NONLINEAR OPTICAL INTERACTIONS IN OPTICAL FIBERS

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We report on our research program aimed at clarifying the physical processes leading
to the nonlinear optical response of silica optical fibers and at studying the implica-
tions of optical nonlinearities on optical pulse propagation and optical switching de-
vices. The dominant physical processes leading to the nonlinear optical response of an
optical fiber are nonresonant electronic polarization, with essentially instantaneous re-
sponse, the Raman interaction, with sub-picosecond response, and electrostriction, with
nanosecond response. We present experimental results that show the consequence of
each of these processes on the propagation of a light pulse through an optical fiber. We
have also performed one of the first direct measurements of the electrostrictive contribu-
tion to the nonlinear refractive index of optical fibers. We measure values ranging from
1.5×10^{-16} to $5.8 \times 10^{-16} \mathrm{cm}^2/\mathrm{W}$, depending on fiber type. These values are comparable
to that of the fast, Kerr nonlinearity (i.e., sum of electronic and Raman contributions) of
$2.5 \times 10^{-16} \mathrm{cm}^2/\mathrm{W}$. The measured electrostrictive nonlinearities are significantly larger
than those predicted by simple models, and the possible explanations of this difference
are discussed.

1. Introduction

There is great interest in the nonlinear optical properties of optical fibers. In this
article we present a brief description of the dominant physical interactions that can
lead to a nonlinear response in optical fibers, and we then describe some recent
research results regarding the influence of optical nonlinearities on the propagation
of light through an optical fiber.

Let us first recall some of the basic properties of optical fiber that are crucial
in determining their nonlinear optical response; additional details are available for
example in the books of Snyder and Love[1] and of Agrawal.[2] A typical single-mode
communication fiber possesses a central core of germania-doped fused silica of di-
ameter of $2a \approx 10~\mu\mathrm{m}$ surrounded by a much wider cladding region of nearly pure
fused silica. The presence of the germania doping raises the refractive index by
approximately 3×10^{-3} with respect to the cladding region. Under such conditions,
the fiber can support the propagation of a single mode field of approximately gaus-
sian cross section and an effective area of $\sim 80~\mu\mathrm{m}^2$. Modern optical fibers can be

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fabricated with extremely low loss, as low as 0.2 dB/km at 1.55 μm, the wavelength of minimum loss for silica fiber. The combination of small mode area and long effective propagation distances allows optical fibers to display large nonlinear effects; for example, a power as low as 100 mW can produce a nonlinear phase shift of π radians, calculated by assuming $L_{\text{eff}} = 20$ km and $n_2 = 3 \times 10^{-16}$cm^2/W through use of the relation

$$\phi_{\text{NL}} = n_2 \frac{2\pi}{\lambda_{\text{vac}}} I L_{\text{eff}} = n_2 \frac{2\pi}{\lambda_{\text{vac}}} \frac{P}{A_{\text{eff}}} L_{\text{eff}} \tag{1}$$

2. Origin of the Nonlinear Response of Optical Fibers

Some of the physical mechanisms that can give rise to optical nonlinearities in optical fibers are summarized in Table 1. The largest contribution is typically that of electronic polarization, which results from the displacement of bound electrons. The Raman response results from the motion of atomic nuclei, and electrostriction is the tendency of materials to become compressed in the presense of an intense laser field. The tensor nature of the nonlinear optical response is quantified in terms of the A and B coefficients of Maker and Terhune,[3] which are defined by the relation

$$\mathbf{P}^{\text{NL}} = A(\mathbf{E} \cdot \mathbf{E}^*)\mathbf{E} + \frac{1}{2}B(\mathbf{E} \cdot \mathbf{E})\mathbf{E}^* \tag{2}$$

where

$$\tilde{\mathbf{P}}^{\text{NL}}(t) = \mathbf{P}^{\text{NL}}e^{-i\omega t} + \text{c.c.} \qquad \tilde{\mathbf{E}}(t) = \mathbf{E}e^{-i\omega t} + \text{c.c} \tag{3}$$

and which can alternatively be expressed by

$$A = 6\chi_{1122} = 6\chi_{1212} \qquad B = 6\chi_{1221} \tag{4}$$

with the convention $\chi = \chi(\omega; \omega, \omega, -\omega)$. Electrostriction had earlier been thought to make a contribution of 20% of that of the electronic response under typical circumstances, but recent measurements[4] suggest that the electrostrictive response can be comparable to the electronic response.

