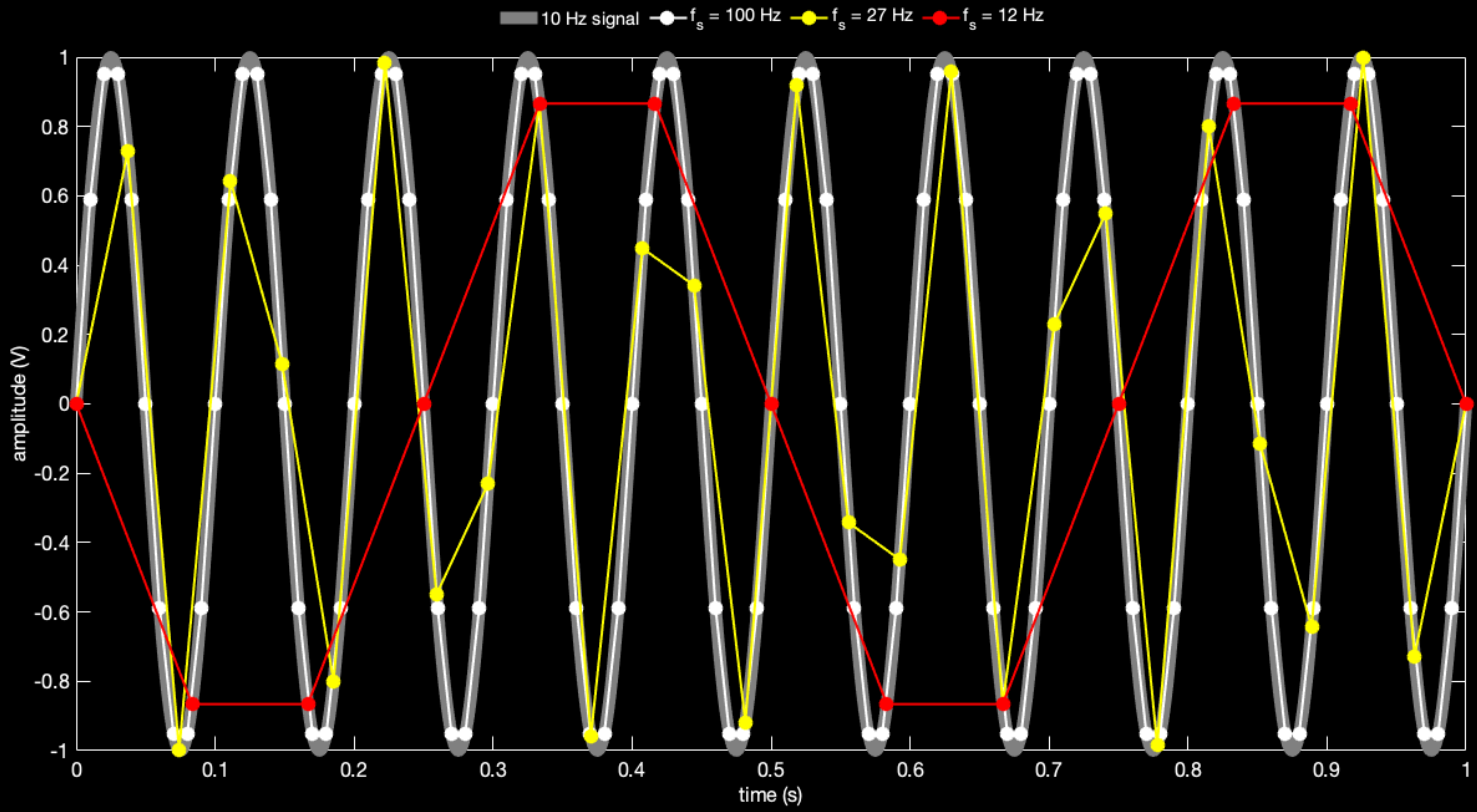


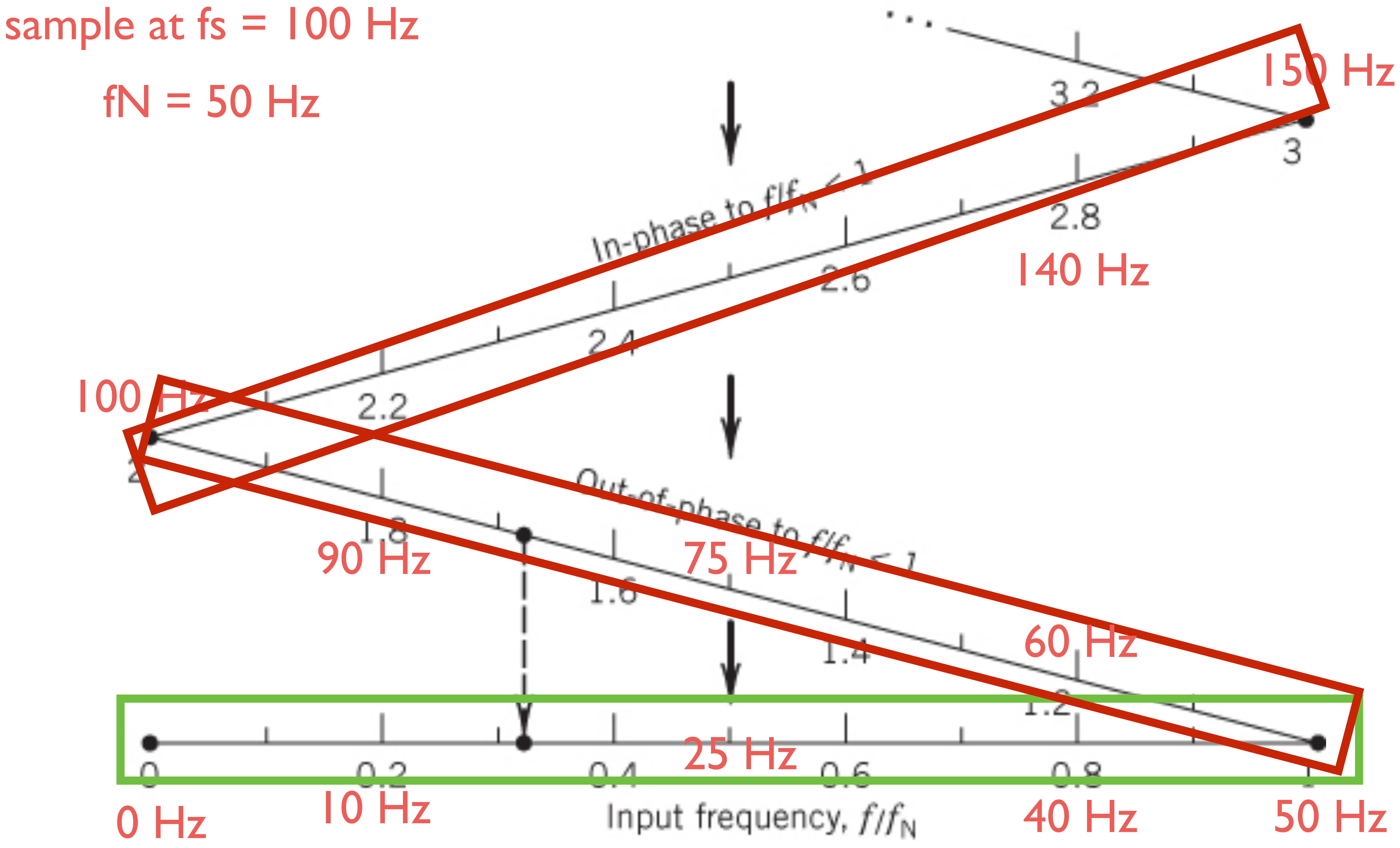
- Often connect via USB; sometimes internal
- Wires from sensors connect to screw terminals
- Range often -5 V to 5 V or 0 V to 10 V
- Special ports for thermocouples
- Analog inputs, digital input/output, occasionally analog outputs
- Varying channel count (1 to ~30), bit depth (8, 12, 16), sampling rates (~100 to ~1000 kS/s)
- Software interface (LabView, Matlab, …)
- National Instruments, Measurement Computing, Keyence, Arduino, …

Ex. 4.5: A 4-bit analog-to-digital converter has an input range of 0 to 10 V. Compute the digital output for an analog input of 6.115 V.

There are 10 kinds of people in the world:
those who understand binary numbers,
and those who don’t.

Aliasing when sampling rate fails the Nyquist criterion

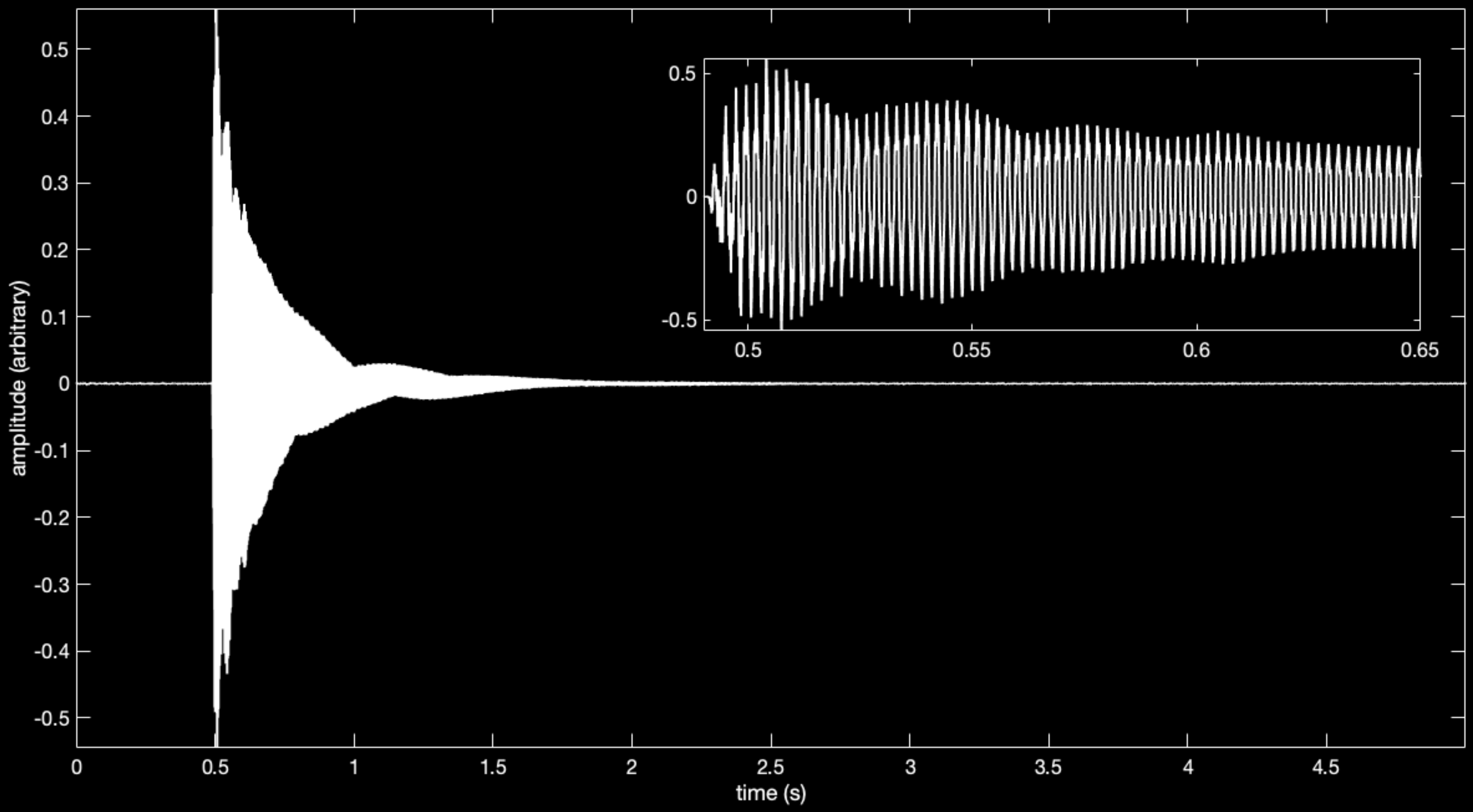



sample at fs = 100 Hz
fN = 50 Hz
150 Hz
3.2
3
In-phase to f/f$_N$ < 1
2.8
140 Hz
2.6
2.4
100 Hz
2.2
90 Hz
1.8
75 Hz
Out-of-phase to f/f$_N$ < 1
1.6
1.4
60 Hz
1.2
25 Hz
0
0.2
0.4
0.6
0.8
0 Hz
10 Hz
Input frequency, f/f$_N$
40 Hz
50 Hz

