## Quantum statistics of the light generated by phase-conjugate resonators

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We examine the quantum noise characteristics of the field produced by a phase-conjugate resonator formed from two mirrors, one of which is a phase-conjugate mirror. We show that the output field on the side of the phase-conjugate mirror can be completely squeezed. Numerical results for the quantum-statistical properties of the forward and backward output fields are given as functions of the reflectivities of the two mirrors.

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There has been considerable work [1] on optical resonators in which one of the mirrors is replaced by a phase-conjugate mirror (PCM). Much of the work has concentrated on the study of the modes in a phaseconjugate resonator. However, if one wants to use such resonators for devices such as lasers, it is of interest to study the quantum noise characteristics of phaseconjugate resonators. Phase-conjugate mirrors have been treated using a quantum-mechanical description [2-4], and there are interesting predictions concerning the extra noise that such mirrors contribute to the conjugated field. One might conclude that such mirrors could hardly be useful if they add an excess amount of noise. However, as our study shows, phase-conjugate resonators could be quite useful in generating nonclassical light with a significant amount of squeezing.

We start by recalling the quantum noise characteristics of an idealized phase-conjugate mirror [Fig. 1(a)]. Consider the phase conjugation of an incident beam of the form

$$\hat{E}_{\text{in}}(\mathbf{r},t) = \hat{E}_{\text{in}}^{(+)} + \text{H.c.} ,$$

$$\hat{E}_{\text{in}}^{(+)} = C(\omega) \hat{a}_{\text{in}} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} ,$$
(1)

where  $\hat{a}_{\rm in}$  represents the annihilation operator for the incident field and  $C(\omega) = -i(2\pi\hbar\omega/V)^{1/2}$ . The field reflected from the phase-conjugate mirror has the form

$$\hat{E}_{\text{out}}(\mathbf{r},t) = C^*(\omega)\hat{b}_{\text{out}}e^{-i(\mathbf{k}\cdot\mathbf{r}+\omega t)} + \text{H.c.} , \qquad (2)$$

where  $\hat{b}_{\rm out}$  is the annihilation operator for the reflected field. The mode operator  $\hat{b}_{\rm out}$  can be related to  $\hat{a}_{\rm in}$  via [2,4]

$$\hat{b}_{\text{out}} = \mu \hat{a}_{\text{in}}^{\dagger} + \nu \hat{b}_{\text{in}}, \quad |\nu|^2 - |\mu|^2 = 1$$
, (3)

where  $\hat{b}_{\rm in}$  is the annihilation operator for the vacuum mode entering from the other side of the PCM. The coefficient  $\mu$  gives the reflection coefficient of the phase-conjugate mirror. Using Eq. (3), Gaeta and Boyd [2] have shown that the P function  $P_{\rm out}(\beta)$  associated with

the  $\hat{b}_{\mathrm{out}}$  can be written in the form

$$P_{\text{out}}(\beta) = \frac{1}{\pi |\mu|^2} \langle (\beta/\mu)^* | \widehat{\rho}_{\text{in}} | (\beta/\mu)^* \rangle . \tag{4}$$

Here  $|(\beta/\mu)^*\rangle$  is a coherent state of the mode  $\hat{a}_{\rm in}$  with amplitude  $(\beta/\mu)^*$ . In deriving Eq. (4), it is assumed that the input mode  $\hat{b}_{\rm in}$  is in the vacuum state. As a result of the positive nature of the P function (4), one concludes that the mode  $\hat{b}_{\rm out}$  does not possess any nonclassical properties. Clearly the phase-conjugate mirror leads to an increase in noise in the mode  $\hat{b}_{\rm out}$  over the input mode. For example, even if both  $\hat{a}_{\rm in}$  and  $\hat{b}_{\rm in}$  are in the vacuum state, then the field associated with mode  $\hat{b}_{\rm out}$  is described by a thermal-like distribution with an average photon number equal to the reflectivity of the phase-conjugate mirror.

(a) 
$$\begin{array}{c|c}
\hat{E}_{f}^{in} & \hat{E}_{b}^{out} \\
\hat{E}_{b}^{out} & \hat{E}_{b}^{in} \\
\hat{E}_{b}^{in} & \hat{E}_{b}^{in} \\
\hline
\\
\hat{E}_{b}^{in} & \hat{E}_{b}^{in} \\
\hline$$

FIG. 1. Schematic illustrations of the fields associated with (a) a phase-conjugate mirror, (b) a phase-conjugate resonator with a perfect mirror, (c) a phase-conjugate resonator with partially reflecting mirror.