Table 1. Physical mechanisms leading to a nonlinear optical response of optical fibers.

Mechanism	Response time	Tensor nature	n_2
Electronic polarization		B/A = 1	$\sim 2 \times 10^{-16}$ cm^2/W
Raman response		material dependent B/A = 0.3 for silica glass	$\sim 0.2 n_2$(electronic)
Electrostriction	~ 1 ns	B = 0	$\sim 0.2 - 1.0\, n_2$(electronic)

Hellwarth and coworkers[5] have shown how to describe the electronic and electrostrictive response in a consistent manner. They show that within the context

of the Born-Oppenheimer approximation for an isotropic material, the material polarization at time t can be expressed as

$$\tilde{\mathbf{P}}(t) = \frac{1}{2}\sigma\tilde{E}^2(t)\tilde{\mathbf{E}}(t) + \tilde{\mathbf{E}}(t)\int_{\infty}^{0} ds\tilde{a}(t-s)\tilde{E}^2(s)$$

$$+ \int_{\infty}^{0} ds\tilde{E}(s)\tilde{b}(t-s)\tilde{\mathbf{E}}(t)\tilde{\mathbf{E}}(s) \tag{5}$$

Here $\sigma = \frac{1}{3}\chi_{1111}$(electronic) describes the "instantaneous" electronic response. The two integral expressions describe the "sluggish" nuclear or Raman response, and show how the applied field at earlier times s can affect the material response. $\tilde{a}(t-s)$ is known as the "isotropic" nuclear response function and $\tilde{b}(t-s)$ is the "anisotropic" nuclear response function. These functions are causal in that they vanish for $s > t$; it is for this reason that we have set the upper limits of integration equal to zero in writing Eq. (5).

It is useful to transform Eq. (5) into the frequency domain. We introduce the transforms

$$a(\Omega) = \int \tilde{a}(\tau)e^{i\Omega\tau}d\tau \qquad b(\Omega) = \int \tilde{b}(\tau)e^{i\Omega\tau}d\tau. \tag{6}$$

The usual frequency domain susceptibilities can be expressed in terms of these new functions. For example, one finds that (with the convention $\chi = \chi(\omega;\omega,\omega,-\omega)$)

$$\chi_{1122} = \chi_{1212} = \sigma + 2a(0) + b(0)$$

$$\chi_{1221} = \sigma + 2b(0) \tag{7}$$

$$\chi_{1111} = 3\sigma + 4a(0) + 4b(0).$$

Expressions for the susceptibility at other frequencies are described for instance by Owyoung[6] and by Buckland.[4]

As mentioned above in the context of Table 1, the Raman contribution has a 75 fs response time. For pulses much larger than 75 fs, and fixed polarization, it is useful to combine the electronic and Raman responses into a "fast" response

$$n_2(\text{fast}) = n_2(\text{electronic}) + n_2(\text{Raman}). \tag{8}$$

Since the electronic and electrostrictive responses have different tensor properties, this relation must be used with caution when polarization effects are important. Let us next see how to specify how polarization effects influence Eq. (8).

3. Polarization Dependence of the Nonlinear Refractive Index for Optical Fibers

In this section, we see how to take explicit account of the polarization properties of light in determining the effective value of the nonlinear refractive for an optical fiber.

This analysis is complicated by the fact that the various contributions have different tensor properties and because the predictions are different depending on whether or not the optical fiber is polarization preserving. For simplicity, we include only the two "fast" contributions (i.e., electronic and Raman) to the nonlinear refractive index.