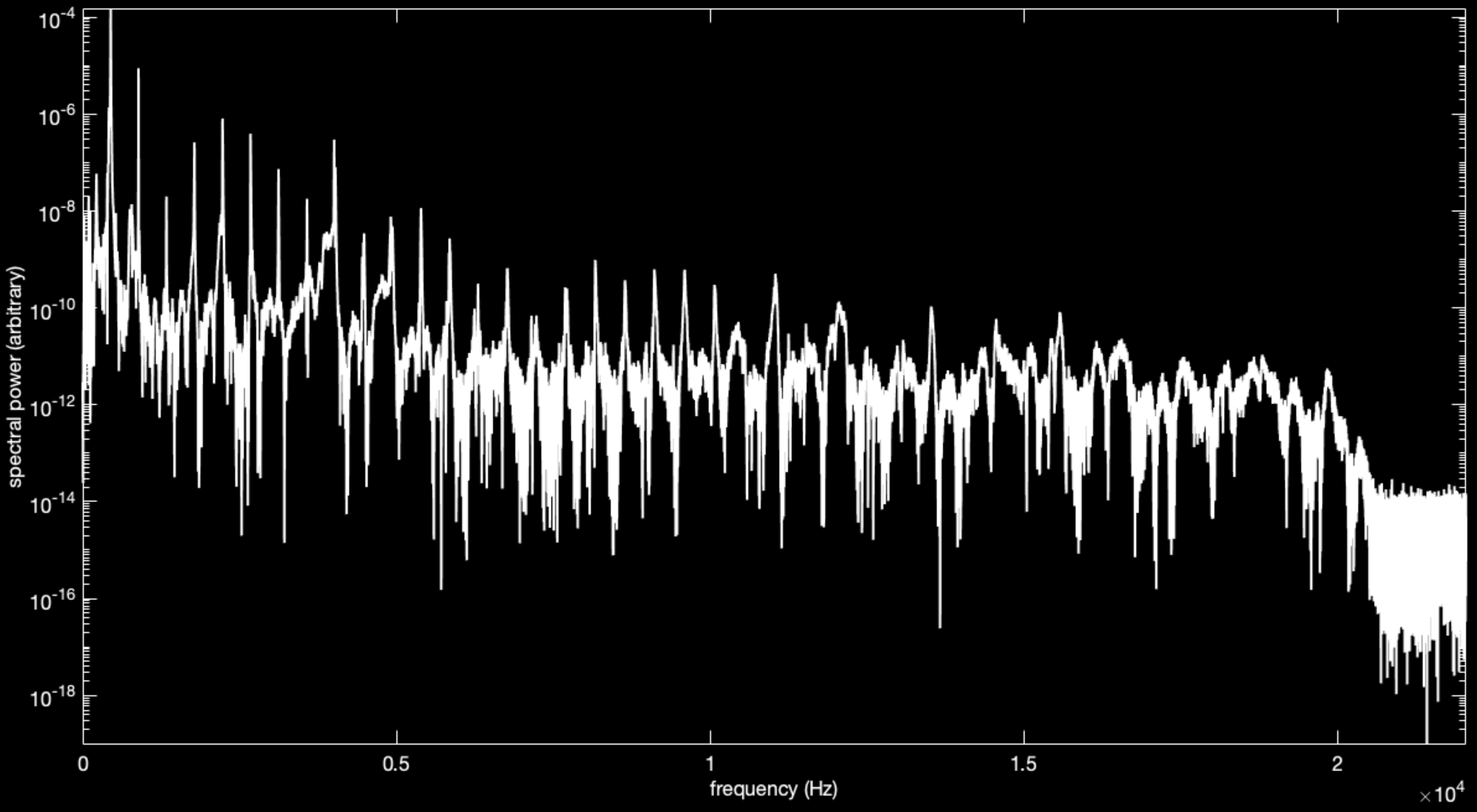
18 kHz tone

440 Hz tone

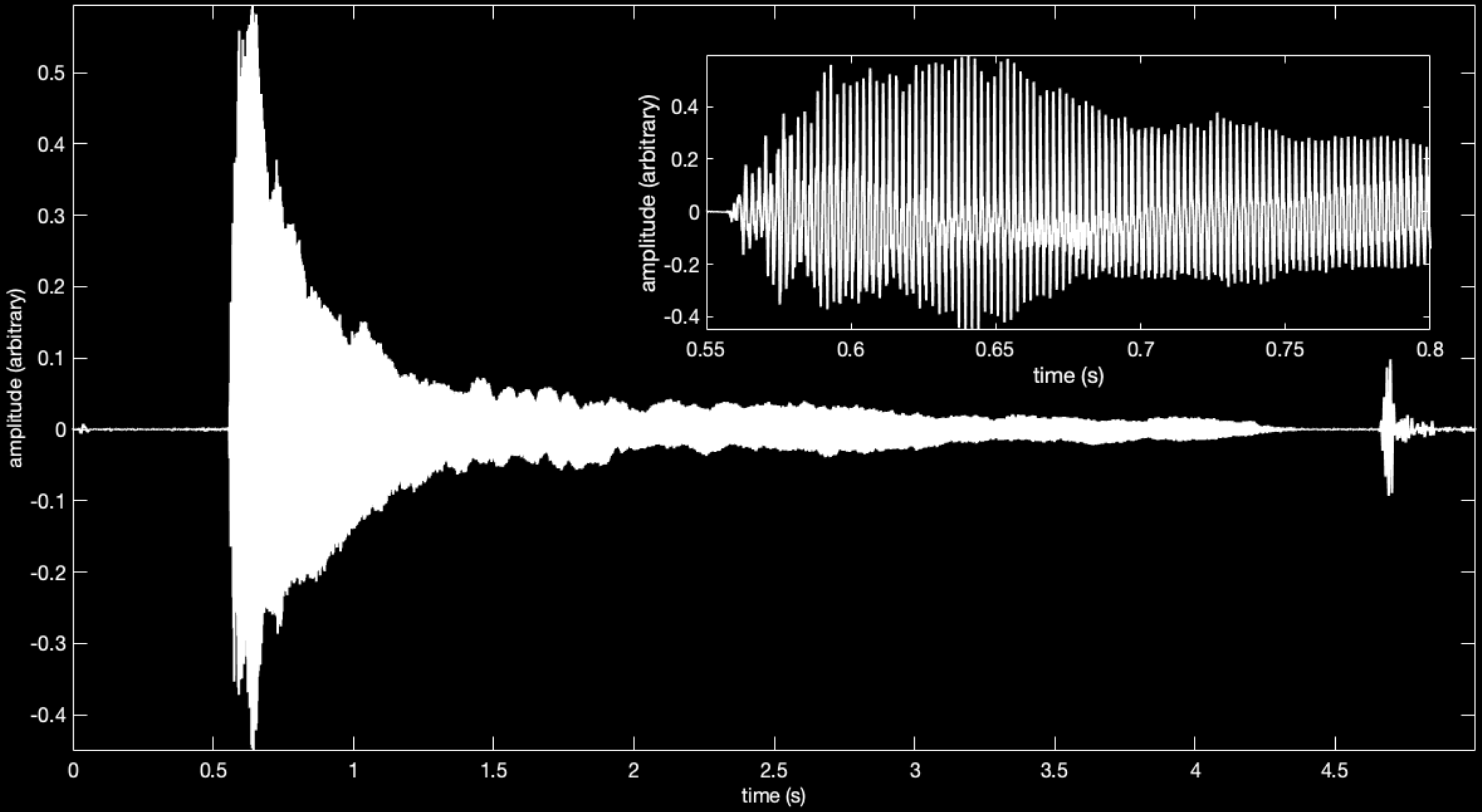
"A" note on ukelele



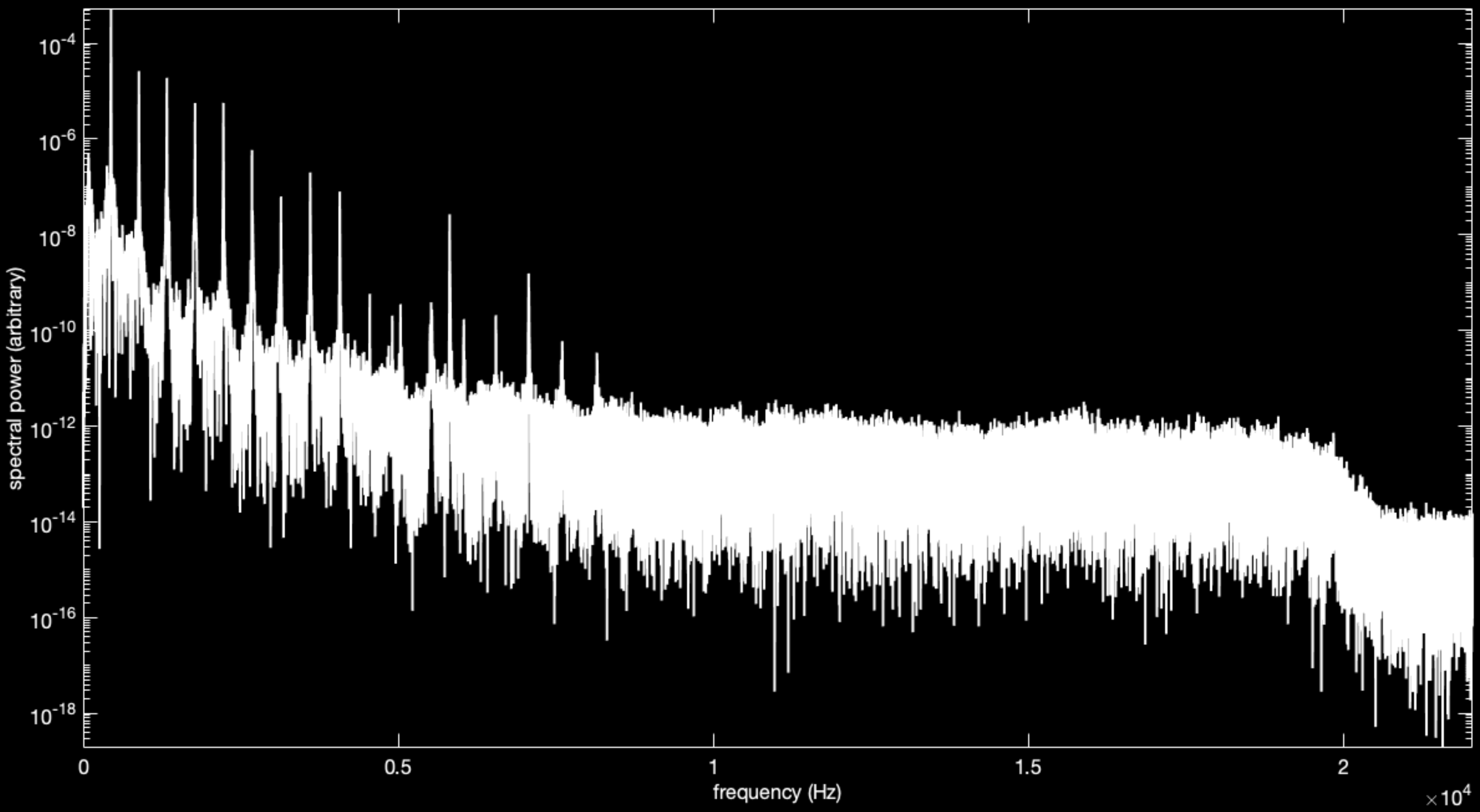
“A” note on ukelele



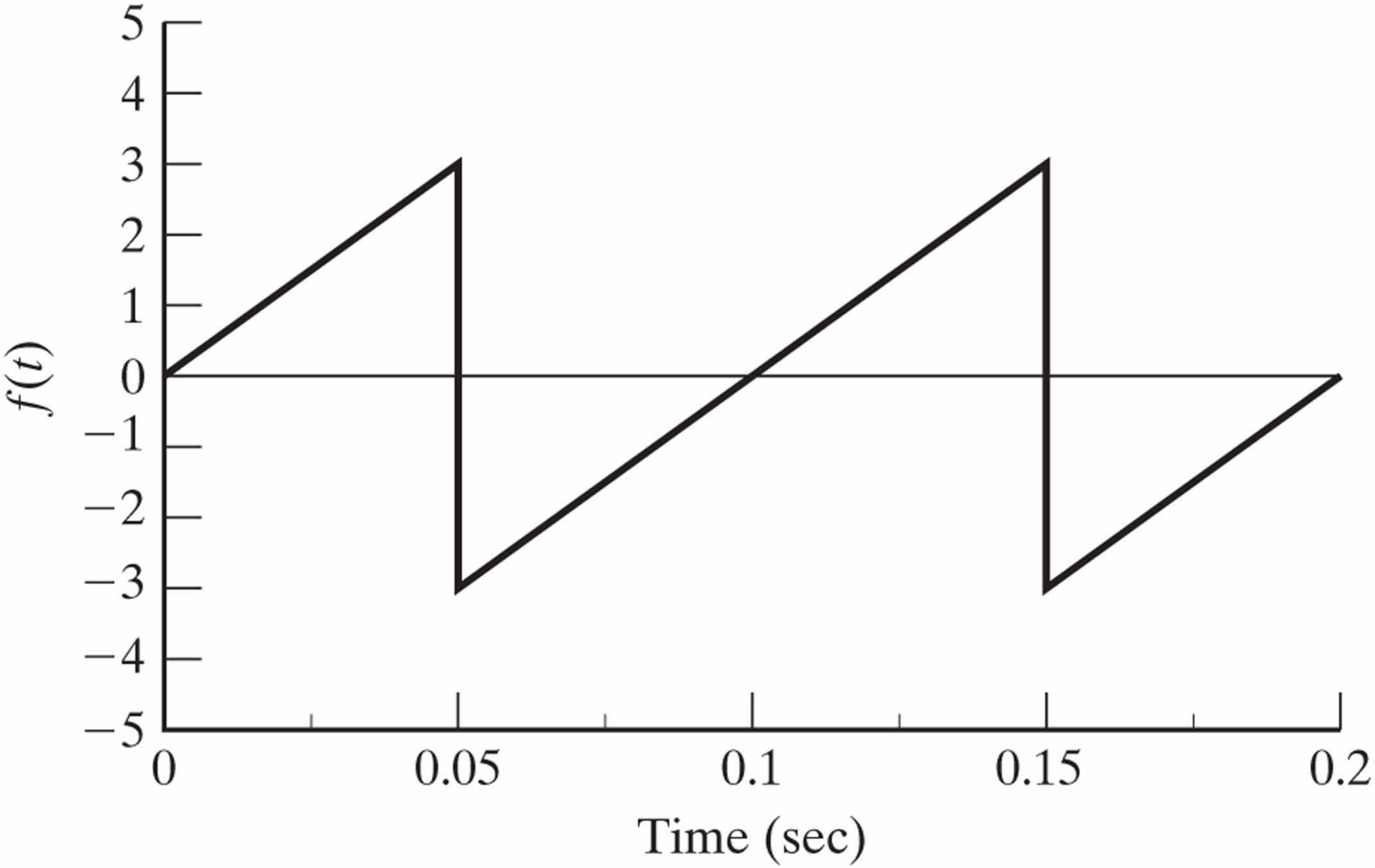
"A" note on piano



"A" note on piano



Ex. 5.2: Determine the amplitudes of the first, second, and third harmonic components of the signal plotted below.