We next consider the noise properties of the phase-conjugate resonator in which a 100%-reflecting mirror is placed a distance L from the phase-conjugate mirror [Fig. 1(b)]. One would expect that the feedback process may eventually lead to a reduction in quantum noise. Let us first consider the semiclassical analysis. For the case of steady-state phase conjugation by degenerate four-wave mixing, the spatial field amplitudes and their spatial evolution [5] can be written as

$$E_f(z) = E_f(0)e^{ikz} , \quad E_b(z) = E_b(L)e^{-ikz} \quad \text{for } 0 \le z \le L ,$$
 (5a)

$$\frac{dE_f}{dz} = i\kappa E_b^*, \quad \frac{dE_b^*}{dz} = -i\kappa E_f \quad \text{for } L \le z \le L + l \ . \tag{5b}$$

Here l denotes the thickness of the phase-conjugate cell and  $\kappa$  denotes the nonlinear coupling constant for four-wave mixing. The boundary conditions to be imposed on the total field amplitude  $E_{tot}(z) = E_f(z) + E_b(z)$  are

$$E_{\text{tot}}(0)=0, \qquad (6a)$$

$$E_{\text{tot}}(L+l) = E_f^{\text{out}} + E_b^{\text{in}} . \tag{6b}$$

Equations (5) and (6) can be solved, leading to

$$E_f(0) = -\frac{\cos \kappa l}{\cos 2\kappa l} E_b^{\text{in}} + \frac{i \sin \kappa l}{\cos 2\kappa l} E_b^{\text{in}*}, \qquad (7)$$

$$E_f^{\text{out}} = -\frac{1}{\cos 2\kappa l} E_b^{\text{in}} + \frac{i \sin 2\kappa l}{\cos 2\kappa l} E_b^{\text{in}*} . \tag{8}$$

Note that the reflectivity of the phase-conjugate mirror is defined by [5]

$$|\mu|^2 = \tan^2 \kappa l \ . \tag{9}$$

A quantum-mechanical treatment of the fields leads to the following expressions for the forward- and backward-traveling fields in terms of the appropriate annihilation and creation operators,

$$\widehat{E}_f(z) = \frac{Ce^{ikz}}{\sqrt{\cos 2\kappa l}} \, \widehat{a}_f \quad \text{for } 0 \le z \le L \ , \tag{10}$$

$$\hat{E}_{f}^{\text{out}} = Ce^{ikz} \hat{a}_{f}^{\text{out}} \quad \text{for } z > L , \qquad (11)$$

where

$$\hat{a}_{f}^{\text{out}} = -\frac{1}{\cos 2\kappa l} \hat{a}_{b}^{\text{in}} - \frac{i \sin 2\kappa l}{\cos 2\kappa l} \hat{a}_{b}^{\text{in}\dagger}, \qquad (12)$$

$$\hat{a}_f = -\frac{\cos\kappa l}{\sqrt{\cos 2\kappa l}} \hat{a}_b^{\text{in}} - \frac{i \sin\kappa l}{\sqrt{\cos 2\kappa l}} \hat{a}_b^{\text{in}\dagger} . \tag{13}$$

Consider now the quantum characteristics of the output mode. Starting from (12) and assuming that  $\hat{a}_b^{\text{in}}$  is initially in vacuum, one can easily prove that the squeezing parameter S is given by

$$S = \langle : (\hat{a}_f^{\text{out}} e^{-i\beta} + \hat{a}_f^{\text{out}\dagger} e^{i\beta})^2 : \rangle$$

$$=2[\sin^2 2\kappa L + \sin 2\kappa L \cos(2\beta + \pi/2)]/\cos^2 2\kappa L \tag{14}$$

$$= \frac{-2\sin 2\kappa L}{1 + \sin 2\kappa L} \quad \text{for } \beta = \pi/2$$
 (15)

= -1 for 
$$\beta = \pi/2$$
,  $\kappa L = \pi/4$ . (16)

Thus, the output field can be fully squeezed in the limit in which the phase-conjugate reflectivity becomes unity, i.e., near the threshold of self-oscillation. This is reminiscent of complete squeezing near the threshold of other systems, such as optical parametric oscillators and second-harmonic generators [6].

