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Noise initiation of stimulated Brillouin scattering

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We describe a theoretical model that shows how stimulated Brillouin scattering (SBS) is initiated by thermally excited acoustic waves distributed within a Brillouin-active medium. This model predicts how the SBS reflectivity, Stokes linewidth, and fluctuations in Stokes intensity depend upon the laser intensity and upon the physical properties of the SBS medium. This model also leads to the prediction that the value of the single-pass gain (i.e., $G=gIL$) at the threshold for SBS is not a universal number, but depends upon the laser frequency and on the properties of the SBS medium. For typical organic liquids at room temperature, G is in the range 20–25.

I. INTRODUCTION

Stimulated Brillouin scattering (SBS) is a process in which a laser and Stokes wave interact with one another in a material medium (see Fig. 1) through the intermediary of a sound wave, leading to the amplification of the Stokes wave and the attenuation of the laser wave.[1–5] The SBS process can be treated theoretically in a very straightforward manner for the case of an SBS amplifier, since the laser and Stokes fields are both applied externally. The theoretical analysis is much more complicated for the case of an SBS generator, because only the laser field is applied externally and the Stokes wave is created within the medium. In this case, the process of SBS is "seeded" by spontaneous Brillouin scattering from thermally excited sound waves or (at low temperatures) by quantum noise.[6–9] It is known empirically that there is a threshold for the occurrence of SBS generation, and that the threshold condition is that the single-pass gain through the Brillouin medium, $G=gI_L(0)L$ [here g is the SBS gain factor that is defined in Eq. (22) below, $I_L(0)$ is the intensity of the incident laser field, and L is the length of the Brillouin medium], must exceed some threshold value G_{th}, which is typically of the order of 25.[10]

It is the intent of the present paper to examine in detail how the SBS process is initiated from noise. In particular, we develop a model which shows how SBS is initiated by a spatially distributed, fluctuating noise source. We compare the predictions of this model with those of a simpler model due to Zel'dovich, Pilipetskii, and Shkunov,[10] (which we will henceforth call the *localized, nonfluctuating source model*) that ascribes the origin of SBS to spontaneous scattering occurring in a thin region located near the rear of the SBS medium (near $z=L$ in the notation of Fig. 1). The predictions of our new model (which we will henceforth call the *distributed, fluctuating source model*) are in good qualitative agreement with those of the localized, nonfluctuating source model, which themselves are in good qualitative agreement with experimental observations. However, we find that the two models lead to measurably different predictions for certain properties, such as the dependence of the SBS reflectivity on the single-pass gain through the interaction region. These differences result both from the distributed nature of the initiation and as a consequence of fluctuations in the initiation process. In addition, our model allows us to treat noise properties of the SBS process, such as the linewidth and intensity fluctuations in the output intensity.

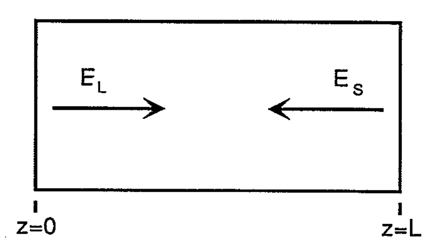

FIG. 1. Geometry of the SBS interaction.

II. THE EQUATIONS DESCRIBING SBS

In this paper, we consider SBS in the backward direction in the geometry shown in Fig. 1. We assume that the total optical field within the Brillouin-active medium,

$$\tilde{E}_{\text{tot}}(z,t)=\tilde{E}_L(z,t)+\tilde{E}_S(z,t)\ , \tag{1}$$

can be represented as the sum of a laser component

$$\tilde{E}_L(z,t)=\tfrac{1}{2}E_L(z,t)e^{i(k_Lz-\omega_Lt)}+\text{c.c.} \tag{2}$$

propagating in the positive z direction, and a Stokes component

$$\tilde{E}_S(z,t)=\tfrac{1}{2}E_S(z,t)e^{i(-k_Sz-w_St)}+\text{c.c.} \tag{3}$$

propagating in the negative z direction. Note that we are ignoring the transverse variation of the laser and Stokes fields. This approximation is known to be highly accurate either for guided propagation through a single-mode optical fiber or for free propagation through a region whose Fresnel number $F=4A/\lambda L$ is close to unity. These fields interact with each other by means of the acoustic disturbance within the medium, which we describe in terms of the variation $\tilde{\rho}(z,t)$ of the density of the medium from its mean value ρ_0. We represent this density variation in terms of its complex amplitude $\rho(z,t)$ according to

$$\tilde{\rho}(z,t)=\tfrac{1}{2}\rho(z,t)e^{i(qz-\Omega t)}+\text{c.c.} , \tag{4}$$

where we have introduced the acoustic frequency $\Omega=\omega_L-\omega_S$ and the acoustic wavenumber $q=k_L+k_S$, which are related by the relation $\Omega=qv$ where v denotes the velocity of sound in the medium. The nature of the coupling of the optical waves is described by the reduced optical wave equations for the laser and Stokes fields[11]

$$\frac{\partial E_L}{\partial z}+\frac{n}{c}\frac{\partial E_L}{\partial t}=\frac{i\gamma\omega_L}{4\rho_0 nc}\rho E_S , \tag{5a}$$

$$\frac{\partial E_S}{\partial z}-\frac{n}{c}\frac{\partial E_S}{\partial t}=\frac{-i\gamma\omega_S}{4\rho_0 nc}\rho^* E_L , \tag{5b}$$

where $\gamma=\rho(\partial\epsilon/\partial\rho)$ is the electrostrictive coupling constant.

