Preventing laser beam filamentation through use of the squeezed vacuum

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We show that the tendency of a laser beam to break into a filamentary structure in passing through a nonlinear optical medium can be largely suppressed by proper tailoring of the vacuum field that interacts with the laser beam. [S1050-2947(99)51304-8]

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The tendency of a laser beam to break up into a filamentary structure as it passes through a laser gain medium or other optical material possessing a third-order optical nonlinearity is a serious problem that can hamper the use of high-power laser beams [1,2]. According to conventional models [3], laser beam filamentation occurs as the result of the growth of small perturbations to the laser wave front. This growth occurs as a consequence of parametric amplification associated with near-forward four-wave-mixing processes [4–6]. The perturbation that initiates filamentation is often a classical wave-front irregularity, although we recently demonstrated [7] that quantum-mechanical zero-point fluctuations can act as the perturbation that initiates filamentation. From this perspective, it appears that laser beam filamentation is a fundamental process, which can occur no matter how regular the laser wave fronts are from a classical perspective. In the present paper, we describe how even this vacuum contribution to the filamentation process can be suppressed by preparing the vacuum state of the electromagnetic field in the proper superposition state, which will minimize the growth of the perturbed solution. Previously, Maillotte *et al.* [8] showed how to reduce the tendency of laser beams to undergo filamentation by imposing transverse spatial structure on the laser beam, and Jain *et al.* [9] showed how to suppress self-action effects by preparing the material system in a quantum superposition state for which the nonlinear response vanishes.

Although the effect we have in mind is a quantum-field effect, some simplification can occur by first treating the classical version of the theory. We let A_0 designate the complex amplitude of a strong plane-wave laser beam, and A_1 and A_2 designate the amplitudes of weak side modes, which represent the perturbation that leads to filamentation. The fields are coupled by means of the four-wave-mixing process [10], illustrated symbolically in Fig. 1. We assume that the side modes propagate at the angle θ for which the gain of the four-wave-mixing process is maximum [4]. The weak-field amplitudes then obey the propagation equations

$$\frac{dA_1}{dz} = \kappa A_2^*, \quad \frac{dA_2}{dz} = \kappa A_1^*, \quad (1)$$

where

$$\kappa = \frac{6\pi i \omega}{nc} \chi^{(3)} A_0^2. \quad (2)$$

These equations can be solved simultaneously to find that

$$A_1(z) = \frac{1}{2}\left[A_1(0) + \frac{\kappa}{g} A_2^*(0)\right]e^{gz}$$
$$+ \frac{1}{2}\left[A_1(0) - \frac{\kappa}{g} A_2^*(0)\right]e^{-gz},$$

$$A_2^*(z) = \frac{1}{2}\left[A_2^*(0) + \frac{g}{\kappa} A_1(0)\right]e^{gz}$$
$$+ \frac{1}{2}\left[A_2^*(0) - \frac{g}{\kappa} A_1(0)\right]e^{-gz}, \quad (3)$$

where $g = |\kappa|$. Note that the exponentially growing part of the solution can be made to vanish if the weak fields at $z = 0$ have the proper amplitudes and phases so that $[A_1(0) + (\kappa/g)A_2^*(0)] = [A_2^*(0) + (g/\kappa)A_1(0)]$ vanishes. If, for simplicity, we choose our phase conventions so that A_0 is real, and assume that $\chi^{(3)}$ is also real (consistent with our assumption of a lossless medium), we see that $\kappa/g = i$, implying that there will be no exponentially growing solution if the weak-field amplitudes at the input to the interaction region are related by the condition

$$\frac{A_2^*(0)}{A_1(0)} = i. \quad (4)$$

We turn next to the quantum-mechanical version of the theory. There has been much previous work on the influence of quantum-mechanical fluctuations on nonlinear optical pro-
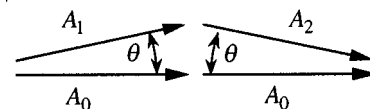

FIG. 1. Forward four-wave-mixing process, which is responsible for laser beam filamentation. The side-mode amplitudes A_1 and A_2 experience growth by means of four-photon parametric amplification.