We next consider the case of a mirror with finite reflectivity  $r^2$  in order to determine how much flexibility one has in the choice of the reflectivity of the ordinary mirror. When the mirror reflectivity is less than unity, then we also have to consider the "open input port" from the left-hand side of the mirror. The arrangement is shown in Fig. 1(c). The boundary conditions at the PCM and at the mirror enable us to write

$$\hat{E}_{f}^{\text{out}} = \mu \hat{E}_{b}^{\text{in}\dagger} + \nu \hat{E}_{f}(0)e^{ikL}, \qquad (17a)$$

$$\widehat{E}_f(0) = t\widehat{E}_f^{\text{in}} + ir\widehat{E}_b(0)e^{ikL}, \qquad (17b)$$

where t and r are the amplitude transmission and reflection coefficients (assumed real), respectively, for the mirror. The solution to Eqs. (17) and the corresponding adjoint equations leads to

$$\hat{E}_{f}^{\text{out}} = \frac{1}{1 - |\mu|^{2} r^{2}} \left[ u (1 + r^{2}) \hat{E}_{b}^{\text{in}\dagger} + i v^{2} r e^{i 2k L} \hat{E}_{b}^{\text{in}} + i \mu v r t e^{i k L} \hat{E}_{f}^{\text{in}\dagger} + v t e^{i k L} \hat{E}_{f}^{\text{in}} \right].$$
(18)

If we let  $\hat{X}_f^{\text{out}}$  be the quadrature

$$\hat{X}_f^{\text{out}} = \hat{E}_f^{\text{out}} e^{-i\beta} + \text{H.c.}$$
 (19)

and we assume that  $\hat{E}_f^{\text{in}}$  and  $\hat{E}_b^{\text{in}}$  are in the vacuum state and in a coherent state, respectively, then one can show that the variance of the quadrature  $\langle (\Delta \hat{X}_f^{\text{out}})^2 \rangle = \langle (\hat{X}_f^{\text{out}})^2 \rangle - \langle \hat{X}_f^{\text{out}} \rangle^2$  is given by

$$\langle (\Delta \hat{X}_f^{\text{out}})^2 \rangle = \frac{|C|^2}{(1-|\mu|^2r^2)^2} [1+2|\mu|^2+4|\mu|^2r^2+2|\mu|^4r^2-|\mu|^4r^4-4|\mu|(|\mu|^2+1)\sin 2\beta'], \qquad (20)$$

where  $\beta' = \beta - kL - \arg(\mu)/2$ . In a similar manner, one can calculate the fluctuations in the quadrature  $\hat{X}_b^{\text{out}}$  of the output field  $\hat{E}_b^{\text{out}}$ . We can find the result

$$\langle (\Delta \hat{X}_{b}^{\text{out}})^{2} \rangle = \frac{|C|^{2}}{(1 - |\mu|^{2} r^{2})^{2}} [1 + 2|\mu|^{2} - 4|\mu|^{2} r^{2} + 2|\mu|^{4} r^{2} - |\mu|^{4} r^{4} - 4|\mu|^{3} t^{2} r \sin 2\beta'] . \tag{21}$$

In Fig. 2 we show the dependence of the quadratures (20) and (21) on the phase-conjugate reflectivity  $|\mu|^2$  and the reflectivity  $r^2$  of the mirror. It is assumed that the phase  $\beta'$  has been adjusted so that these variances are minimum. Figure 2 shows that the output field on the side of the PCM is squeezed significantly over a very wide range of parameters. The maximum squeezing occurs when the phase-conjugate reflectivity  $|\mu|^2$  is equal to  $r^2$ , which can be understood from the fact that, under this condition, approximately equal components of the field and its Hermitian conjugate are present in the output field. Nevertheless, the output-field quadrature  $\hat{X}_b^{\text{out}}$  on the side of the ordinary mirror is never squeezed.