The density disturbance is described by the acoustic wave equation

$$\frac{\partial^2\tilde{\rho}}{\partial t^2}-\Gamma'\frac{\partial}{\partial t}\frac{\partial^2\tilde{\rho}}{\partial z^2}-v^2\frac{\partial^2\tilde{\rho}}{\partial z^2}=-\frac{\gamma}{8\pi}\frac{\partial^2}{\partial z^2}\tilde{E}^2_{\text{tot}}+\tilde{f} , \tag{6}$$

where $\tilde{f}$ represents a Langevin noise source that describes the thermal (or quantum) excitation of acoustic waves and which leads therefore to the initiation of the SBS process. In the first term on the right-hand side of this equation, we drop the terms at zero frequency and at twice the optical frequency (which do not lead to a Brillouin-resonant excitation of the medium) so that the right-hand side becomes

$$\frac{\gamma q^2}{16\pi}\{E_L E_S^* e^{i(qz-\Omega t)}+\text{c.c.}\}+\tilde{f} .$$

We next introduce Eq. (4) into Eq. (5) and make the slowly varying amplitude approximation to obtain

$$\frac{\partial\rho}{\partial t}+v\frac{\partial\rho}{\partial z}+\tfrac{1}{2}\Gamma\rho=\frac{i\gamma q^2}{16\pi\Omega}E_L E_S^*+f , \tag{7}$$

where $\Gamma=\Gamma' q^2$ is the phonon (intensity) decay rate and where we have introduced the amplitude f of the Langevin force, which is related to the original force $\tilde{f}$ by

$$\tilde{f}=-i\Omega f e^{i(qz-\Omega t)}+\text{c.c.} \tag{8}$$

We can usually ignore the second term on the left-hand side in Eq. (7), which describes the effects of phonon propagation. By doing so we obtain the equation

$$\frac{\partial\rho}{\partial t}+\tfrac{1}{2}\Gamma\rho=\frac{i\gamma q^2}{16\pi\Omega}E_L E_s^*+f . \tag{9}$$

III. FORM OF THE LANGEVIN NOISE TERM

We next examine the statistical properties of the Langevin noise term appearing in Eq. (9). To do so, it is convenient to turn to a discrete description of the acoustic field. We divide the interaction region into subregions of length Δz, and let ρ_i denote the amplitude of the acoustic disturbance averaged over one such subregion. In the absence of the electrostrictive driving term, this quantity obeys the equation

$$\frac{d\rho_i}{dt}+\tfrac{1}{2}\Gamma\rho_i=f_i(t) , \tag{10}$$

where $f_i(t)$ represents the quantity $f(z,t)$ averaged over subregion i. We make the standard physical assumptions that f_i is a Gaussian random variable with zero mean such that $\langle f_i\rangle=0$ and is δ correlated in the sense that

$$\langle f_i(t)f_j^*(t')\rangle=\tilde{Q}\delta_{ij}\delta(t-t') . \tag{11}$$

We next determine the value of the parameter $\tilde{Q}$ which characterizes the strength of the fluctuations in f_i. By introducing the formal solution of Eq. (10) into the left-hand side of Eq. (11), we find that

$$\Gamma\langle\rho_i(t)\rho_j^*(t)\rangle=\tilde{Q}\delta_{ij} . \tag{12}$$

We next calculate the density correlation $\langle\rho_i(t)\rho_j^*(t)\rangle$ by considering the energy stored in a thermally excited acoustic wave. The energy density of a sound wave is given by[12]

$$\langle u\rangle=\tfrac{1}{2}\rho_0\langle V^2\rangle+\tfrac{1}{2}v^2\langle\tilde{\rho}^2\rangle/\rho_0 , \tag{13}$$

and hence the energy in slab i whose volume is $A\Delta z$ is equal to $\langle u\rangle A\Delta z$ or to

$$E_i=(\tfrac{1}{2}\rho_0\langle V^2\rangle+\tfrac{1}{2}v^2\langle\tilde{\rho}_i^2\rangle/\rho_0)A\,\Delta z . \tag{14}$$