Claude Debussy's "Rêverie"

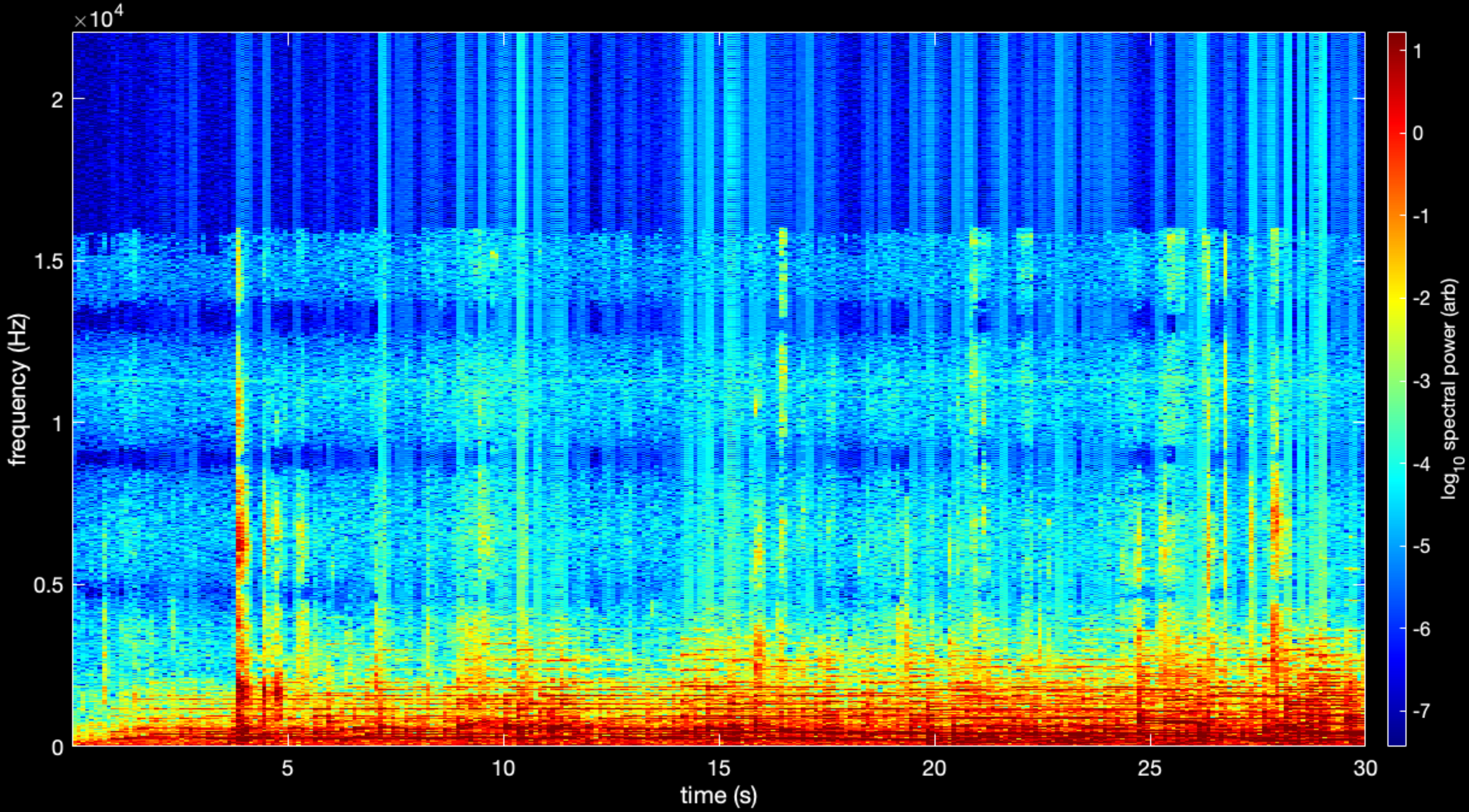


FIGURE 1. Schematic of the experimental set-up (*a*) and the UDV Sensor positions for (*b*) the axial velocity and (*c*) the chord velocity measurements. (*d*) Photograph of the experimental set-up without sidewall insulation in place. Image credit: Y. Xu (UCLA).

Vogt, Horn, & Aurnou *J. Fluid Mech.* 2021

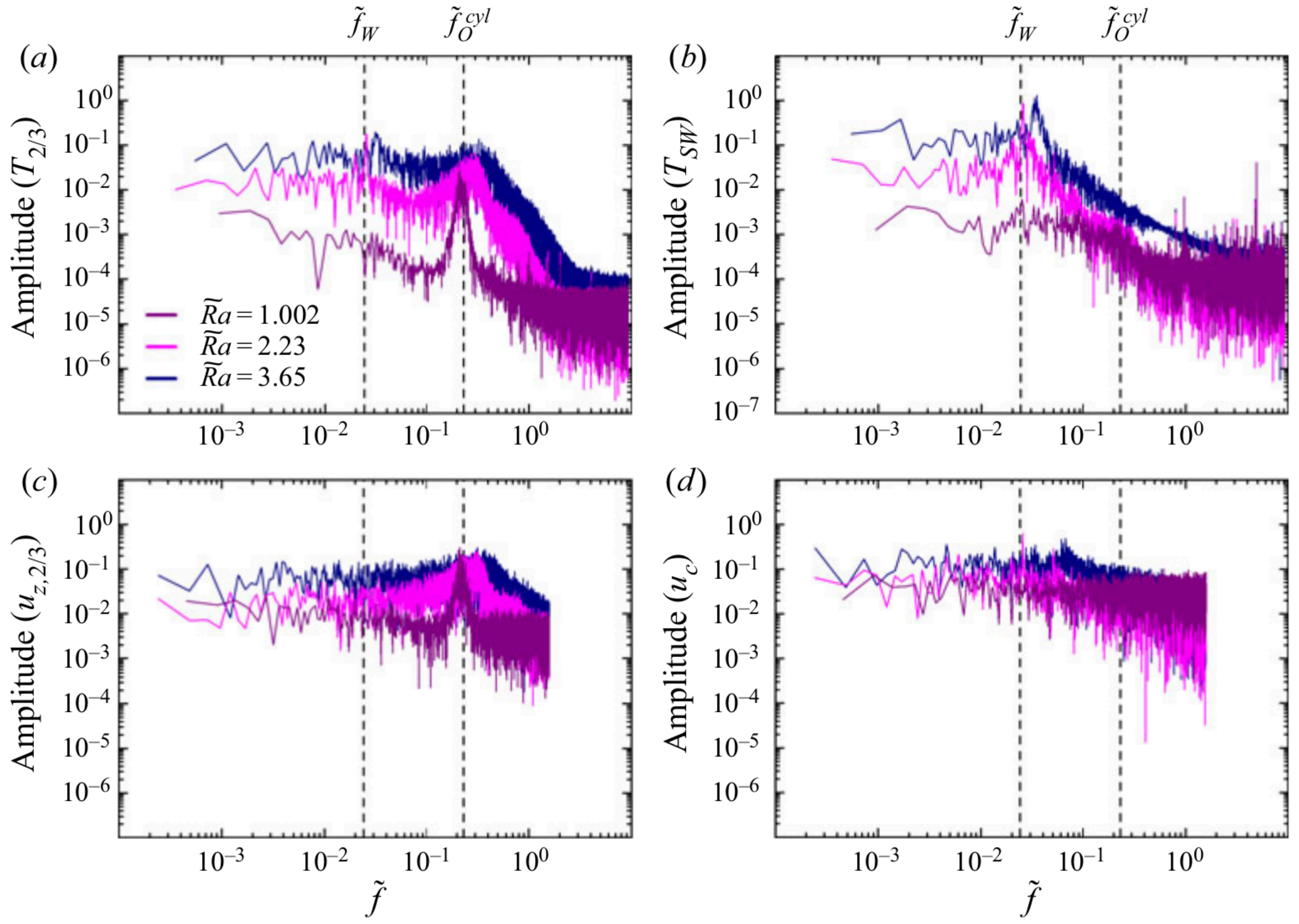


FIGURE 7. Amplitude of the Fourier transforms of temperature and velocity signals versus normalized frequency $\tilde{f} = f/f_\Omega$. The Ekman number is $Ek = 5 \times 10^{-6}$ and the supercriticality $\widetilde{Ra}$ is indicated by the line colour. All spectra are evaluated on the midplane, $z/H = 1/2$. (*a*) Temperature spectra measured with a thermistor situated within the fluid bulk at $r/R = 2/3$. (*b*) Temperature spectra measured on the cylindrical tank's outer sidewall at $r/R = 1.05$. (*c*) Vertical velocity spectra measured at $r/R = 2/3$. (*d*) Chord velocity spectra evaluated in the vicinity the the sidewall. Vertical dashed lines indicate the onset frequency for wall modes $\tilde{f}_W = 0.024$ and bulk oscillations $\tilde{f}_O^{cyl} = 0.274$.