We next examine the noise properties of the off-axis fields. Consider the situation shown in Fig. 3. We now have to include modes propagating symmetrically about the axis normal to the ordinary mirror. Using the notations of Fig. 3, we write the boundary conditions as

$$\hat{E}_{1f}(0) = t\hat{E}_{1f}^{\text{in}} + ir\hat{E}_{2h}(0) , \qquad (22a)$$

$$\hat{E}_{2h}(L) = \mu \hat{E}_{2f}^{\text{in}\dagger} + \nu \hat{E}_{2h}(0)$$
, (22b)

$$\hat{E}_{2f}(0) = t\hat{E}_{2f}^{\text{in}} + ir\hat{E}_{1h}(0) , \qquad (22c)$$

$$\hat{E}_{1h}(L) = \mu \hat{E}_{1f}^{\text{in}\dagger} + \nu \hat{E}_{1h}(0) . \tag{22d}$$

It should be noted that the parameters r, t,  $\mu$ , etc., now correspond to off-axis reflections and transmissions. These equations and their complex conjugates can be solved for  $\hat{E}_{1f}^{\text{out}}$  and  $\hat{E}_{2f}^{\text{out}}$ . The quantum properties of the generated fields can be studied in terms of the quantum properties of the input fields. In order to find the squeezing characteristics of the output field, we consider the linear combination of  $\hat{E}_{1f}^{\text{out}}$  and  $\hat{E}_{2f}^{\text{out}}$ ,

$$\hat{E}_{12}^{\text{out}} = \frac{1}{\sqrt{2}} (\hat{E}_{1f}^{\text{out}} + i\hat{E}_{2f}^{\text{out}}) . \tag{23}$$

This linear combination could be the output from a 50-50 beamsplitter in which  $\hat{E}_{1f}^{\text{out}}$  and  $\hat{E}_{2f}^{\text{out}}$  are incident on different input ports. We define the quadrature of  $\hat{E}_{12}^{\text{out}}$  in

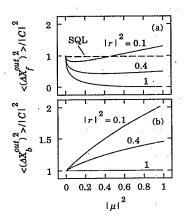


FIG. 2. Variance in the quadrature components of the output fields as functions of the phase-conjugate reflectivity  $|\mu|^2$  for various mirror reflectivities (a) on the side of the phase-conjugate mirror, and (b) on the side of the ordinary mirror. The variances are normalized to the standard quantum-noise limit (SQL).

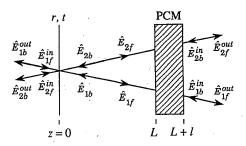


FIG. 3. Schematic illustration of the phase-conjugate resonator and the off-axis fields.

the same way as in (19) but with  $\hat{E}_f^{\text{out}}$  replaced by  $\hat{E}_{12}^{\text{out}}$ . By using Eqs. (22) and (23), we find that the fluctuations in the quadrature component are the same as given in Eq. (20). The only difference is that the reflection coefficients  $\mu$  and r are for off-axis fields. Thus, the results of Fig. 2 are also applicable to off-axis modes.

We next discuss the photon statistics of the field  $\hat{E}_f^{\text{out}}$ . Equation (18) in terms of the single-mode operators can be written as

$$\hat{a}_{f}^{\text{out}} = d_{1}\hat{a}_{h}^{\text{in}} + d_{2}\hat{a}_{h}^{\text{in}\dagger} + d_{3}\hat{a}_{f}^{\text{in}} + d_{4}\hat{a}_{f}^{\text{in}\dagger}. \tag{24}$$

Since  $\hat{a}_b^{\text{in}}$  and  $\hat{a}_f^{\text{in}}$  are independent modes, mean values such as  $\langle \hat{a}_f^{\text{out}} \rangle$  and  $\langle (\hat{a}_f^{\text{out}})^2 \rangle$ , etc., can be easily obtained. One can in principle construct the density matrix or the Wigner function  $\Phi$  for the field  $\hat{a}_b^{\text{out}}$ . Assuming that  $\hat{a}_f^{\text{in}}$  is in the vacuum state and  $\hat{a}_b^{\text{in}}$  is in a coherent state, then the Wigner function  $\Phi(\alpha, \alpha^*)$  for the  $\hat{a}_f^{\text{out}}$  mode can be shown to be (see Appendix)

$$\Phi(\alpha, \alpha^*) = \frac{2}{\pi Q} \exp\{-[(\sinh x)e^{-i\theta}(\alpha - \alpha_0)^2 + (\sinh x)e^{i\theta}(\alpha^* - \alpha_0^*)^2 + 2(\cosh x)|\alpha - \alpha_0|^2]/Q\}, \quad (25)$$