According to the equipartition theorem, each term in Eq. (14) contributes an amount $\frac{1}{2}kT$ to the energy. We hence find that $\langle\tilde{\rho}_i^2\rangle=kT\rho_0/v^2A\,\Delta z$. However, since $\langle\tilde{\rho}_i^2\rangle=\frac{1}{2}\langle\rho_i(t)\rho_i^*(t)\rangle$, we find that

$$\langle\rho_i(t)\rho_i^*(t)\rangle=\frac{2kT\rho_0}{v^2A\,\Delta z} . \tag{15}$$

By comparison with Eq. (12), we see that the strength parameter is given by

$$\tilde{Q}=\frac{2kT\rho_0\Gamma}{v^2A\,\Delta z} . \tag{16}$$

If we now return to a continuum description of the acoustic field, we find that the Langevin source term obeys the relations

$$\langle f(z,t)\rangle=0 \tag{17a}$$

and

$$\langle f(z,t)f^*(z',t')\rangle=Q\delta(z-z')\delta(t-t') , \tag{17b}$$

where

$$Q=\tilde{Q}\Delta z=\frac{2kT\rho_0\Gamma}{v^2A} . \tag{18}$$

We can convert this last formula into its quantum-mechanical analog by replacing kT by $\hbar\Omega(\bar{n}+1)$, where $\bar{n}=(e^{\hbar\Omega/kT}-1)^{-1}$ gives the mean number of phonons per mode of the acoustic field. We thereby find that

$$Q=\frac{2\rho_0\Gamma\hbar\Omega(1+\bar{n})}{v^2 A} . \tag{19}$$

This expression for Q along with Eqs. (5), (9), and (17) completely describes the SBS process including its initiation by noise.

IV. ANALYTIC SOLUTION OF THE SBS EQUATIONS IN THE CONSTANT PUMP LIMIT

For the case of small conversion of the laser field into the Stokes field, the laser amplitude E_L can be taken to be a constant. For this case and in the long-time limit, $\Gamma\tau\gg 1$, Eqs. (5b) and (9) can be solved formally to obtain an explicit expression for the temporal and spatial evolution of the Stokes field amplitude:[13]

$$\begin{aligned} E_S(z,\tau)=&E_S(L,\tau)\\ &+\left[\frac{\Gamma G(L-Z)}{4L}\right]^{1/2}\int_0^\tau d\tau' e^{-\Gamma(\tau-\tau')/2}E_S(L,\tau')I_1\{[G\Gamma(L-z)(\tau-\tau')/L]^{1/2}\}/(\tau-\tau')^{1/2}\\ &+\frac{i\gamma\omega_S}{4\rho_0 nc}E_L\int_0^\tau d\tau'\int_L^z dz' e^{-\Gamma(\tau-\tau')/2}f^*(z',\tau')I_0\{[G\Gamma(z-z')(\tau-\tau')/L]^{1/2}\} , \end{aligned} \tag{20}$$

where $I_j(x)$ is the modified Bessel function of order j, $r=t-zn/c$ is the local time variable, and where we have introduced the quantity

$$G=\frac{\gamma^2 n\omega_S^3|E_L|^2L}{4\pi\rho_0 c^3\Omega\Gamma} , \tag{21}$$

which we shall see below is the logarithm of single-pass intensity amplification. It is also convenient to introduce the SBS gain factor g, defined through the relation $G=gIL$, where $I=(nc/8\pi)|E_L|^2$ is the laser intensity. We hence find that the gain factor is given by the expression

$$g=\frac{\gamma^2\omega_S^2}{\rho_0 nc^3 v\Gamma} . \tag{22}$$

In the absence of an input Stokes field, only the laser term in Eq. (20) gives a nonvanishing contribution to the expectation value of the Stokes intensity. (The same conclusion holds quantum mechanically for the normally ordered intensity when the Stokes input is in the vacuum state.) We then find, letting $\tau\to\infty$, and making explicit use of the statistical properties of the Langevin noise term as given by Eqs. (17) and (19), that the Stokes intensity is given by

$$\langle|E_S(0)|^2\rangle=\left[\frac{\gamma\omega_S}{4\rho_0 nc}\right]^2|E_L|^2Q\int_0^\infty d\tau'\int_0^L dz' e^{-\Gamma\tau'}I_0^2[(G\Gamma z'\tau'/L)^{1/2}] . \tag{23}$$

The integrations can be performed explicitly, leading to the result

$$\langle|E_S(0)|^2\rangle=\left[\frac{\gamma\omega_S}{4\rho_0 nc}\right]^2|E_L|^2\frac{QL}{\Gamma}\times e^{G/2}[I_0(G/2)-I_1(G/2)] . \tag{24}$$