The primary value of the nonlinear refractive index (e.g., the one that is quoted most commonly in the liturature) refers to the case of a single-frequency beam of linearly polarized light in a polarization preserving fiber, that is

$$n_2^{\parallel}(\text{fast}) = \frac{12\pi^2\chi_{1111}^{(3)}(\omega;\omega,\omega,-\omega)}{n_0^2 c}.$$ (9)

Under other circumstances, we write

$$n_2(\text{eff}) = \kappa n_2^{\parallel}(\text{fast})$$ (10)

where κ is a correction factor that accounts for the tensor and frequency dependence of the nonlinear optical interaction. We note that, as a formality, $\kappa^{\parallel} = 1$. For example, we may be interested in determining how the propagation of the x-polarized component of a beam of light is influenced by the y-polarized component of a beam of the same frequency. To treat this case, we first calculate the total x component of the nonlinear polarization. Since in general

$$P_i(w) = 3\Sigma_{jkl}\chi_{ijkl}^{(3)}(\omega;\omega,\omega,-\omega)E_j(\omega)E_k(\omega)E_l^*(\omega)$$ (11)

we find that (with the frequency dependence implicit)

$$P_x = 3\chi_{1111}^{(3)}|E_x|^2 E_x + 3[\chi_{1122}^{(3)} + \chi_{1212}^{(3)}]|E_y|^2 E_x + 3\chi_{1221}^{(3)} E_y^2 E_x^*.$$ (12)

Here the first term describes $n_2^{\parallel}$ (fast), as described in Eq. (9), the second term describes $n_2^{\perp}$ (eff), and the third term can usually be ignored, because it describes a four-wave mixing interaction, that will usually not be phase matched in an optical fiber. By comparing the magnitude of the first and second terms, we find that

$$\begin{aligned}\kappa^{\perp} &= \frac{\chi_{1122}^{(3)} + \chi_{1212}^{(3)}}{\chi_{1111}} = \frac{2\sigma + 4a(0) + 2b(0)}{3\sigma + 4a(0) + 4b(0)} \\ &= \frac{2}{3}\frac{(1 + \rho\mu)}{(1 + \mu)}\end{aligned}$$ (13)

where $\mu = \chi_{1111}^{(3)}(\text{Raman})/\chi_{1111}^{(3)}(\text{electronic})$ and $\rho = (3/2)(1 + b(0)/2a(0))/(1 + b(0)/a(0))$. Note that in the absence of the Raman contribution, $\kappa^{\perp} = 2/3$, which is a well known result.

We now consider the example of propagation through a non-polarization preserving fiber, in which case the polarization evolves randomly with distance in the

fiber. We first note that at any point in the fiber we can express the change in refractive index for x and y polarized light as

$$\Delta n_x = n_2^{\parallel}(\text{fast})I_0[\kappa_{\parallel}f + \kappa_{\perp}(1 - f)] \qquad (14)$$

$$\Delta n_y = n_2^{\parallel}(\text{fast})I_0[\kappa_{\perp}f + \kappa_{\parallel}(1 + f)] \qquad (15)$$

where f is the fraction of the intensity in polarization x. In a fiber with random birefringence, we cannot predict f as a function of distance, but we can describe f in terms of a probability density function $p(f)$. In this case the measured change in refractive index is predicted to be

$$\langle \Delta n \rangle = \int_0 [\Delta n_x f + \Delta n_y (1 - f)] p(f) df \qquad (16)$$

Numerical simulations suggest[8] that non-polarization preserving fibers are described by the function $p(f) = 1$, that is, all fractional distributions of intensity between the two orthogonal components are equally likely. In this case, the integral in Eq. (15) is readily performed to obtain

$$\kappa(\text{random polarization evolution}) = \frac{2}{3}\kappa_{\parallel} + \frac{1}{3}\kappa_{\perp} = \frac{2}{3} + \frac{2}{9}\frac{(1 + \rho\mu)}{(1 + \mu)} \qquad (17)$$

One additional special case is that of an unpolarized incident light beam. In the formalism of Eq. (15), this situation corresponds to $p(f) = \delta(f - \frac{1}{2})$, because for an unpolarized beam of light, half of the intensity will reside in each polarization any choice of polarization bases set. If we evaluate Eq. (15) for this choice of $p(f)$, we obtain

$$\kappa(\text{unpolarized}) = \frac{1}{2}\kappa_{\parallel} + \frac{1}{2}\kappa_{\perp} = \frac{5}{6}. \qquad (18)$$

Values of κ for other circumstances and for all three contributions to the nonlinear refractive index have been tabulated by Buckland.[7]

4. Measurement of the Electrostrictive Contribution to the Nonlinear Optical Response of Optical Fibers

In this section we briefly review the theoretical understanding of the electrostrictive contribution of the nonlinear optical response of optical fibers and we summarize recent experimental measurements of this effect.