where

$$\begin{split} &\langle \hat{a}_{f}^{\text{ out}} \rangle \!=\! \alpha_{0} \!=\! d_{1} \alpha_{b}^{\text{in}} \!+\! d_{2} \alpha_{b}^{\text{in}}^{*} \;, \\ &\langle (\hat{a}_{f}^{\text{ out}})^{2} \rangle \!=\! -\alpha_{0}^{2} \!=\! -\frac{Q}{2} (\sinh x) e^{i\theta} \!=\! d_{1} d_{2} \!+\! d_{3} d_{4} \;, \quad (26) \\ &\langle \hat{a}_{f}^{\text{ out}} \!\!\! \uparrow \!\!\! \hat{a}_{f}^{\text{ out}} \rangle \!=\! \frac{Q}{2} \!\cosh\! x \!+\! |\alpha_{0}|^{2} \!-\! \frac{1}{2} \!=\! |d_{2}|^{2} \!+\! |d_{4}|^{2} \;. \end{split}$$

Note that the condition for squeezing in the quadrature  $(\hat{a}_f^{\text{out}}e^{i\beta} + \text{H.c.})$  is

$$Q\left[\cosh x - \sinh x \cos(\theta - 2\beta)\right] < 1. \tag{27}$$

Note further that in general the parameter  $Q \neq 1$ . This parameter would be unity if  $d_3 = d_4 = 0$  and  $|d_1|^2 - |d_2|^2 = 1$ . Thus, the deviation of Q from unity is a measure of the quantum noise entering from the left side of the mirror as a result of its reflectivity not being equal to unity.

The Gaussian character of the Wigner function for the field  $\hat{a}_f^{\text{out}}$  is useful in the study of the photon statistics and higher-order squeezing [7]. For example, using the moment theorem for Gaussian processes, one can show that

$$\langle (\Delta \hat{X}_f^{\text{out}})^{2N} \rangle = \frac{2N!}{2^N N!} \langle (\Delta \hat{X}_f^{\text{out}})^2 \rangle^N . \tag{28}$$

Thus, squeezing to second order implies to all even orders. The distribution of photon numbers p(n) can be obtained from the results of Agarwal and Adam [8]. If both  $\hat{a}_{f}^{\text{in}}$  and  $\hat{a}_{b}^{\text{in}}$  are in the vacuum state, then

$$p(n) = \frac{2}{(Q^2 + 2Q \cosh x + 1)^{1/2}} \left[ \frac{Qe^x - 1}{Qe^x + 1} \right]^n \times \sum_{m=0}^{n} { -\frac{1}{2} \choose m} {n \choose n - m} \left[ \frac{4Q \sinh x}{Q^2 + 2Q \sinh x - 1} \right]^m.$$
(29)

Figure 4(a) shows a plot of the photon-number distribution (29) for the case in which the ordinary mirror is not present (i.e., r=0) and  $|\mu|^2=5$ . As predicted [2], the resulting distribution is identical to that of a thermal source in which the expected value of the number of photons is equal to 5. Figure 4(b) is a plot of the distribution for the case of a perfect mirror (i.e., r=1) and  $|\mu|^2=0.5$ . The probability for observing n photons is nonzero only for the case in which n is even. In this case, the output field from the phase-conjugate resonator is in the squeezed vacuum, which is known to have p(n)=0 for an odd number of photons [9]. This result can be understood from the fact that the four-wave-mixing process within the phase conjugator produces photons only in pairs, and all the photons must eventually leave the resonator on the side of the PCM. In Fig. 4(c), the effect of having a partially reflecting mirror ( $r^2=0.7$  and  $|\mu|^2=0.7$ ) is to allow for a nonzero probability of having an odd number of photons, which is simply a result of photons within the resonator leaking out from the side of the ordinary mirror.

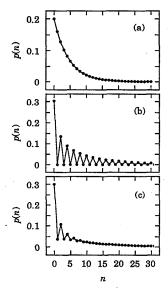


FIG. 4. Photon-number distributions for the field on the side of the PCM for the cases, (a)  $|\mu|^2=5$ ,  $r^2=0$ , (b)  $|\mu|^2=0.5$ ,  $r^2=1$ , (c)  $|\mu|^2=0.7$ ,  $r^2=0.7$ .

In conclusion, we have calculated the quantum statistics for the fields emitted by a phase-conjugate resonator. The output field on the side of the PCM is predicted to show squeezing, whereas the output field on the side of the ordinary mirror is never squeezed. In addition, the photon distribution for the output field on the PCM side is predicted to contain only an even number of photons for the case in which the ordinary mirror is 100% reflecting.