Through use of Eqs. (18), (22) and (24), we now find that the SBS reflectivity defined by $R=\langle|E_S(0)|^2\rangle/|E_L(0)|^2$ is given by

$$R=Ye^{G/2}[I_0(G/2)-I_1(G/2)] , \tag{25}$$

where we have introduced the parameters

$$Y=(\bar{n}+1)X,\quad X=g\hbar\omega_S\Gamma\frac{L}{4A} . \tag{26}$$

Equations (25) and (26) give the formal solution to the equations describing the initiation of SBS by a distributed noise source. This solution is displayed graphically in Fig. 2 as the curve labeled *distributed fluctuating source.* For comparison, the growth rate $\exp(G)$ that would be experienced by a localized source near $z=L$ is shown as the curve labeled *localized source.* In plotting this figure, we have chosen the amplitude of the localized source such that the two curves agree in the limit of low gain. In this limit ($G\ll 1$), the SBS reflectivity predicted by Eq. (25) becomes simply $R=Y$. In this case the reflected field originates entirely from the scattering of the laser field by spontaneously (i.e., thermally) generated phonons, and hence the reflectivity is independent of the laser intensity. In the opposite limit of high gain ($G\gg 1$) the reflectivity becomes

$$R=\frac{Y}{\sqrt{\pi}}\frac{e^G}{G^{3/2}} . \tag{27}$$

This result shows that, due to the distributed nature of the noise source that initiates SBS, the effective amplification of the SBS generation process is reduced by

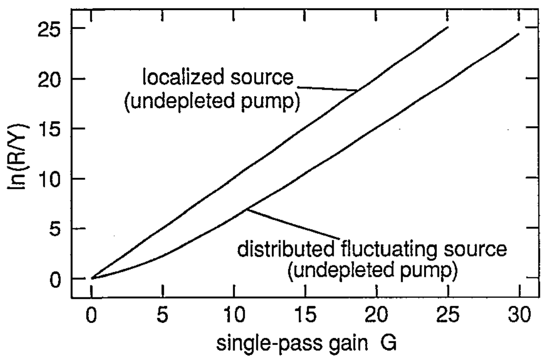

FIG. 2. SBS reflectivity R, as predicted by two models that ignore pump depletion effects, plotted as a function of the single-pass gain $G=gI_L L$. The reflectivity is normalized by the noise initiation parameter $Y=(\bar{n}+1)X$. For all values of G, the SBS reflectivity as predicted by the *distributed fluctuating source* model [i.e., Eq. (25)] increases with G less rapidly than $\exp(G)$, which is the growth rate that would be experienced by a *localized source* placed near the rear of the cell.

an amount $\sqrt{\pi}G^{3/2}$ from the single-pass amplification factor $\exp(G)$ that would be experienced by a localized source near $z=L$.

Our result given by Eqs. (25) and (26) shows that the reflectivity can be expressed as the product of a factor that depends only on the total gain G through the interaction region and another factor Y that depends on the physical and geometrical characteristics of the Brillouin-active medium. For the case in which the interaction region has a Fresnel number of unity, the geometrical factor $L/4A$ that appears in Eq. (26) should be set equal to $1/\lambda$, where λ is the wavelength of the radiation in the medium. In such a case, the factor X scales as the fourth power of the laser frequency, since g is independent of laser frequency and Γ is proportional to ω_S^2. In addition, the factor $(\bar{n}+1)$ is proportional to ω_S^{-1} in the (usual) limit $\bar{n}\gg 1$. As an example, for the case of SBS in CS_2 at a wavelength of 1.06 μm at room temperature, the factor X is equal to 6.65×10^{-14} and the factor $(\bar{n}+1)$ is equal to 1623 and therefore Y is equal to 1.08×10^{-10}.

We can use the results given in Eqs. (25) and (26) to provide a quantitative estimate of the threshold for SBS. We define threshold as the condition that R attain some prescribed value $R_{\rm th}$, which is usually taken to be of the order of 1%. We can then solve Eq. (25) numerically using the relevant value of Y to find the value of G for which R is equal to $R_{\rm th}$. For the example of SBS in CS_2 at room temperature at a wavelength of 1.06 μm, we find that G must equal 23.66 for $R_{\rm th}=1\%$ and that G must equal 26.11 for $R_{\rm th}=10\%$. For the case of SBS in CS_2 at room temperature excited by a frequency-doubled Nd:YAG laser, we find that $X=1.06\times 10^{-12}$ and $\bar{n}=812$, and hence that the threshold value of G is 21.44 for $R_{\rm th}=1\%$ and 23.9 for $R_{\rm th}=10\%$. For other standard Brillouin liquids, the product $Y=(\bar{n}+1)X$ is smaller than that for CS_2 by a factor of at most 3, and hence the value of G at threshold will be raised by an increment of at most 1. Our theory hence predicts that at room temperature the threshold for SBS in organic liquids corresponds to G in the range 20–25.