Electrostriction is the tendency of materials to become compressed in the presence of a high intensity laser field. Simple models of this effect predict that the nonlinear refractive index will be given by[9]

$$n_2(\text{str}) = \frac{\gamma_e^2}{4c\rho_0 n^2 v^2} \qquad (19)$$

where c is the speed of light in vacuum, ρ_0 is mean density of the material, n is the refractive index, v is the speed of sound, and γ_e is the electrostrictive coefficient defined as

$$\gamma_e = \rho_0 \frac{\partial \epsilon}{\partial \rho} . \tag{20}$$

The value of γ_e can be derived theoretically by assuming the validity of the Lorentz–Lorenz law with a constant (i.e., density independent) value of the molecular polarizability. This argument leads to the prediction $\gamma_e = 1.5$ for silica glass. Alternatively, the value of γ_e can be deduced from measured[10] values of the stress-optic coefficients, also leading to the value $\gamma = 1.5$. In either case the predicted value of $n_2(\text{str})$ is $0.6 \times 10^{-16} \text{cm}^2/\text{W}$, approximately 20% of $n_2(\text{electronic})$ for silica glass.

In 1995, Kato et al.[11] performed a measurement of n_2 of dispersion-shifted fibers using a novel modulation technique and measured a value of $3.3 \times 10^{-16} \text{cm}^2/\text{W}$, which is higher than the accepted value[12] of 2.52×10^{-16}. Buckland and Boyd[13] proposed that this discrepancy was due to an electrostrictive contribution to n_2, which contributed under the experimental conditions of Kato et al. using cw laser beams and relatively slow (MHz) modulation, but not under more conventional experimental conditions using short (e.g., picosecond) pulse excitation. Buckland and Boyd also developed a model which showed that when proper account was taken of the unpolarized light used in the experiment of Kato et al., the discrepancy could be resolved by assuming that $n_2(\text{str})$ is 0.17 times as large as $n_2(\text{electronic})$.

In order to test this theoretical model, we have recently performed a series of experiments[4] to determine the frequency response of n_2 as measured by the modulation technique. Our experimental setup is shown in Fig. 1. The pump beam is derived from a semiconductor diode laser whose drive current is modulated at a frequency ranging from 10 MHz to 1 GHz. The laser output is then amplified by an erbium doped fiber amplifier before being combined with the depolarized output of a separate semiconductor diode laser that acts as the probe beam. The two beams co-propagate through the fiber under test. The transmitted probe beam is then separated from the pump beam and is allowed to fall onto a fast photodetector whose output is sent to an rf spectrum analyzer. The depth of modulation impressed upon the probe beam provides a measure of the effective nonlinear coefficient of the optical fiber.