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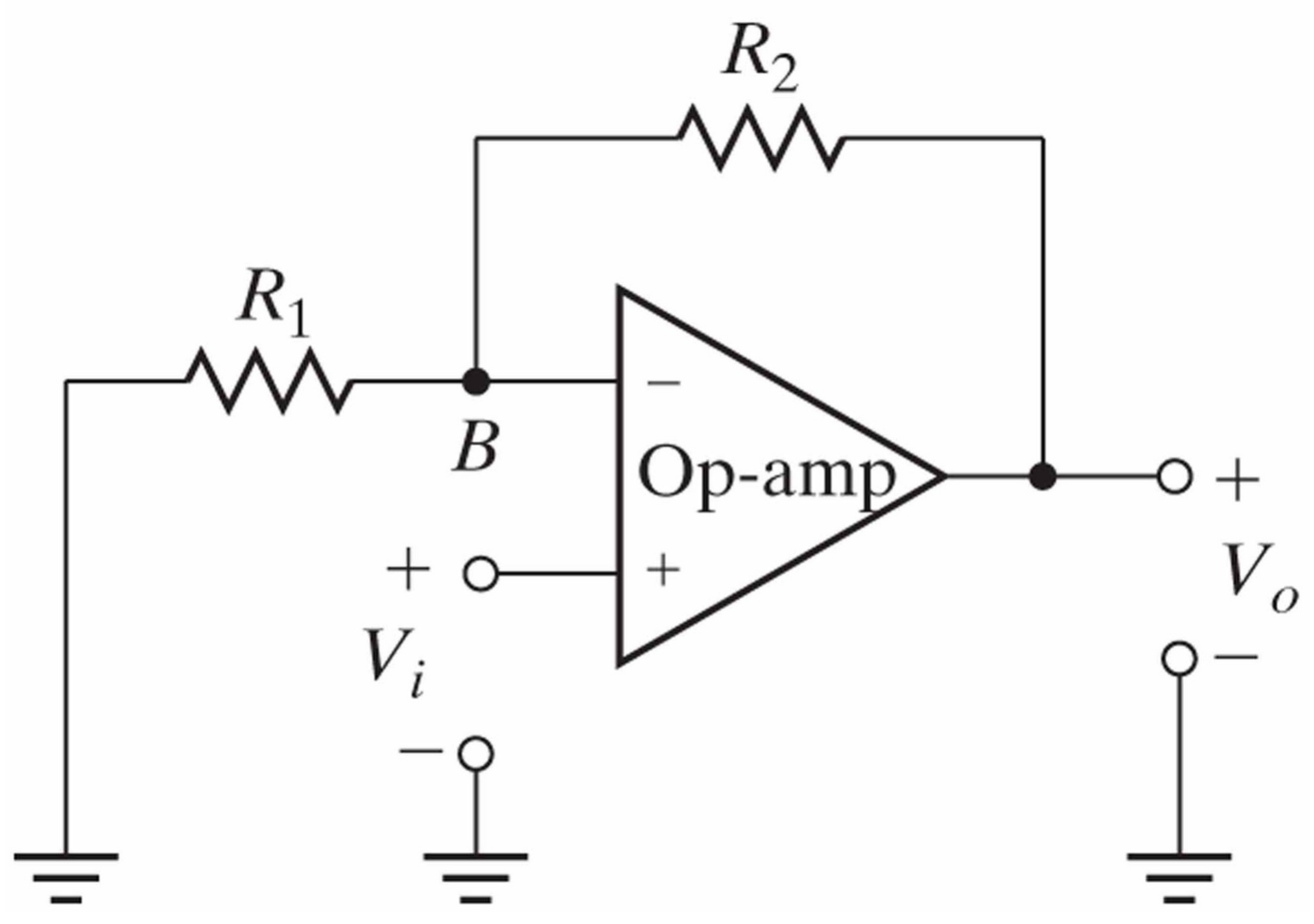
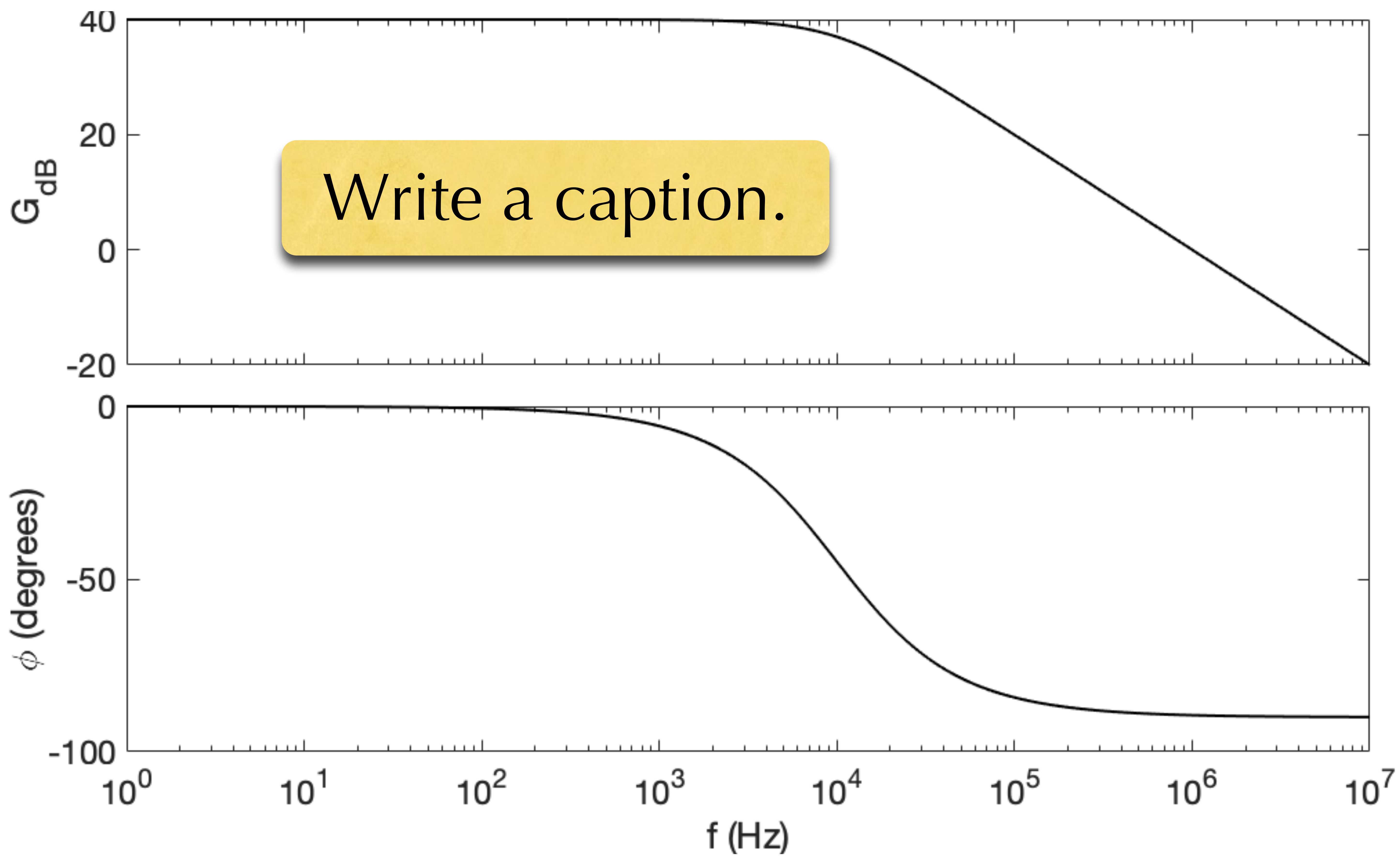
A philosophy of captions

- Start with a brief and literal statement of what the figure shows (e.g., "Distributions of normalized resistance for different vascular arrangements.").
- Give the reader all information necessary to understand what you're plotting and how: colors, plot symbols, math symbols, acronyms, etc. But information that is self-evident in the plots, e.g. because it's on legends or labels, need not be repeated.
- End the caption with a brief statement of the take-home message (e.g., "Though all distributions overlap with the observed normalized resistance, the hexagonal pseudorandom perturbed arrangement gives the closest match").
- Don't write "A figure showing…" or "Plots of …" because that's obvious.
- In the body text, avoid writing about colors or symbols; just write about the quantities.
- Keep details about the methods that produced the plots and the analysis of the results in the body text, not the caption…
  - … unless you're writing with a tight word limit on body text (e.g. *Nature* or *Science*) and must therefore pack words into captions.

HW2 #1: The noninverting amplifier shown below is to be constructed with a μA741C op-amp. It is to have a gain of 100. Sketch the Bode plots for this amplifier using specific numerical values.





Abstract: Oscillatory thermal-inertial flows in liquid metal rotating convection

We present the first detailed thermal and velocity field characterization of convection in a rotating cylindrical tank of liquid gallium, which has thermophysical properties similar to those of planetary core fluids. Our laboratory experiments, and a closely associated direct numerical simulation, are all carried out in the regime prior to the onset of steady convective modes. This allows us to study the oscillatory convective modes, sidewall modes and broadband turbulent flow that develop in liquid metals before the advent of steady columnar modes. Our thermo-velocimetric measurements show that strongly inertial, thermal wind flows develop, with velocities reaching those of non-rotating cases. Oscillatory bulk convection and wall modes coexist across a wide range of our experiments, along with strong zonal flows that peak in the Stewartson layer, but that extend deep into the fluid bulk in the higher supercriticality cases. The flows contain significant time-mean helicity that is anti-symmetric across the midplane, demonstrating that oscillatory liquid metal convection contains the kinematic components to sustain system-scale dynamo generation.

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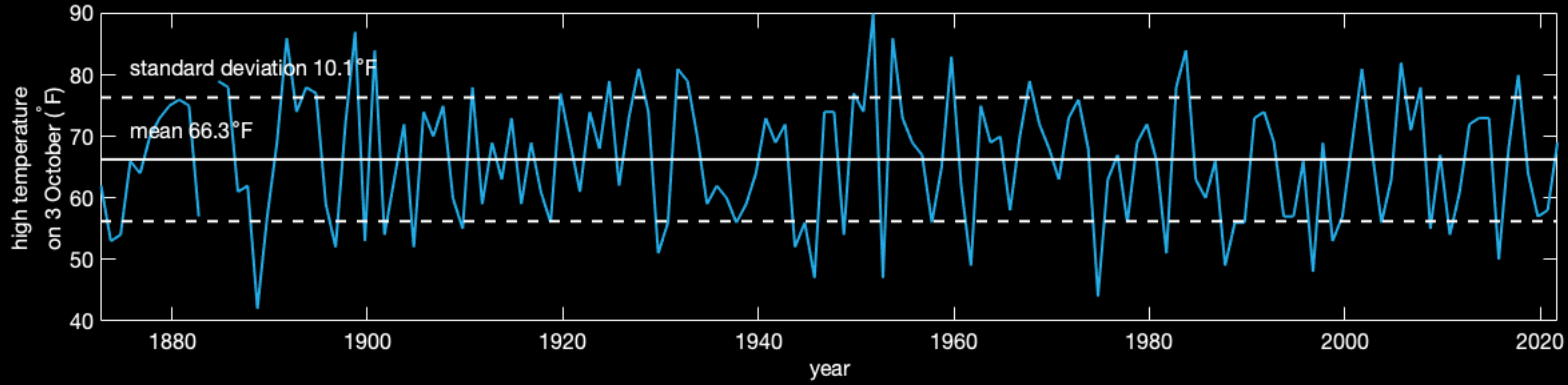
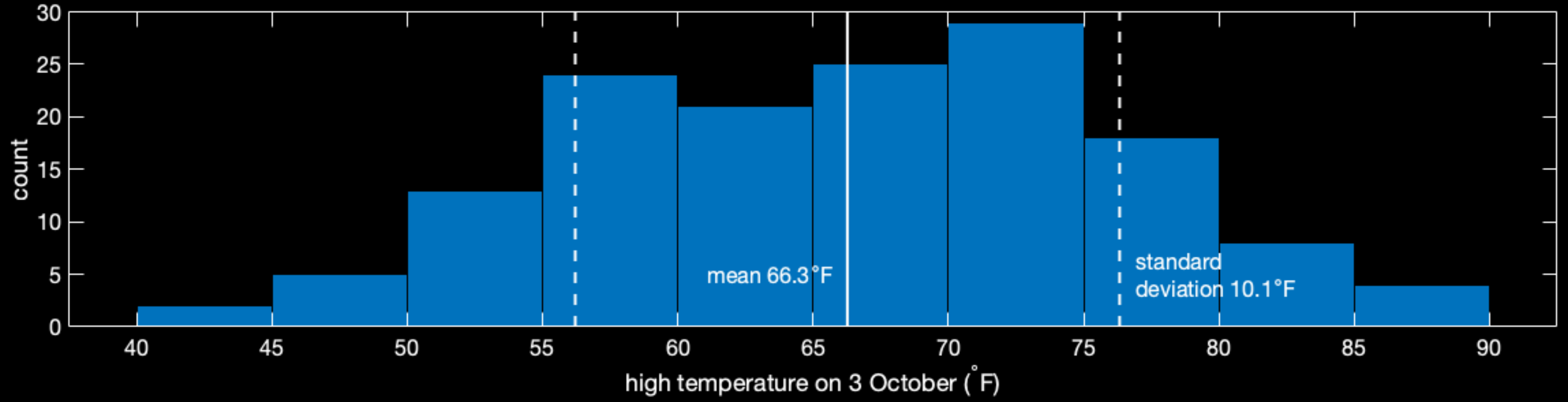
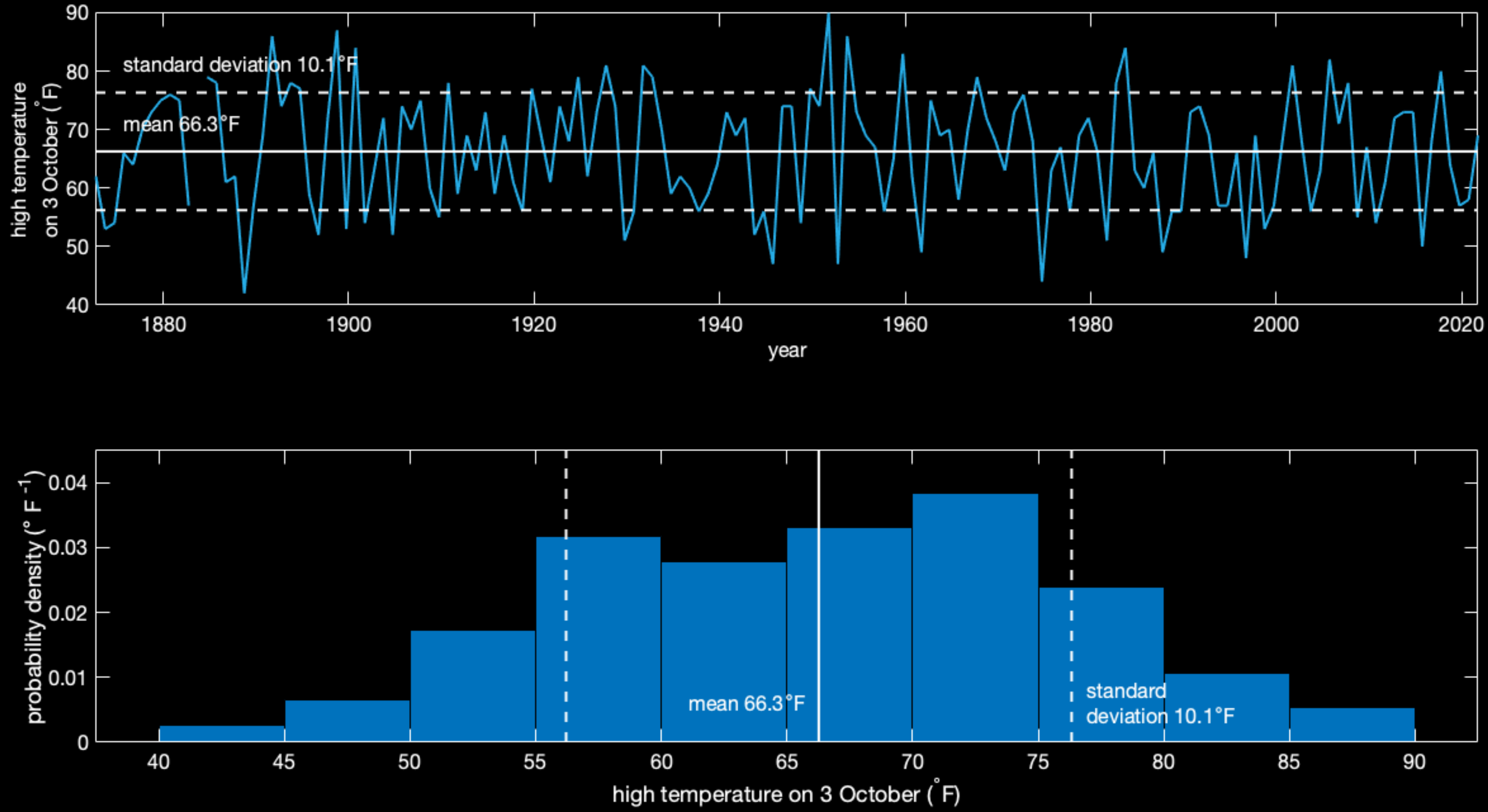
Rochester's weather on 3 October





Rochester's weather on 3 October



Ex. 6.1: The life x of a given type of ball bearing can be characterized by a probability density function

$$f(x) = \begin{cases} 0, & x < 10 \text{ h} \\ \dfrac{200}{x^3}, & x \geq 10 \text{ h} \end{cases}$$

If we pick a random bearing from this batch, what is the probability that its life will exceed 20 h? Be exactly 20 h?