The theoretical prediction for the numerical value of the gain at threshold is very different for the case of SBS in a single-mode optical fiber. In such a case, the parameter L appearing in Eq. (6) represents the physical length of the fiber and A represents the physical area of the region of the fiber in which the radiation is confined. Consequently the factor $L/4A$ can be much larger than the value $1/\lambda$ which applies to free propagation through a region of unit Fresnel number. For this reason the gain at threshold can be much lower for an optical fiber than for the case of an organic liquid. For one particular case, Ippen and Stolen[14] report a value of G at threshold of 20.

V. LOCALIZED NONFLUCTUATING SOURCE MODEL OF THE INITIATION OF SBS

In this section we summarize briefly the localized, nonfluctuating source model of Zel'dovich, Pilipetskii, and Shkunov[10] for the initiation of SBS. In subsequent sections, the predictions of this model will be compared with those of the present calculation. The model of Zel'dovich, Pilipetskii and Shkunov considers SBS under steady-state conditions; in such a case it is adequate to describe SBS in terms of coupled intensity equations for the laser and Stokes fields, which are given by

$$\frac{dI_L}{dz}=-gI_L I_S \ , \tag{28a}$$

$$\frac{dI_S}{dz}=-gI_L I_S \ , \tag{28b}$$

where g is the SBS gain factor given by Eq. (22). We solve these equations subject to the boundary conditions that $I_L(0)$ is specified and that $I_S(L)=fI_L(L)$. The boundary condition at $z=L$ is a statement that SBS is initiated by spontaneous Brillouin scattering occurring in a small region near the rear of the interaction volume; f then represents the fraction of the transmitted laser intensity $I_L(L)$ that is backscattered to form the Stokes seed. Under typical conditions involving SBS in organic liquids, f is of the order of 10^{-12}. To good approximation (i.e., ignoring terms of order f) the solution to the coupled intensity equations (28) for these boundary conditions is[10]

$$G=\frac{\ln R+G_{\rm ref}}{1-R} \ , \tag{29}$$

where, as before, $G=gI_L(0)L$ is the single-pass gain, $R=I_S(0)/I_L(0)$ is the SBS reflectivity, and where $G_{\rm ref}=-\ln f$. This functional form is plotted in Fig. 3(a) for the typical case of $G_{\rm ref}=28.5$ as the curve labeled *localized, nonfluctuating source.* We see that the reflectivity R is much less than unity for $G<G_{\rm ref}$ and increases rapidly with G for $G>G_{\rm ref}$. For this reason, $G_{\rm ref}$ is a measure of the threshold gain required for the occurrence of SBS.

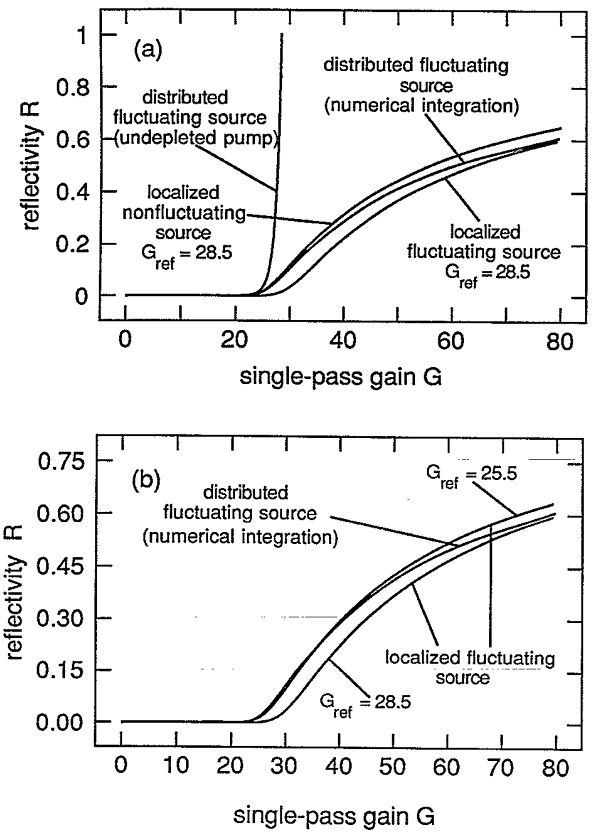

FIG. 3. SBS reflectivity R plotted as a function of the single-pass gain G, according to the theoretical models described in the text.

VI. NUMERICAL INTEGRATION OF THE SBS EQUATIONS

In order to study the properties of SBS in the general case where the SBS reflectivity is not necessarily much smaller than unity, we have performed a numerical integration of the complete set of coupled equations describing the laser, Stokes, and phonon fields [Eqs. (5) and (9)] with the Langevin source term given by Eqs. (17) and (19). We divide the interactions region into 800 subregions and integrate for a length of time sufficient to ensure that transients associated with the turn on of the laser field have died away. Unlike the case of the constant-pump theory, where universal results such as those shown in Fig. 2 could be obtained, in the present case the predictions depend upon the values of the initiation parameter $Y=(\bar{n}+1)X$ and on the product ΓT_t of the phonon damping rate Γ and the transit time $T_t=nL/c$ through the medium. The results of such calculations have been used to obtain Figs. 3–6, which are described in detail below.