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## APPENDIX: WIGNER FUNCTION FOR THE OUTPUT FIELD

In this appendix we show how Eq. (25) can be obtained from Eq. (24). Let us define the characteristic function for the output field  $\hat{a}_f^{\text{out}}$  via

$$C(\beta, \beta^*) = \langle \exp(\beta \hat{a}_f^{\text{out}\dagger} - \beta^* \hat{a}_f^{\text{out}}) \rangle . \tag{A1}$$

The Wigner function is then the Fourier transform of (A1), that is,

$$\Phi(\alpha, \alpha^*) = \frac{1}{\pi} \int d^2\beta \, C(\beta, \beta^*) e^{\beta \alpha^* - \beta^* \alpha} \,. \tag{A2}$$

We substitute the expression for  $\hat{a}_f^{\text{out}}$  [Eq. (24)] into Eq. (A1) and use the fact that the modes  $\hat{a}_f^{\text{in}}$  and  $\hat{a}_b^{\text{in}}$  are independent, to obtain

$$C(\beta, \beta^{*}) = \langle \exp[-\beta^{*}(d_{3}\hat{a}_{f}^{\text{in}} + d_{4}\hat{a}_{f}^{\text{in}}) + \beta(d_{3}^{*}\hat{a}_{f}^{\text{in}} + d_{4}^{*}\hat{a}_{f}^{\text{in}})] \rangle$$

$$\times \langle \exp[-\beta^{*}(d_{1}\hat{a}_{b}^{\text{in}} + d_{2}^{*}\hat{a}_{b}^{\text{in}})] \rangle$$

$$+ \beta[(d_{1}^{*}\hat{a}_{b}^{\text{in}} + d_{2}^{*}\hat{a}_{b}^{\text{in}})] \rangle . \tag{A3}$$

Equation (A3) can be written in terms of the product of the characteristic functions for the two input modes,

$$C(\beta, \beta^*) = C_f^{\text{in}}(\beta d_3^* - \beta^* d_4, \beta^* d_3 - \beta d_4^*) \times C_h^{\text{in}}(\beta d_1^* - \beta^* d_2, \beta^* d_1 - \beta d_2^*) . \tag{A4}$$

Note further that

$$C(\gamma, \gamma^*) = \langle \exp(\gamma \hat{\alpha}^{\dagger} - \gamma^* \hat{\alpha}) \rangle$$

$$= \exp(\frac{1}{2} |\gamma|^2) \langle \exp(\gamma \hat{\alpha}^{\dagger}) \exp(-\gamma^* \hat{\alpha}) \rangle$$

$$= \exp(\frac{1}{2} |\gamma|^2) \langle \exp(\gamma \alpha^*) \exp(-\gamma^* \alpha) \rangle$$
 (A5)

for the case in which the mode  $\hat{a}$  is in the coherent state  $|\alpha\rangle$ . Using Eqs. (A4) and (A5) yields the following expression for the characteristic function for the output field  $\hat{a}_f^{\text{out}}$ :

$$C(\beta, \beta^*) = \exp\left[-\frac{1}{2}(|\beta d_3^* - \beta^* d_4|^2 + |\beta d_1^* - \beta^* d_2|^2) - (\beta^* d_1 - \beta d_2^*)\alpha_b^{\text{in}} + (\beta d_1^* - \beta^* d_2)\alpha_b^{\text{in}}^*\right],$$
(A6)

where we assume that the field mode  $\hat{a}_f^{\text{in}}$  is in the vacuum state and that the field mode  $\hat{a}_b^{\text{in}}$  is in the coherent state  $|\alpha_b^{\text{in}}\rangle$ . Note that the expression in Eq. (A6) is Gaussian in  $\beta$  and  $\beta^*$ , and hence its Fourier transform, defined by Eq.

(A2), will be Gaussian in  $\alpha_b^{\text{in}}$  and  $\alpha_b^{\text{in}*}$ . It is straightforward to obtain the Fourier transform by using

$$\int d^{n}x \exp\left\{-\frac{1}{2}\sum_{i,j}x_{i}x_{j}A_{ij}+i\sum_{i}h_{i}x_{i}\right\} = \frac{(2\pi)^{n/2}}{\sqrt{\det(A)}}\exp\left[-\frac{1}{2}\sum_{i,j}h_{i}h_{j}(A^{-1})_{ij}\right], \quad (A7)$$

and the result can be written in the form given by Eq. (26).

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