Ex. 6.1: The life x of a given type of ball bearing can be characterized by a probability density function

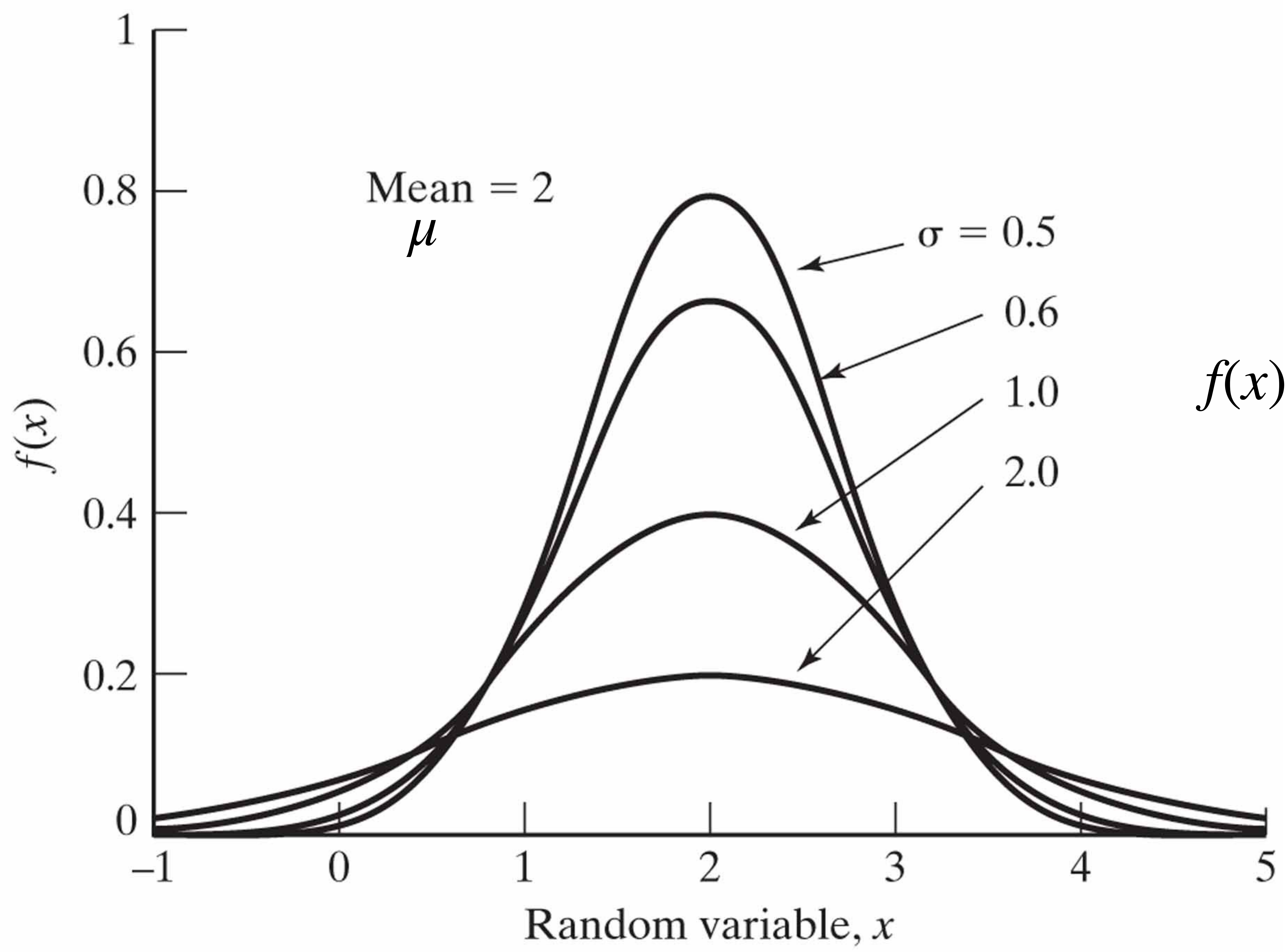
$$f(x) = \begin{cases} 0, & x < 10 \text{ h} \\ \dfrac{200}{x^3}, & x \geq 10 \text{ h} \end{cases}$$

Calculate the expected life of a bearing.

Ex. 6.4: For a given batch of light bulbs, 10% are defective. You buy 4. What are the probabilities that 4, 1, or 0 of them are defective?

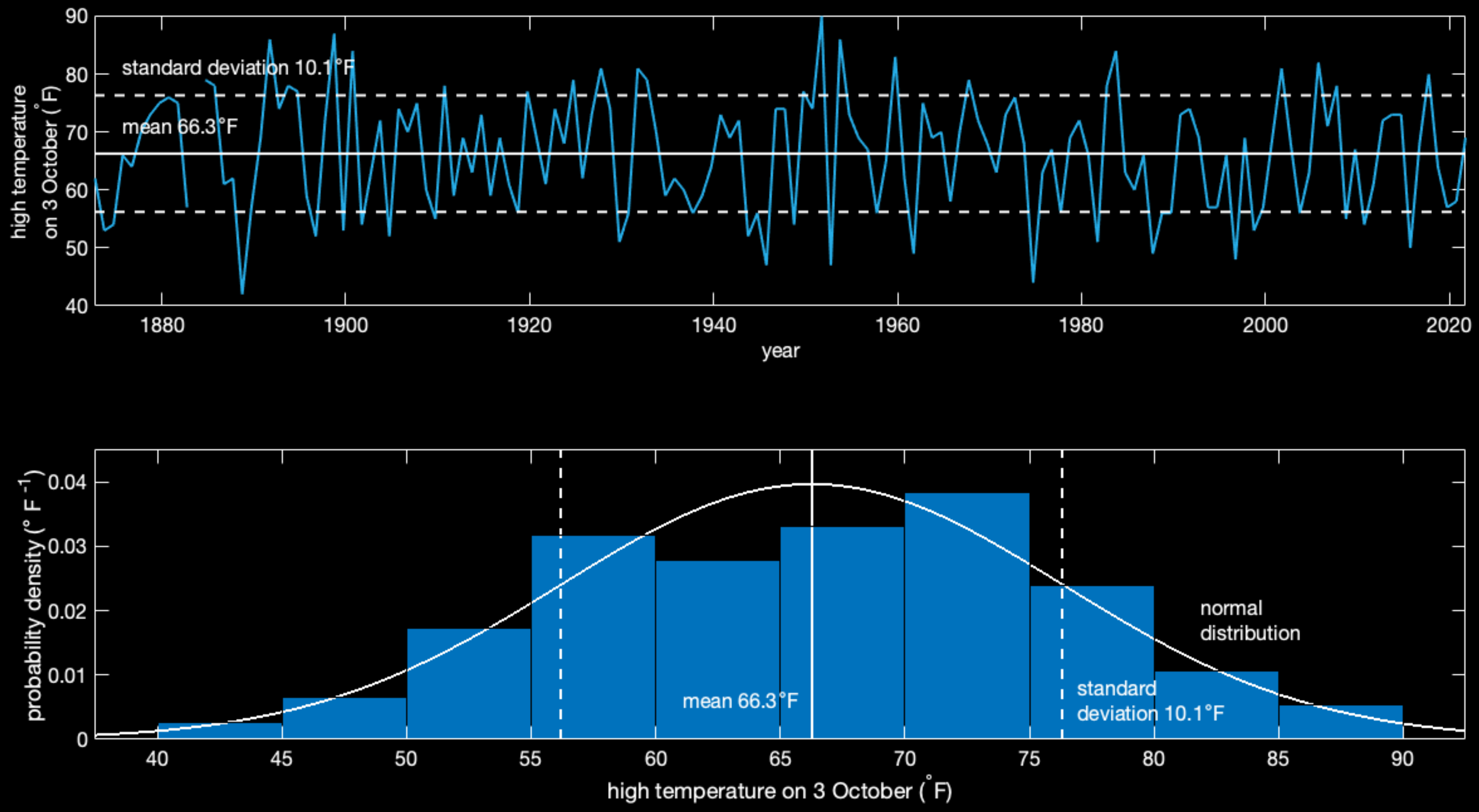
Ex. 6.8: On average, welded pipes have 0.5 defects per linear meter. What is the probability of finding a single defect in a 0.5-m section? What is the probability of finding more than one defect in a 0.5-m section?

Normal (Gaussian) distribution



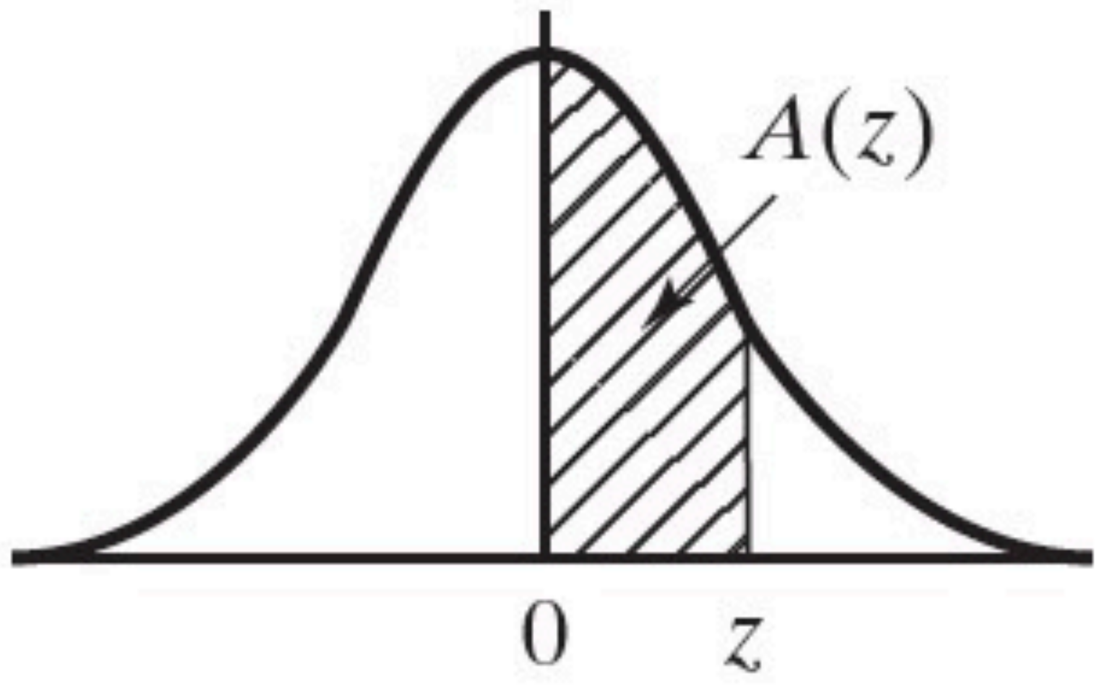
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Rochester's weather on 3 October



Normal distribution (Wheeler & Ganji Table 6.3)

TABLE 6.3 Area Under the Normal Distribution From $z = 0$ to z



Second decimal point is in top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621

Normal distribution (Wheeler & Ganji Table 6.3)

1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Writing an abstract

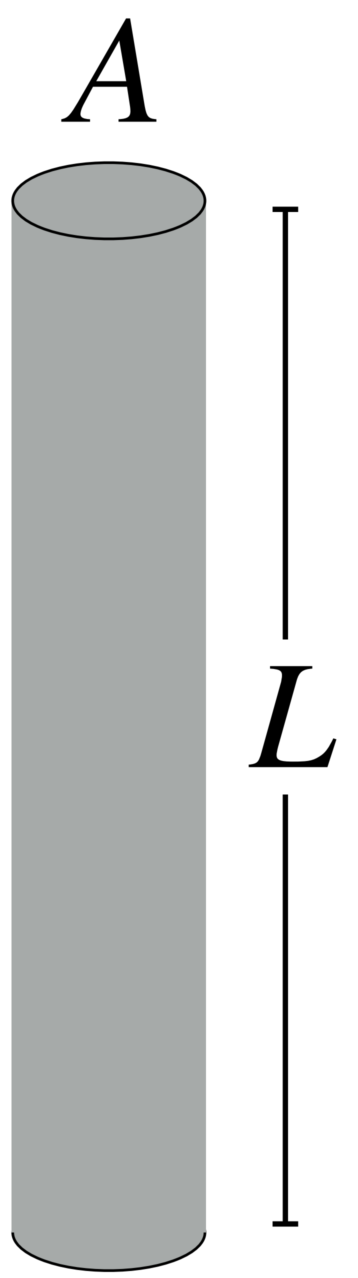
An abstract is a one-paragraph summary of your report.