VII. COMPARISON OF VARIOUS MODELS FOR THE INITIATION OF SBS

Figure 3 shows the dependence of the SBS reflectivity R on the single-pass gain G according to various models for the initiation of SBS. In all cases we consider SBS in carbon disulfide at room temperature, in which case the initiation parameter Y defined by Eq. (26) is equal to 1.08×10^{-10}. The curves labeled *distributed, fluctuating source (numerical integration)* give the results obtained by

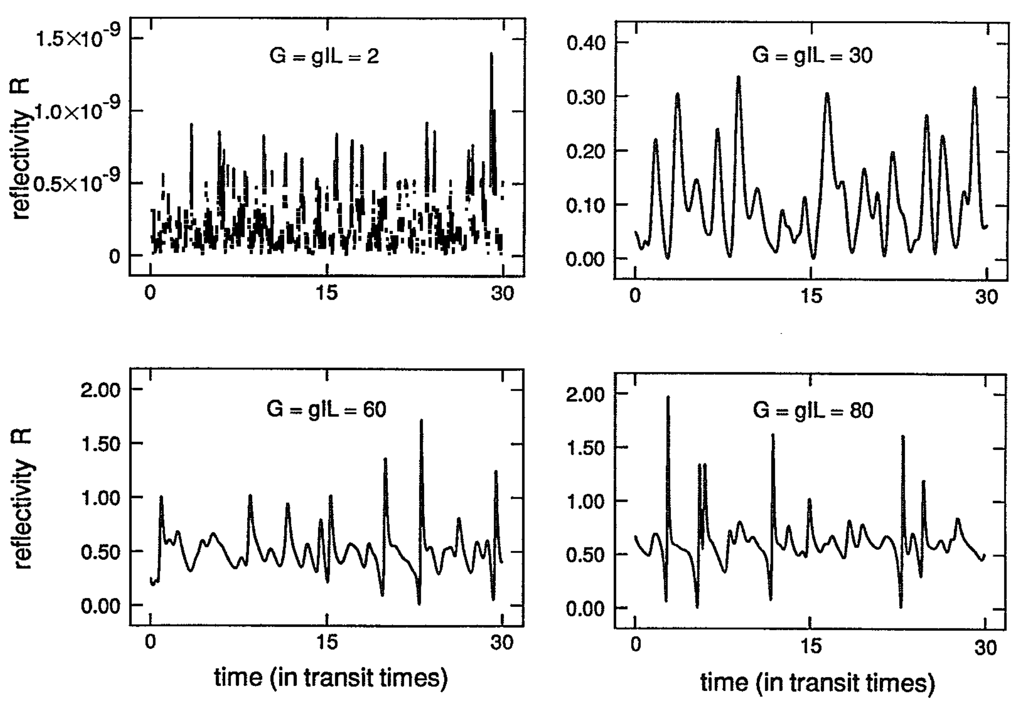

FIG. 4. Fluctuations in the Stokes output intensity, as predicted by the numerical integration of the equations of the *distributed fluctuating source model* described in the text. As the single-pass gain $G=gI_L(0)L$ is increased, the mean reflectivity increases and the fluctuations tend to become suppressed due to pump depletion effects. The case $\Gamma T_t=20$ is shown.

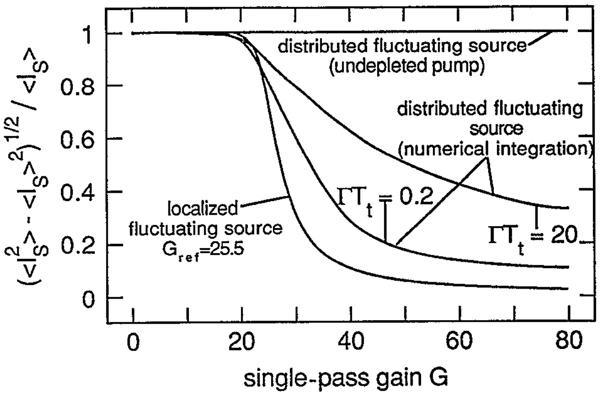

FIG. 5. Fluctuations in the Stokes output intensity, as measured by the square root of the normalized variance of the Stokes intensity, plotted as a function of the single-pass gain G, for the theoretical models described in the text.