cesses [11–20]. Here we examine the possibility of suppressing filamentation for the actual physical situation in which the electromagnetic field is required to obey the laws of quantum mechanics. Consequently, we treat the weak-field amplitudes as quantum-mechanical operators. Also, for generality, we allow the two side modes to be at different frequencies and we include the effects of wave-vector mismatch and of the nonlinear phase shift induced by the strong incident wave on each of the side modes. We begin by expressing the positive-frequency part of the total field as

$$\hat{E}^{(+)}(\mathbf{r},t) = [\mathcal{E}_0 + \hat{\mathcal{E}}_1(\mathbf{r},t)]e^{i\gamma_0 z}e^{ik_0 z - i\omega_0 t}. \tag{5}$$

Here we are using the same notation as in Ref. [7]. In particular, $\mathcal{E}_0$ denotes the amplitude of the strong pump field, which we treat classically and assume to have frequency ω_0 and wave vector $k_0 + \gamma_0$, where $k_0 = n_0\omega_0/c$ is its linear contribution, and $\gamma_0 = n_2 I_0\omega_0/c$ is its nonlinear contribution with $n_2 = (12\pi^2/n_0^2 c)\chi^{(3)}$ and $I_0 = (n_0 c/2\pi)|\mathcal{E}_0|^2$. The perturbation, which we treat as a quantum-mechanical operator, is denoted by $\hat{\mathcal{E}}_1$ and is conveniently decomposed in terms of its frequency components ω and transverse wave-vector components $\mathbf{q}$ as

$$\hat{\mathcal{E}}_1(\mathbf{r},t) = \int d^2 q \int_0^\infty d\omega N(\omega)\hat{a}(\mathbf{q},\omega;z)$$
$$\times e^{i\mathbf{q}\cdot\mathbf{r}}e^{i[k_z(\omega)-k_0]z - i(\omega-\omega_0)t}.$$

The mode amplitudes are denoted by $\hat{a}(\mathbf{q},\omega;z)$ and satisfy the usual commutation relation $[\hat{a}(\mathbf{q},\omega;z),\hat{a}^\dagger(\mathbf{q}',\omega';z)] = \delta^2(\mathbf{q}-\mathbf{q}')\delta(\omega-\omega')$. We have also introduced the mode normalization factor $N(\omega) = \sqrt{\hbar\omega^2 n^2(\omega)/4\pi^2 k_z(\omega)c^2}$.

We introduce next the field $\hat{E}^{(+)}(\mathbf{r},t)$ into the wave equation

$$\nabla^2\hat{E}^{(+)} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\hat{E}^{(+)} = \frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\hat{P}^{(+)}, \tag{6}$$

where $\hat{P}^{(+)}$ is the sum of the linear and nonlinear (i.e., $3\chi^{(3)}\hat{E}^{(-)}\hat{E}^{(+)}\hat{E}^{(+)}$) contributions of the material polarization. We linearize this equation in the perturbation and make the paraxial and slowly varying amplitude approximations. We thereby derive coupled amplitude equations that take on the form [7]

$$\frac{d\hat{a}}{dz} = i\beta\hat{a} + i\delta\hat{b}^\dagger e^{-i\Delta z},$$

$$\frac{d}{dz}(\hat{b}^\dagger e^{-i\Delta z}) = -i(\beta+\Delta)\hat{b}^\dagger e^{-i\Delta z} - i\delta\hat{a}, \tag{7}$$

where $\hat{a} = \hat{a}(\vec{q},\omega;z)$, $\hat{b}^\dagger = \hat{a}^\dagger(-\vec{q},2\omega_0-\omega;z)$, and where the coefficients that appear in these equations are given by

$$\beta = \frac{-1}{2[k_z(\omega)+\gamma_0]}$$
$$\times\left\{q^2 + [k_z(\omega)+\gamma_0]^2 - k^2(\omega) - 4k_0\gamma_0\left(\frac{\omega}{\omega_0}\right)^2\right\},$$

$$\delta = \frac{\gamma_0 k_0}{k_z(\omega)+\gamma_0}\left(\frac{\omega}{\omega_0}\right)^2,$$

and

$$\Delta = \Delta k(\omega) \equiv k_z(\omega) + k_z(2\omega_0-\omega) - 2k_0. \tag{8}$$