- Basic introduction to the broad topic, readable by any colleague (1-2 sentences)
- Detailed motivation for solving the specific problem at hand, readable by engineers (2-3 sentences)
- Problem statement (1 sentence)
- Statement of your findings (e.g., "Here we show…") (1 sentence)
- Implications of the result for the specific problem at hand (1-2 sentences)
- Broader implications for possible future problems, readable by any colleague (1-2 sentences)

Ex. 6.9: A random variable has normal distribution with mean 10.0 and standard deviation 1.0. Find the probability that a single measurement falls between 8 and 9.55.

Strain gages

- Resistance of a wire



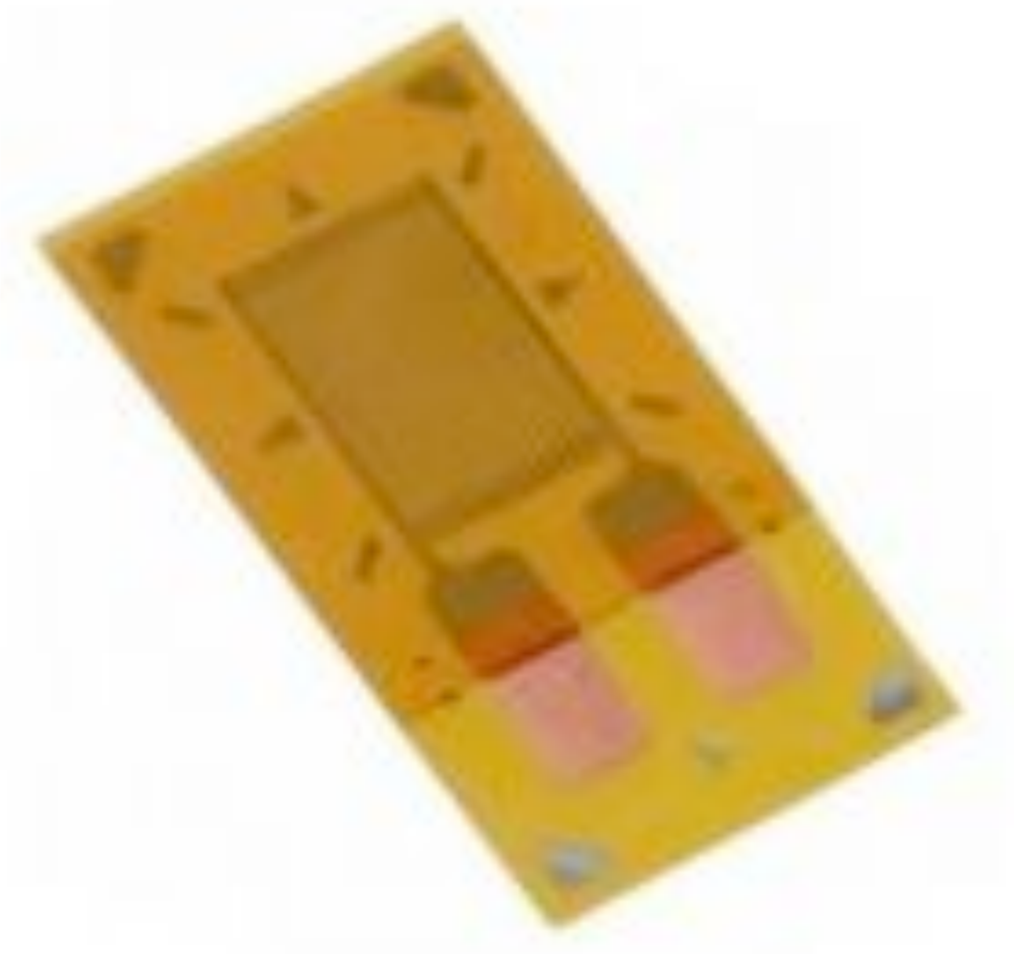
resistivity $1.68 \times 10^{-8}\ \Omega \cdot \mathrm{m}$ for Cu

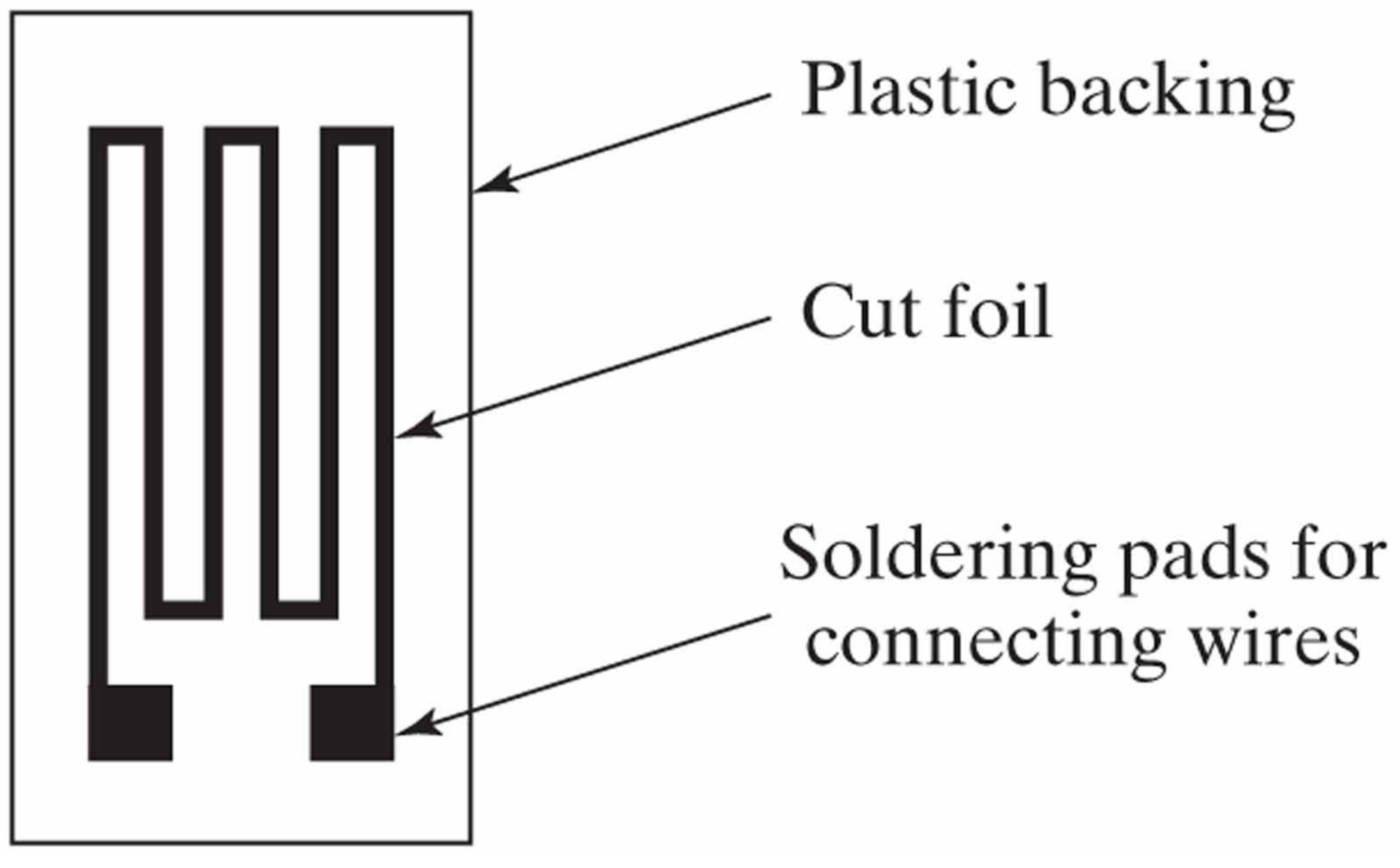
resistance of a wire; length

$$R = \frac{\rho L}{A}$$

cross-sectional area

- Depends linearly on L — and therefore strain ϵ (and stress σ)
- Resistance of N wires side-by-side scales as $N\epsilon$ — stronger signal



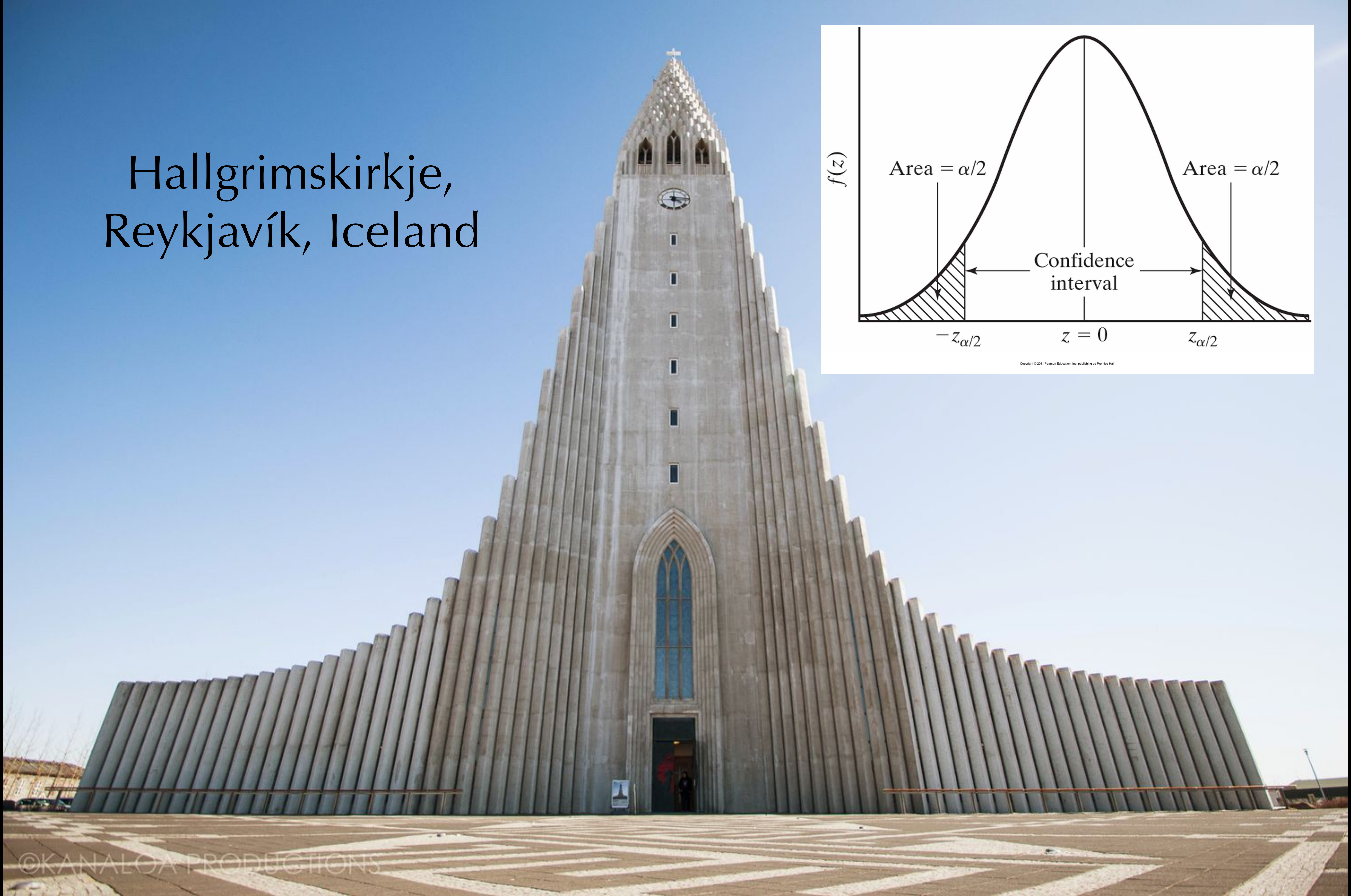


- Glued to beam / strut / surface whose strain is to be measured (not easy)
- $R = 120\ \Omega$ resistance varying $\delta R \approx \pm 0.1\ \Omega$ with strain; use Wheatstone bridge!
- Key parameter: gage factor GF:

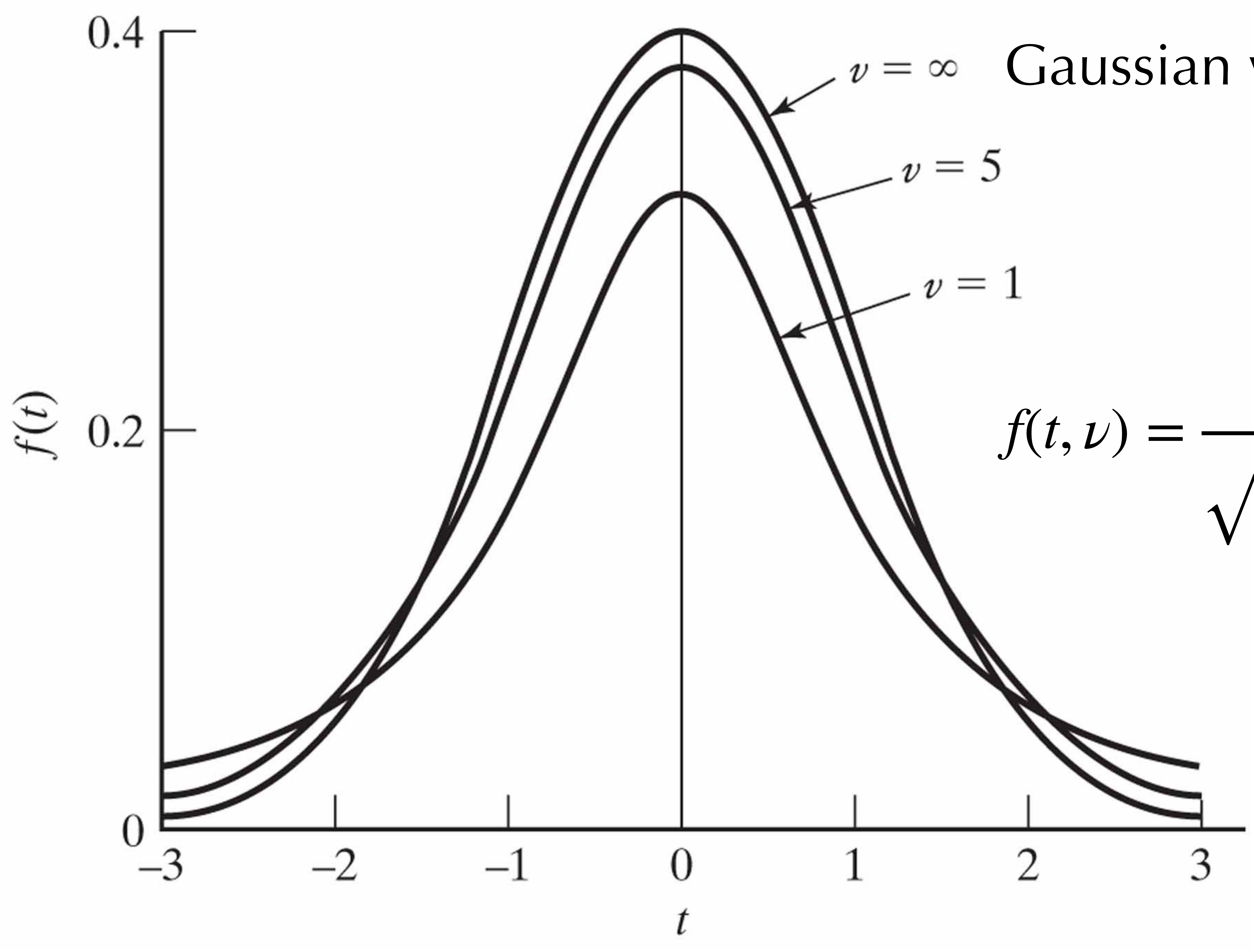
$$\frac{\delta R}{R} = GF\,\epsilon$$

Ex. 6.14: From a batch of resistors, you measure 36 resistances, finding the average to be $25\ \Omega$ and the standard deviation to be $0.5\ \Omega$. Determine the 90% confidence interval of the mean resistance of the batch.

Hallgrimskirkje, Reykjavík, Iceland



Student's t distribution



Gaussian when $\nu = \infty$.

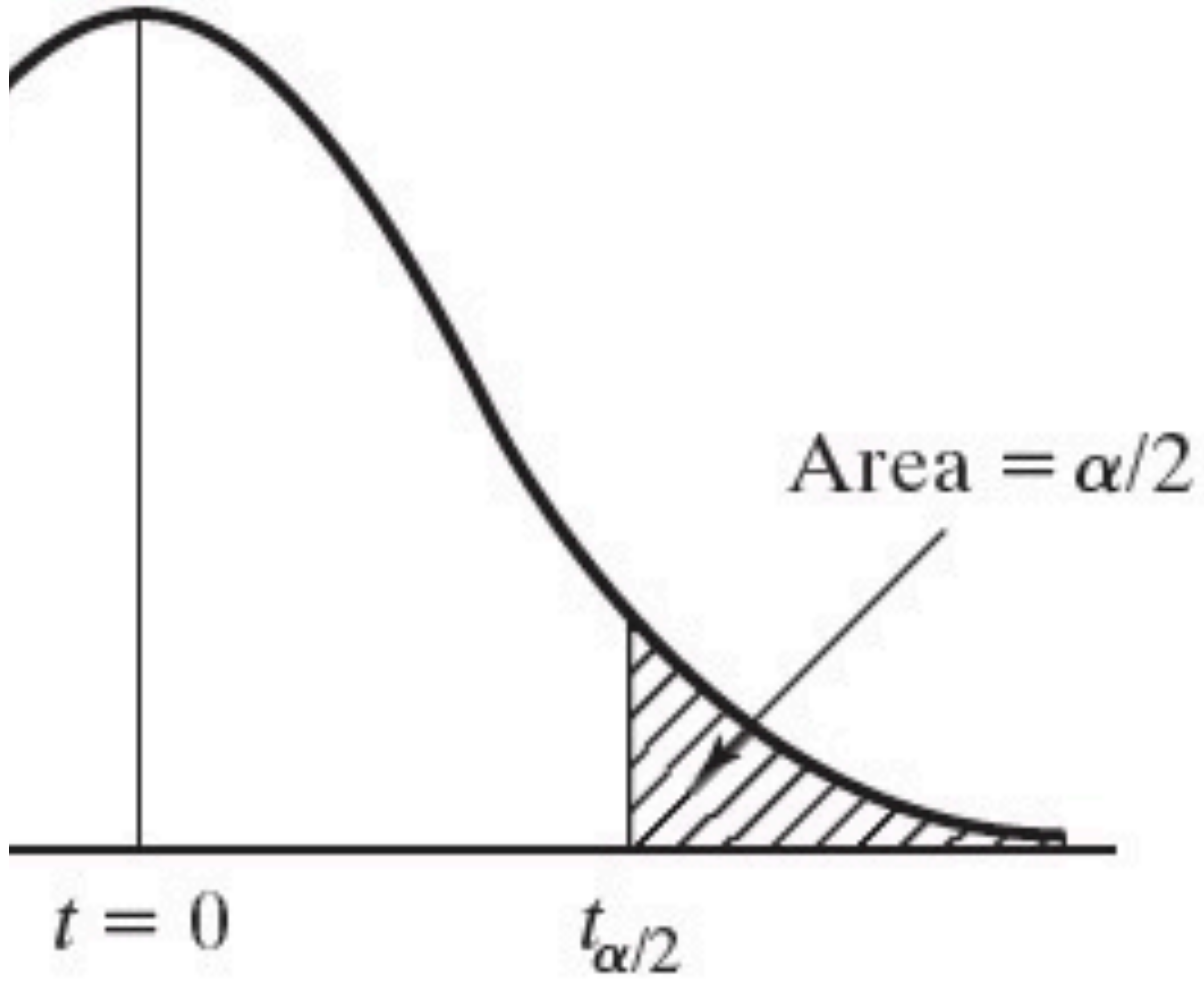
$$f(t,\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\left(\frac{\nu}{2}\right)\left(1+\frac{t^2}{\nu}\right)^{(\nu+1)/2}}$$

Student's distribution (Wheeler & Ganji Table 6.6)

degrees of freedom v	Area (probability) $\alpha/2$ 0.100	0.050	0.025	0.010	0.005
1	3.078	6.314	12.706	31.823	63.658
2	1.886	2.920	4.303	6.964	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.054
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947

$t_{\alpha/2}$

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}}$$



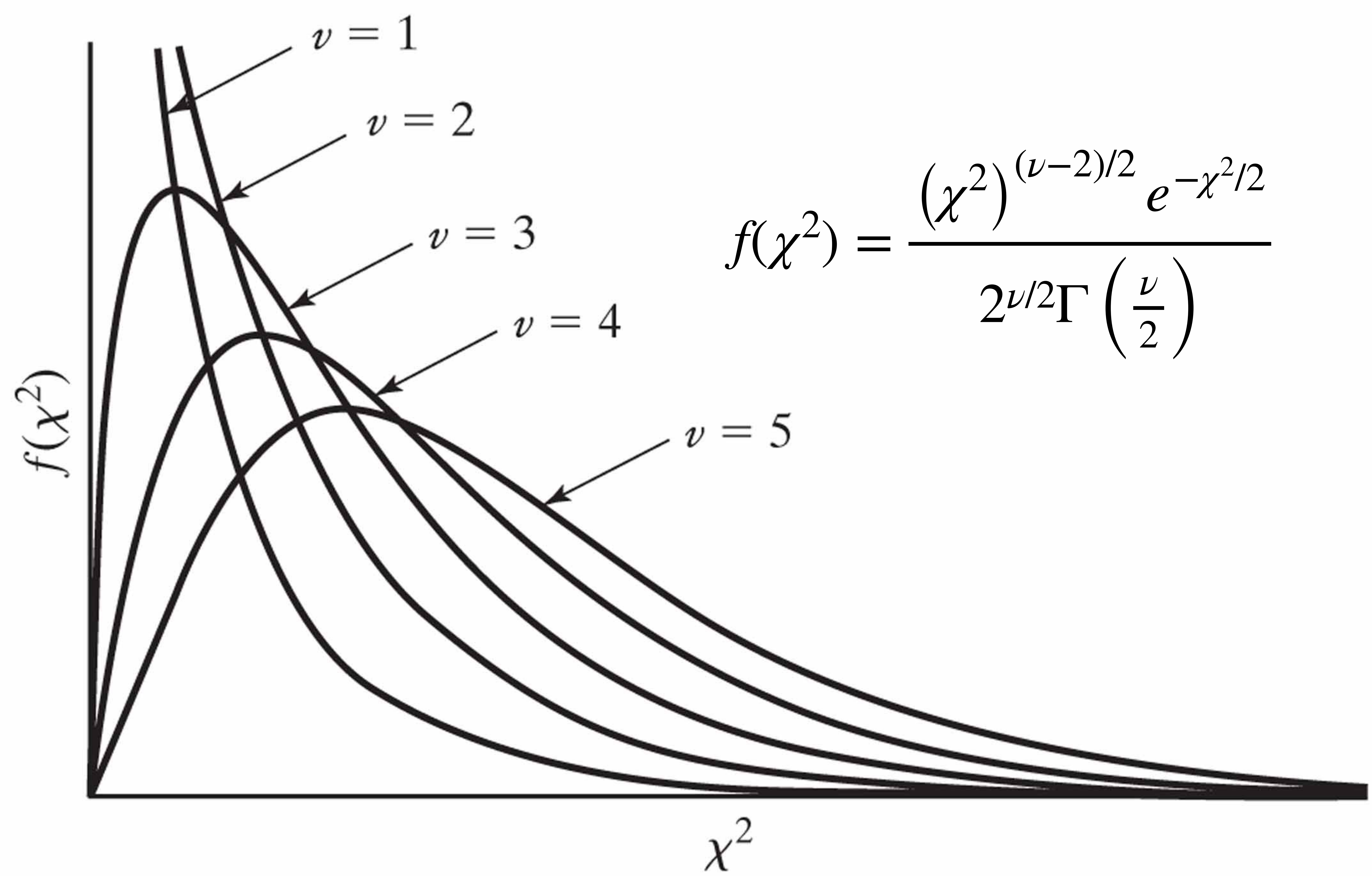
Area of tail, not hump!

Student's distribution (Wheeler & Ganji Table 6.6)

16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
∞	1.283	1.645	1.960	2.326	2.576

Ex. 6.15 (updated): A manufacturer of 3D printers would like to estimate the mean failure time of a competitor's product with 95% confidence. Six systems are tested, and their failure times (in hours) are 1250, 1320, 1542, 1464, 1275, and 1383. Estimate the population mean and 95% confidence interval of the mean.