the numerical integration of the SBS equations as described in Sec. IV, for the case $\Gamma T_t=20$. (We have found that the predicted reflectivity does not depend sensitively on the value of the parameter ΓT_t.) By comparison, the curve labeled *distributed fluctuating source (undepleted pump)* in part (a) of the figure gives the predictions of Eq. (25). The results of this model agree well with those of the numerical model for values of G near and below threshold, but predict that the reflectivity increases much too rapidly and without bound for values of G above threshold. Also shown in part (a) as the curve labeled *localized, nonfluctuating source* are the predictions of Eq. (29). We see that this theory also overestimates the reflectivity of the SBS process. The primary reason for the discrepancy is that this theory assumes that the SBS process is initiated by a nonfluctuating source. Due to the saturable nature of the SBS gain, a fluctuating input will experience smaller time-averaged gain than a stationary input. That this is the case can be seen from the results given in the curve labeled *localized, fluctuating source.* This curve was obtained by averaging the reflectivity predicted by Eq. (29) over the input intensity distribution of a thermal source whose mean intensity is the same as that used in obtaining the curve labeled *localized, nonfluctuating source.* This calculation was performed using the method outlined in the Appendix. We see that the primary effect of including source fluctuations in the localized source model is to shift the curve slightly to the right. We can shift the curve back to the left by using a smaller value of G_{ref} in the theoretical model. This dependence on G_{ref} is illustrated in part (b) of the figure. We see from part (a) that the *localized nonfluctuating* source model with $G_{ref}=28.5$ agrees well with the results of the full numerical calculation near threshold, and we see from part (b) that the *localized fluctuating* source model with $G_{ref}=25.5$ also agrees well near threshold; however, in neither case can we obtain good agreement for all values of G.

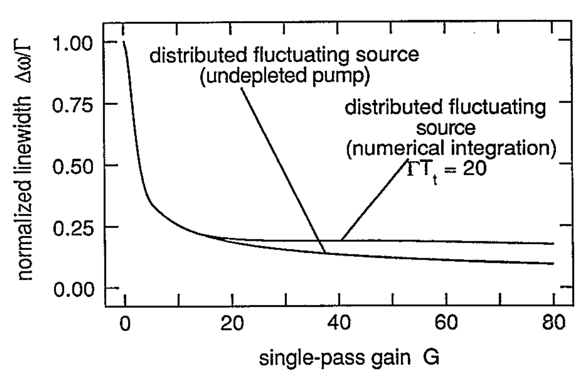

FIG. 6. Gain narrowing of the Stokes output spectrum. The Stokes linewidth normalized by the linewidth for spontaneous scattering is plotted as a function of the single-pass gain G. The analytic predictions of the *distributed, fluctuating source model with an undepleted pump* and the predictions of the numerical results of the *distributed fluctuating source* model are shown.

VIII. FLUCTUATIONS IN THE STOKES OUTPUT INTENSITY

Since the SBS process is initiated by noise, the Stokes output intensity is expected to be a fluctuating quantity. Figure 4 shows the time evolution of the output intensity as predicted by our numerical integration of the SBS equations. The case $\Gamma T_t=20$ is treated. We see that the fluctuations in the output intensity tend to be suppressed through the use of a large value of the single-pass gain G. This behavior occurs as a result of the saturable nature of the SBS gain due to pump depletion effects. We also see that the time scale of the fluctuations increases as G is increased, due to spectral gain narrowing of the Stokes radiation. In the limit of high gain, the fluctuations that do occur have a characteristic, bidirectional, spiked shape that is reminiscent of the phase waves that have been predicted to occur in stimulated Raman scattering.[15] We have determined that these features are associated with an abrupt change in the phase of the Stokes wave by approximately π radians. The nature of these features is described further by Gaeta.[16] Our results are similar to those of Wandzura[17] and to Dianov *et al.*[9]

We characterize the strength of the Stokes intensity fluctuations by calculating the variance of the Stokes intensity, which is defined by

$$\mathrm{var}(I_S)=\langle I_S^2\rangle-\langle I_S\rangle^2 . \tag{30}$$

In Fig. 5 we show the dependence of the square root of the normalized variance, $[\mathrm{var}(I_S)/\langle I_S\rangle^2]^{1/2}$ on the single-pass gain G. This quantity is equal to one according to the undepleted pump model. When pump depletion is included into the model, this quantity is seen to be equal to one for small values of G and to decrease monotonically for increasing values of G. This behavior can be understood as follows: In the limit of no pump depletion, the Stokes output is due simply to the linear amplification of light scattered from thermally generated phonons. The Stokes output hence has fluctuations characteristic of thermal light, for which the normalized variance is unity. In the saturation region these fluctuations tend to be damped out due to saturation of the amplification process. The curve labeled *localized fluctuating source* was calculated using the method shown in the Appendix; this curve shows the most rapid falloff with increasing values of G. The other two curves, labeled *distributed fluctuating source,* were obtained using the numerical technique described above. These curves show that the fluctuations in the Stokes output are reduced less dramatically through use of a small value of ΓT_t. The reason for this behavior is that the limit of small ΓT_t corresponds to a medium for which the transit time T_t is much shorter than the time scale Γ^{-1} on which the fluctuations in the Stokes intensity occur. In such a case, the degree of pump depletion can change appreciably over the time duration of a fluctuation, and this effect tends to smooth out the fluctuations in the Stokes output intensity.[16]

