

FIG. 1. (Color) Example basis modes. Each tracked particle is shown with a dot whose color indicates the local value of the mode (in arbitrary units). The total observation area is shown. [(a)-(d)] Streamfunction modes. [(e)-(h)] Boundary modes.

Scale-local velocity fields from particle-tracking data

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Modern flow measurement techniques like Lagrangian particle tracking produce rich data sets. Those data, though less structured than the results of numerical simulations, allow for similarly detailed examination of the fluid dynamics. To this end, we have developed a technique for the fully resolved reconstruction of the flow field that is localized in spatial scale.

We produce quasi-two-dimensional flow by electromagnetically forcing a thin layer of salt water 4 mm deep and with transverse dimensions 86 cm \times 86 cm. Fluorescent tracer particles follow the flow and are imaged by a high-speed 4 megapixel digital camera. We extract individual particle trajectories from the movies, recording particle positions and velocities.¹ Since velocities are measured only at the particle locations, care is necessary in postprocessing, particularly when calculating spatial gradients. Instead of interpolating onto a regular grid, we project the measurements onto a set of numerically constructed two-dimensional incompressible basis functions, minimizing the least-square error. This basis is composed of two types of modes.² Streamfunction modes contain the bulk of the flow, and velocity

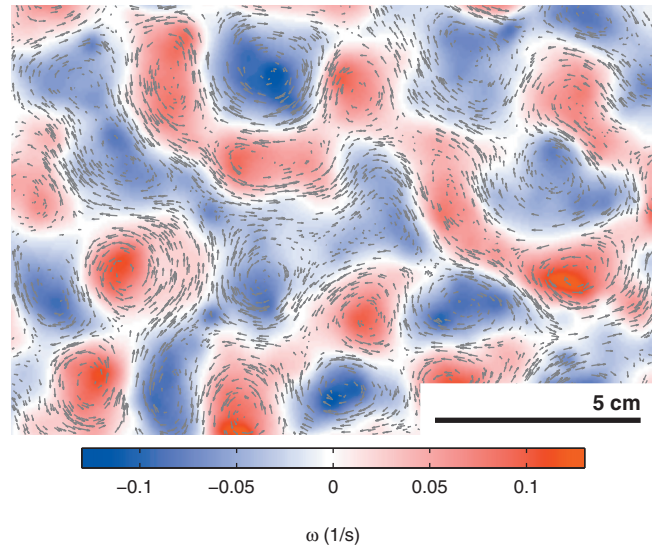


FIG. 2. (Color) Reconstructed velocity field. The location and velocity of each tracked particle are shown with arrows, and the color indicates the vorticity ω . The region shown is the central 20% of the total observation area.

potential modes account for the inflow and outflow at the open boundaries of the observation region. Four of each type are shown in Fig. 1.

This method allows us to reconstruct the full velocity field without explicit smoothing, eliminating outliers while preserving a much larger fraction of the kinetic energy than is kept with interpolation. An example snapshot is shown in Fig. 2. The root-mean-square velocity is 0.50 cm/s and the forcing scale is $L_f = 2.54$ cm, giving a Reynolds number of $Re = 100$.

Our technique also allows us to examine the flow on different length scales, since each basis mode has a well defined scale. Scale-local velocity and vorticity fields can therefore be reconstructed directly, without devising filters. Examples of these fields are shown in Fig. 3.

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²F. Lekien, C. Coulliette, R. Bank, and J. Marsden, *J. Geophys. Res.* **109**, C12004, doi:10.1029/2004JC002323 (2004).

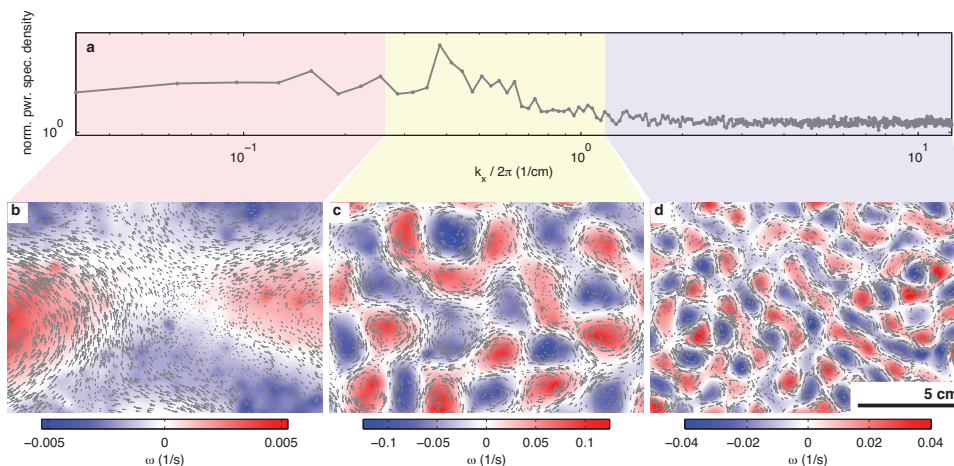


FIG. 3. (Color) Dynamics at different length scales. (a) Power spectrum. (b) Flow at scales $L < 2L_f/3$. (c) Flow at scales $3L_f/3 < L < 2L_f/3$. (d) Flow at scales $L > 3L_f/3$. Parts (b)-(d) show the same region as in Fig. 2.