These equations can be solved straightforwardly, for example, through use of the Laplace transform method. One finds that the solution possesses parts varying as $\exp(\Lambda_+ z)$ and $\exp(\Lambda_- z)$, where

$$\Lambda_\pm = -\tfrac{1}{2}i\Delta \pm \sqrt{\delta^2 - (\beta+\tfrac{1}{2}\Delta)^2} \equiv -\tfrac{1}{2}i\Delta \pm \Lambda_0. \tag{9}$$

Note that (for a fixed value of δ) the gain factor Λ_+ is maximized by setting the phase mismatch Δ equal to 2β. Explicitly, the exponentially growing part of the solution for the side mode $\hat{a}$ is given by

$$\hat{a}(z) = (1/2\Lambda_0)\{[\Lambda_0 + i(\beta+\tfrac{1}{2}\Delta)]\hat{a}_0 + i\delta\hat{b}_0^\dagger\}\exp(\Lambda_+ z), \tag{10}$$

and thus the intensity associated with this part of the solution is given by

$$\langle\hat{a}^\dagger(z)\hat{a}(z)\rangle = [|\Lambda_0 + i(\beta+\tfrac{1}{2}\Delta)|^2\langle\hat{a}_0^\dagger\hat{a}_0\rangle + \delta^2\langle\hat{b}_0\hat{b}_0^\dagger\rangle$$
$$+ \{i\delta[\Lambda_0 - i(\beta+\tfrac{1}{2}\Delta)]\langle\hat{a}_0^\dagger\hat{b}_0^\dagger\rangle + \text{H.c.}\}]$$
$$\times\frac{\exp(2\Lambda_+ z)}{4\Lambda_0^2}. \tag{11}$$

Note that if the input fields are in the ordinary (i.e., nonsqueezed) vacuum state so that $\langle\hat{a}_0^\dagger\hat{a}_0\rangle = 0$, $\langle\hat{a}_0^\dagger\hat{b}_0^\dagger\rangle = 0$, and $\langle\hat{b}_0\hat{b}_0^\dagger\rangle = 1$, the generated intensity becomes

$$\langle\hat{a}^\dagger(z)\hat{a}(z)\rangle = \frac{\delta^2}{4\Lambda_0^2}\exp(2\Lambda_0 z). \tag{12}$$

Let us now examine whether it is possible to suppress this growing solution by preparing input fields in the proper quantum-mechanical superposition state. We first note that if we treat Eq. (10) as a classical equation, the condition for suppressing filamentation is

$$(\tfrac{1}{2}i\Delta + i\beta + \Lambda_0)\langle a\rangle + i\delta\langle b^\dagger\rangle = 0 \tag{13}$$

or

$$\frac{\langle a\rangle}{\langle b^\dagger\rangle} = \frac{-\delta}{\tfrac{1}{2}\Delta + \beta - i\Lambda_0}, \tag{14}$$

so that [through the use of Eq. (9)]