IX. SPECTRUM OF THE STOKES RADIATION

We define the spectral density $S(\omega)$ of the Stokes radiation as the Fourier transform of the field autocorrelation function, that is, as

$$S(\omega)=\int_{-\infty}^{\infty} e^{-i\omega\tau'}\langle E_S(O,\tau+\tau')E_S^*(0,\tau)\rangle d\tau' . \quad (31)$$

Through use of expression (20) for $E_S(z,\tau)$, we find by direct integration that the spectrum is given by[18]

$$S(\omega)=\frac{8\pi\hbar\omega_S(\bar{n}+1)}{ncA\Gamma}\left[\exp\left[\frac{G(\Gamma/2)^2}{\omega^2+(\Gamma/2)^2}\right]-1\right] . \quad (32)$$

In the low-gain limit ($G \ll 1$), Eq. (32) becomes

$$S(\omega)=\frac{8\pi\hbar\omega_S(\bar{n}+1)\Gamma G}{ncA}\left[\frac{1}{\omega^2+(\Gamma/2)^2}\right] , \quad (33)$$

which has the form of the Lorentzian line shape characteristic of spontaneous Brillouin scattering. In the high-gain limit ($G \gg 1$), Eq. (32) becomes

$$S(\omega)=\frac{8\pi\hbar\omega_S(\bar{n}+1)}{ncA\Gamma}e^{G}\exp[-(4G\omega^2/\Gamma^2)] , \quad (34)$$

which has the form of a Gaussian line shape. The full width at half maximum (FWHM) width of the spectrum in this limit is given by

$$\Delta\omega=\Gamma(\ln 2/G)^{1/2} . \quad (35)$$

The narrowing of the spectrum with increasing values of G is known as gain narrowing. The dependence of the Stokes linewidth on the single-pass gain G as predicted by the general result of Eq. (32) is shown in Fig. 6 by the curve labeled *distributed fluctuating source (undepleted pump).*

Also shown in Fig. 6 is the curve labeled *distributed fluctuating source.* This curve was obtained from the width of the power spectrum $S(\omega)$ estimated as the squared modulus of the Fourier transform of a segment of the Stokes field strength $E_S(t)$ obtained by numerical integration of the SBS equations. We see that the Stokes spectrum narrows with increasing gain G less rapidly than the dependence predicted by the constant-pump theory.

X. CONCLUSIONS

We have developed a model that shows how SBS is initiated by spatially distributed thermal fluctuations of the density of the Brillouin medium. Under conditions near and below the threshold for SBS, pump depletion effects are negligible, and we have obtained an analytic expression showing how the SBS reflectivity depends on the single-pass gain G through the medium. From this formula, we obtain an explicit prediction for the value of G at the threshold. These predicted values are in good agreement with the empirically known values. We have also conducted numerical studies of the complete set of coupled equations that describe the SBS process including thermal initiation and pump depletion effect. We find that the output intensity is a fluctuating quantity and that the relative magnitude of these fluctuations is decreased through use of a large value of the single-pass gain. We have also compared the results of our model to those of the simpler model of Zel'dovich, Pilipetskii, and Shkunov (which ignores the distributed nature of the initiation), and we find that the two models are in good qualitative but not quantitative agreement.

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APPENDIX

In this appendix, we present a generalization of the localized source model of Zel'dovich, Pilipetskii, and Shkunov that allows the Stokes seed to be a stochastic quantity. Our starting point is the relation (29) between the reflectivity R and the single-pass gain G:

$$G=\frac{\ln R+G_{\text{ref}}}{1-R} , \quad \text{(A1)}$$

where

$$G_{\text{ref}}=-\ln f \quad \text{(A2)}$$

and where

$$f=I_s(L)/I_L(L) . \quad \text{(A3)}$$

Unlike in the original treatment of Zel'dovich, Pilipetskii, and Shkunov, we allow f to be a stochastic quantity. We assume that the probability distribution of f is that characteristic of a thermal source with mean value equal to f_0:

$$P(f)=\frac{1}{f_0}\exp(-f/f_0) . \quad \text{(A4)}$$

The fluctuating seed induces the fluctuations of the reflectivity R. The probability distribution of the reflectivity R is given by

$$Q(R)=\frac{1}{f_0}\exp[-f(R)/f_0]\left|\frac{df}{dR}\right|$$
$$=\frac{1}{f_0}\exp\left[\frac{-Re^{GR-G}}{f_0}\right](1+GR)e^{GR-G}\ . \qquad \text{(A5)}$$

Through use of this expression we can compute the expectation values of various functions of the reflectivity. The curves labeled *fluctuating localized source* in Figs. 3 and 5 were obtained in this manner.

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