Typical experimental results are shown in Fig. 2, where the effective values of n_2 measured by the modulation technique are plotted against the pump modulation frequency. The top graph corresponds to a standard single-mode fiber. The solid curve is a theoretical fit to the data, based on the model of reference[13] with the relative strengths of the electrostrictive and fast nonlinearities given by $n_2(\text{str})/2$ $\kappa_{\text{eff}} n_2(\text{fast}) = 1.58$. Since $\kappa_{\text{eff}} = 2/3$ for our experimental conditions using a depolarized probe beam, one finds that $n_2 \ (\text{str})/n_2(\text{fast}) = 2.1$. Thus, for a fiber of this construction, the electrostrictive nonlinearity is more than two-times larger than the fast component, that is, is approximately ten times larger than the value expected on the basis of the accepted value of the electrostrictive coefficient.
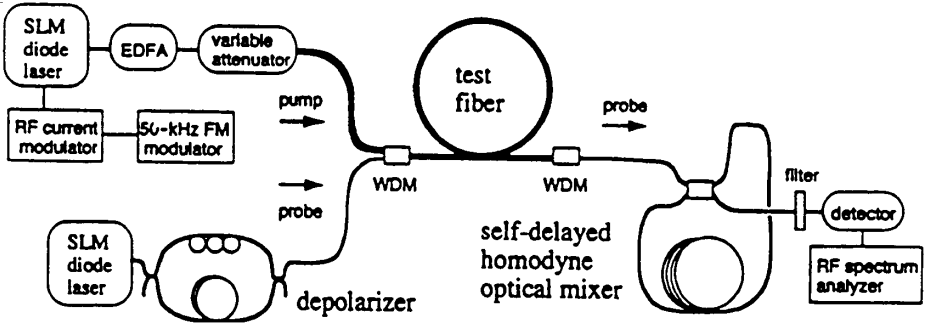
Fig. 1. Experimental apparatus used to measure n_2 through frequency-modulated cross-phase modulation. SLM, single-longitudinal-mode laser; EDFA, erbium doped fiber amplifier; WDM, wavelength division multiplexer.
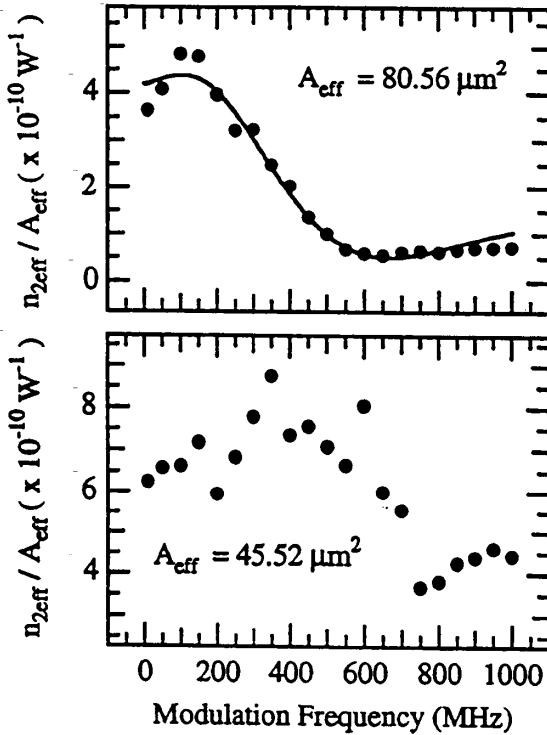

Fig. 2. Measured nonlinear coefficient $n_{2\text{eff}}(f)/A_{\text{eff}}$ versus pump modulation frequency for a standard single-mode fiber (top) and a dispersion-shifted fiber (bottom). The solid curve in the top plot is a theoretical fit to the data.

The bottom graph in Fig. 2 corresponds to a dispersion-shifted fiber. Here the scatter of the data, due presumably to the influence of acoustic reflections, prevents us from obtaining a good theoretical fit to the data. However, the ratio of $n_2(\text{eff})$ at low and high frequencies provides an estimate of the quantity $1 + n_2(\text{str})/2$

$\kappa_{\text{eff}}n_2(\text{fast})$. From this reasoning we find that $n_2(\text{str})/2\kappa_{\text{eff}}n_2(\text{fast}) = 0.41$ or that $n_2(\text{str})/n_2(\text{fast}) = 0.55$, which is some four-times smaller than that deduced above for the standard single-mode fiber. Values of $n_2(\text{str})/n_2(\text{fast})$ for other fiber designs have been measured and summarized by Buckland.[7]

In summary, our measurements indicate that the electrstrictive contribution to the nonlinear refractive index is much larger than previously believed, and varies considerable with fiber design. We do not fully understand the origin of this enhanced electrostrictive response. We surmise that maybe the origin of these results lies in a modification of the elastic properties glass that has beem formed into optical fibers, for instance as the result of stresses that are produced during the fabrication of the optical fiber.

Acknowledgements

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