$$\left|\frac{\langle a\rangle}{\langle b^\dagger\rangle}\right| = 1. \tag{15}$$

We examine next the quantum-mechanical conditions that must be satisfied in order to suppress filamentation. We assume that the phases of the input side-mode fields have been adjusted so that the cross term in Eq. (11) is real and negative, so that the intensity can be expressed as
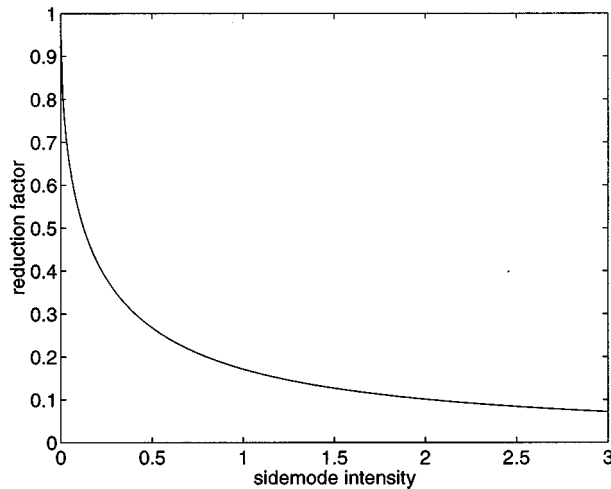
FIG. 2. Dependence of the filamentation reduction factor $\mathcal{R}$ on the side-mode intensity $\langle \hat{a}_0^\dagger \hat{a}_0 \rangle$ under the conditions specified in the text.

$$\langle \hat{a}^\dagger(z)\hat{a}(z) \rangle = \frac{\delta^2}{4\Lambda_0^2} [\langle \hat{a}_0^\dagger \hat{a}_0 \rangle + \langle \hat{b}_0 \hat{b}_0^\dagger \rangle$$
$$-2|\langle \hat{a}_0 \hat{b}_0 \rangle|]\exp(2\Lambda_0 z). \qquad (16)$$

If the input fields are in a perfectly squeezed state in the sense that

$$\langle \hat{a}_0^\dagger \hat{a}_0 \rangle = \langle \hat{b}_0^\dagger \hat{b}_0 \rangle \quad \text{and} \quad |\langle \hat{a}_0 \hat{b}_0 \rangle| = \sqrt{\langle \hat{a}_0^\dagger \hat{a}_0 \rangle \langle \hat{b}_0 \hat{b}_0^\dagger \rangle}, \tag{17}$$

we find that

$$\langle \hat{a}^\dagger(z)\hat{a}(z) \rangle = \frac{\delta^2}{4\Lambda_0^2} [\sqrt{\langle \hat{a}_0^\dagger \hat{a}_0 \rangle} - \sqrt{1 + \langle \hat{a}_0^\dagger \hat{a}_0 \rangle}]^2 \exp(2\Lambda_0 z). \tag{18}$$

By comparison with Eq. (12) we see that the intensity of the generated light can be smaller in this case by a factor $\mathcal{R}$ given by

$$\mathcal{R} = [\sqrt{\langle \hat{a}_0^\dagger \hat{a}_0 \rangle} - \sqrt{1 + \langle \hat{a}_0^\dagger \hat{a}_0 \rangle}]^2. \tag{19}$$

The variation of the reduction factor $\mathcal{R}$ with the side-mode intensity $\langle \hat{a}_0^\dagger \hat{a}_0 \rangle$ is plotted in Fig. 2. If the input fields are in the vacuum state so that $\langle \hat{a}_0^\dagger \hat{a}_0 \rangle = \langle \hat{b}_0^\dagger \hat{b}_0 \rangle = 0$, we find from Eq. (19) that $\mathcal{R} = 1$. However, for populated input modes there can be appreciable reduction of the intensity of the generated light. In fact, for $\langle \hat{a}_0^\dagger \hat{a}_0 \rangle$ large, we find that $\mathcal{R}$ takes on the asymptotic form $\mathcal{R} = 1/4\langle \hat{a}_0^\dagger \hat{a}_0 \rangle$.

In summary, we have shown that the tendency of laser beams to undergo filamentation can be suppressed by preparing the spatial side modes of the incident laser field so that they satisfy the condition expressed by Eq. (17). Fields possessing quantum correlations of this sort can be generated by various nonlinear optical interactions including four-wave-mixing processes, although it is not clear at present how to generate fields that obey this relation over a broad range of frequencies and propagation directions. However, it should be possible to satisfy this condition over some range of wave vectors that spans the region of maximum parametric gain, leading to an appreciable reduction in the threshold for the filamentation process. The calculation presented here considers only the expectation value of the intensity of the generated field. Consideration of the fluctuations in the intensity, resulting from quantum effects and their influence on the threshold for filamentation, remains an interesting unanswered question and a possible direction for further work.

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