χ^2 distribution



$$f(\chi^2) = \frac{\left(\chi^2\right)^{(\nu-2)/2} e^{-\chi^2/2}}{2^{\nu/2}\Gamma\left(\frac{\nu}{2}\right)}$$

χ^2 distribution (Wheeler & Ganji Table 6.7)

degrees of freedom	Area (probability)		Area of right hand tail				$1 - \alpha/2$ and $\alpha/2$			
$v\downarrow$	0.995	0.990	0.975	0.950	0.900	0.100	0.050	0.025	0.010	0.005
1	0.000039	0.000157	0.000982	0.003932	0.015791	2.706	3.841	5.024	6.635	7.879
2	0.0100	0.0201	0.0506	0.1026	0.2107	4.605	5.991	7.378	9.210	10.597
3	0.0717	0.1148	0.2158	0.3518	0.5844	6.251	7.815	9.348	11.345	12.838
4	0.2070	0.2971	0.4844	0.7107	1.0636	7.779	9.488	11.143	13.277	14.86
5	0.4118	0.5543	0.8312	1.1455	1.6103	9.236	11.070	12.832	15.086	16.750
6	0.6757	0.8721	1.2373	1.6354	2.2041	10.645	12.592	14.449	16.812	18.548
7	0.9893	1.2390	1.6899	2.1673	2.8331	12.017	14.067	16.013	18.475	20.278
8	1.3444	1.6465	2.1797	2.7326	3.4895	13.362	15.507	17.535	20.090	21.955
9	1.7349	2.0879	2.7004	3.3251	4.1682	14.684	16.919	19.023	21.666	23.589
10	2.1558	2.5582	3.2470	3.9403	4.8652	15.987	18.307	20.483	23.209	25.188
11	2.6032	3.0535	3.8157	4.5748	5.5778	17.275	19.675	21.920	24.725	26.757
12	3.0738	3.5706	4.4038	5.2260	6.3038	18.549	21.026	23.337	26.217	28.300
13	3.5650	4.1069	5.0087	5.8919	7.0415	19.812	22.362	24.736	27.688	29.819
14	4.0747	4.6604	5.6287	6.5706	7.7895	21.064	23.685	26.119	29.141	31.319
15	4.6009	5.2294	6.2621	7.2609	8.5468	22.307	24.996	27.488	30.578	32.801
16	5.1422	5.8122	6.9077	7.9616	9.3122	23.542	26.296	28.845	32.000	34.267
17	5.6973	6.4077	7.5642	8.6718	10.0852	24.769	27.587	30.191	33.409	35.718
18	6.2648	7.0149	8.2307	9.3904	10.8649	25.989	28.869	31.526	34.805	37.156
19	6.8439	7.6327	8.9065	10.1170	11.6509	27.204	30.144	32.852	36.191	38.582

χ^2

Ex. 6.17: To estimate the uniformity of the diameter of ball bearings in a production batch, a sample of 20 is chosen and carefully measured. The sample mean is 0.32500 inches, and the sample standard deviation is 0.00010 inches. Obtain a 95% confidence interval for the standard deviation of the production batch.

Thompson's τ (Wheeler & Ganji Table 6.8)

TABLE 6.8 Values of Thompson's τ

Sample size		Sample size	
n	τ	n	τ
3	1.150	22	1.893
4	1.393	23	1.896
5	1.572	24	1.899
6	1.656	25	1.902
7	1.711	26	1.904
8	1.749	27	1.906
9	1.777	28	1.908
10	1.798	29	1.910
11	1.815	30	1.911
12	1.829	31	1.913
13	1.840	32	1.914
14	1.849	33	1.916
15	1.858	34	1.917
16	1.865	35	1.919
17	1.871	36	1.920
18	1.876	37	1.921
19	1.881	38	1.922
20	1.885	39	1.923
21	1.889	40	1.924

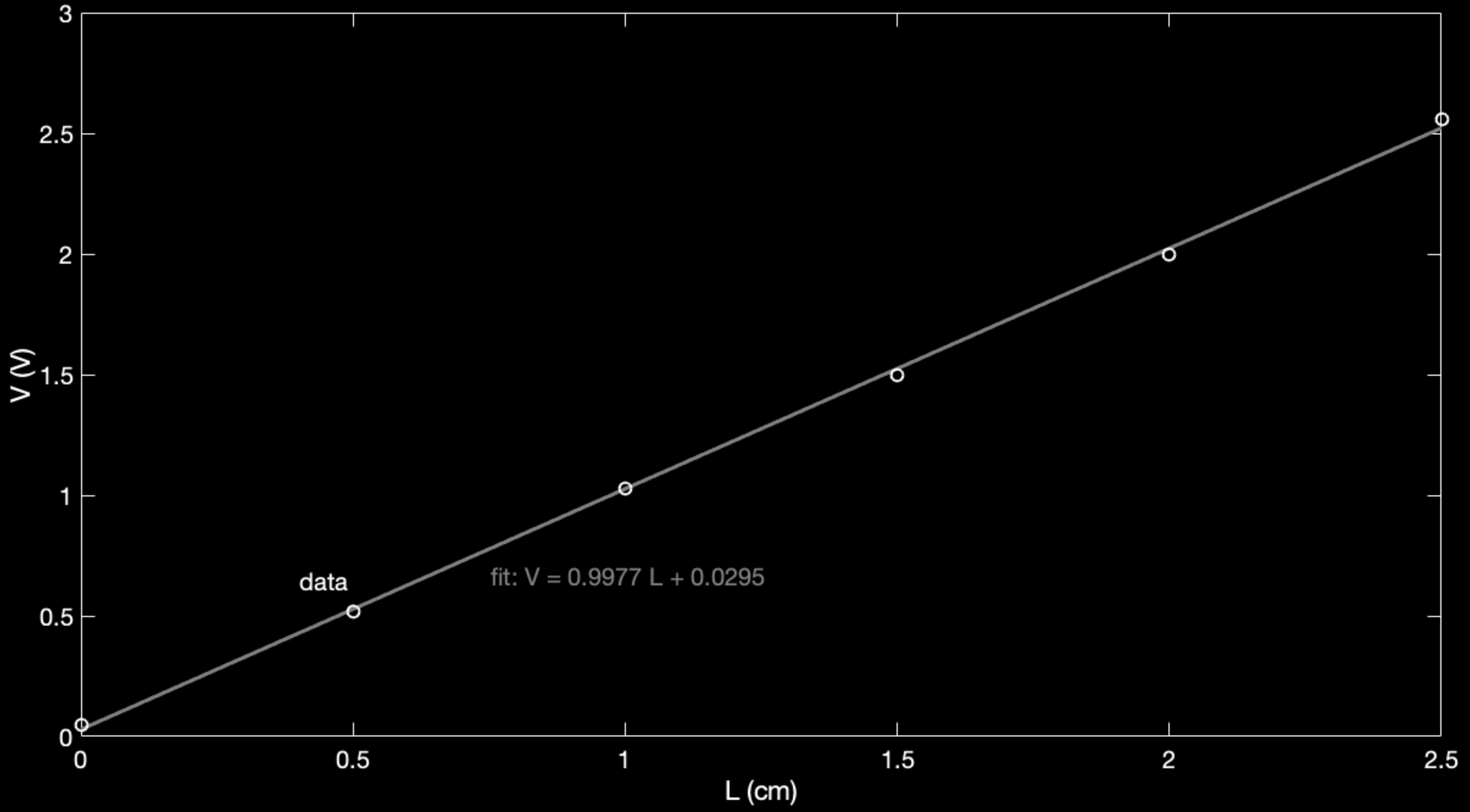
Source: ASME (1998).

Ex. 6.18: Nine voltage measurements in a circuit have produced the following values: 12.02, 12.05, 11.96, 11.99, 12.10, 12.03, 12.00, 11.95, and 12.16 V. Determine whether any of the values can be rejected.

Ex. 6.20: A linear variable differential transformer (LVDT) is a transducer for measuring displacements, which outputs a voltage. Five displacements L_i and corresponding voltages V_i are listed below. Calculate the best linear fit and coefficient of determination.

L_i (cm): 0.00, 0.50, 1.00, 1.50, 2.00, 2.50

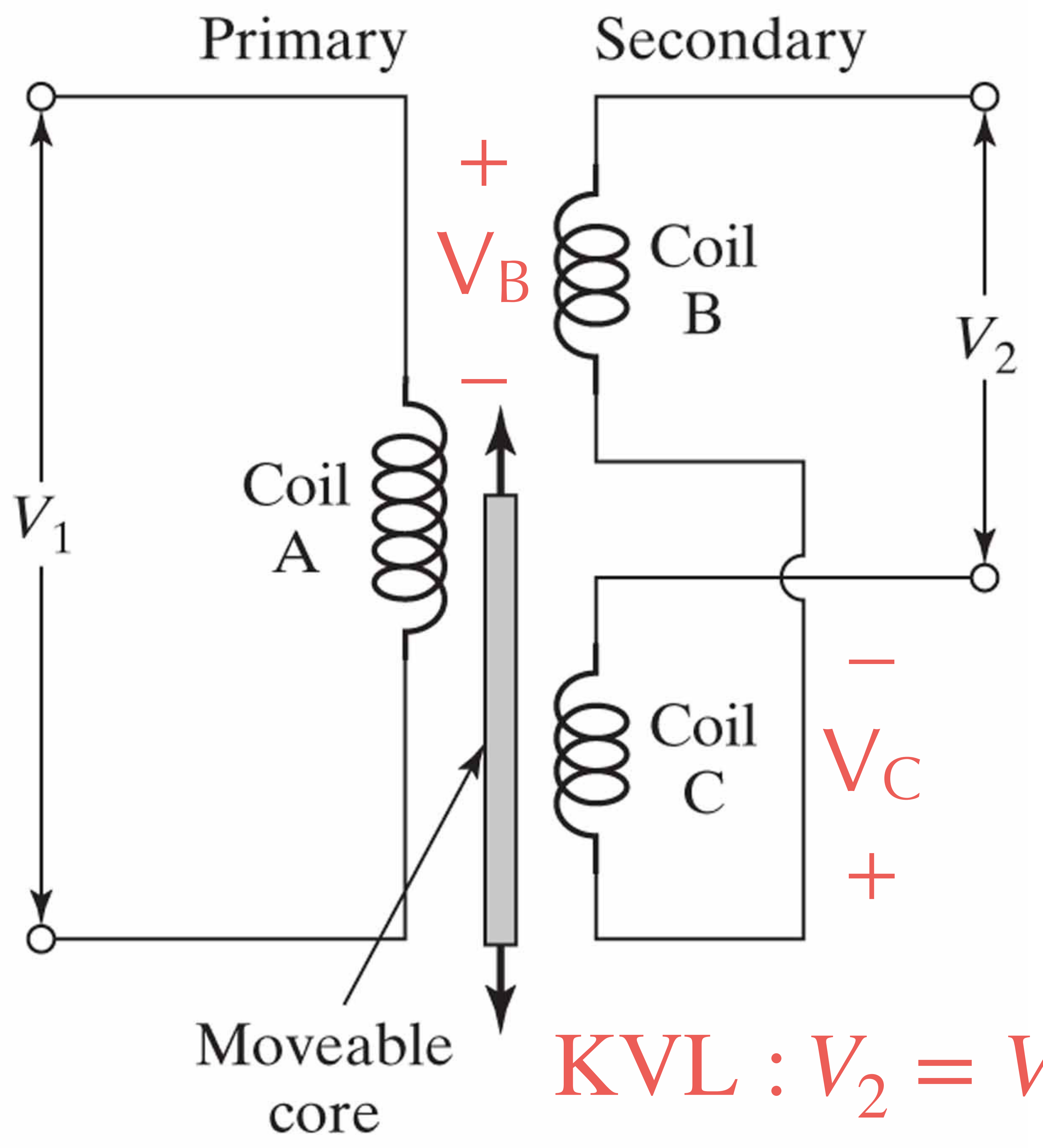
V_i (V): 0.05, 0.52, 1.03, 1.50, 2.00, 2.56



3
2.5
2
1.5
1
0.5
0
V (V)
data
fit: V = 0.9977 L + 0.0295
0
0.5
1
1.5
2
2.5
L (cm)

Linear variable differential transformers (LVDTs)

- Produces voltage linearly related to displacement
- Drive AC current through Coil A
- Resulting AC magnetic field induces AC currents in B and C
- Field and voltage drop concentrate near ferromagnetic core
- Voltages add; additional circuitry gives DC
- Range up to ~cm, resolution < µm, linearity < 0.5%
- Core mass can cause mechanical loading
- High-frequency signal requires high-frequency excitation



KVL : $V_2 = V_B + V_C$

Ex. 7.1: A circuit's power consumption is given by $P = IV$, where the measured current and voltage are $I = 10 \pm 0.2$ A and $V = 100 \pm 2$ V, both at 95% confidence. Determine the 95% confidence interval for P.

Ex. 7.2: A manometer is a device in which pressure can be determined by measuring the height of a column of fluid. You are choosing the fluid for a manometer that has 0.1 mm uncertainty in reading the scale, and you need an accuracy of 0.1% of the maximum reading, 10 kPa. If the fluid has nominal density 2500 kg/m^3, what uncertainty in the density is acceptable?

Ex. 7.7: The manufacturer gives this data about a pressure transducer: range ±3000 kPa, sensitivity ±0.25% of full scale, linearity ±0.15% of full scale, hysteresis ±0.10% of full scale (all at 95% confidence level). To test the transducer, many measurements were made in a tank held at 1500 kPa; the resulting standard deviation was 10.0 kPa. Many tests of the data transmission system resulted in a 5.0 kPa standard deviation. The A/D converter produces a 3.0 kPa random uncertainty. Calculate the random, systematic, and total uncertainty of the pressure measurement.

Ex. 7.10: The thermal efficiency of a natural gas internal combustion engine is $\eta = P m_f^{-1} H^{-1}$, where P is the power, m_f is the gas mass flow rate, H is heating value. To establish the mean efficiency, five engines were tested, yielding 0.30, 0.305, 0.308, 0.306, and 0.302. The average values of P, m_f, and H are 50 kW, 0.2 kg/minute, and 49,180 kJ/kg, respectively. Their systematic uncertainties, with 95% confidence, are 0.2 kW, 0.003 kg/minute, and 1500 kJ/kg, respectively. Calculate the mean value and uncertainty of the efficiency.

Procedure for uncertainty analysis

1. Define the measurement process: independent parameters, functional relationship
2. List all elemental error sources, considering calibration, data acquisition, data reduction, methods, etc.
3. Estimate elemental errors, including systematic uncertainties and standard deviations. Be consistent with confidence level.
4. Calculate systematic and random uncertainty for each measured variable.
5. Calculate systematic and random uncertainty for each result.
6. Calculate total uncertainty for each result.