International Journal of Nonlinear Optical Physics
Vol. 1, No. 3 (1992) 581–594

STIMULATED BRILLOUIN SCATTERING IN THE PRESENCE OF EXTERNAL FEEDBACK

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Received 18 June 1991

We investigate theoretically and experimentally the process of stimulated Brillouin scattering (SBS) in the presence of weak feedback in a single-mode optical fiber. The threshold for the occurrence of Brillouin oscillation can be considerably lower than the threshold for normal SBS. The spectrum of the emitted Stokes light exhibits an extreme narrowing near and above the oscillation threshold. The Stokes intensity is found to exhibit various types of periodic and stable behavior.

In stimulated Brillouin scattering (SBS), strong coupling between a laser and Stokes field leads to gain for the Stokes field. By enclosing the Brillouin medium within an optical cavity, Brillouin oscillation can occur if the round-trip gain for the Stokes wave exceeds its losses. Optical fibers are well suited to the study of Brillouin oscillation, because SBS can be generated under continuous-wave (CW) excitation with milliwatt of laser power.[1] Several workers [2–5] have studied Brillouin oscillation from optical fibers in the presence of weak external feedback. Bar-Joseph et al.[2] and Dianov et al.[3] have found that the output Stokes intensity could become temporally unstable and exhibit steady oscillations. Botineau et al.[4] and Johnstone et al.[5] include the nonlinear Kerr effect in their theoretical models and find further evidence of temporally unstable behavior. For the case in which the Brillouin oscillator is operated in a ring configuration with large external feedback (e.g. >50%), the oscillator can be termed a Brillouin fiber ring laser.[6] The output Stokes wave from a Brillouin laser can have a spectral width as narrow as 2 kHz,[7] and can exhibit soliton-like behavior.[8]

In this paper, we describe the results of our investigation of the behavior of SBS in the presence of weak feedback. In Sec. 1, the threshold for Brillouin oscillation is calculated. We find that, even for the case in which the feedback is weak, the threshold can still be considerably lower than the threshold for usual SBS. In addition, the predicted spectrum of the Stokes light undergoes extreme narrowing at the threshold for Brillouin oscillation. In Sec. 2, theoretical results are presented that show the temporal behavior of the Stokes intensity when the system is above the Brillouin-oscillation threshold. The evolution of the output Stokes intensity is found to exhibit both stable and oscillatory behavior. Experimental studies of SBS

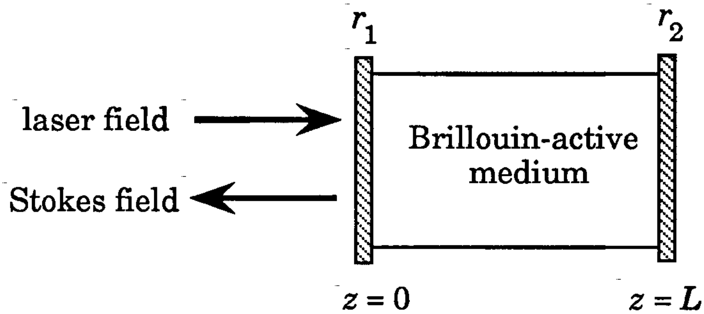

Fig. 1. Schematic illustration of the geometry for SBS in the presence of reflecting boundaries; r_1 and r_2 are the amplitude reflectivities.

in an optical fiber with weak feedback are described in Sec. 3. Brillouin oscillation is found to occur at a threshold considerably lower than that for normal SBS. The spectrum is observed to undergo an extreme narrowing and becomes much narrower than the gain-narrowed linewidth associated with usual SBS. In Sec. 4, experimental observations of the temporal evolution of the Stokes output intensity are presented. The behavior is found to drift between a wide range of oscillatory, behavior and a nonfluctuating stable output.

1. Spectrum and Threshold of SBS with External Feedback

The geometry of the interaction that we have studied is shown in Fig. 1, where r_1 and r_2 are the amplitude reflectivities of the boundaries at $z=0$ and $z=L$, respectively. The boundary at $z=L$ reflects the laser field to form a counterpropagating wave traveling in the -z direction. Under these conditions, the generation of a field component at the antiStokes frequency is possible due to a four-wave mixing process between the Stokes field and forward- and backward-traveling laser fields.[9] However, in the present treatment the generation of the antiStokes field is ignored. This assumption is valid as long as the intensity of the backward-traveling laser field is low, or as long as the length of the Brillouin-active medium is sufficiently large that the total phase mismatch associated with the four-wave mixing process (i.e. $\Delta kL = 2n\Omega L/c$, where Ω is the Brillouin frequency) is much greater than unity.[10] We also assume that the reflectivities of the boundaries are much smaller than unity, and for this reason the Brillouin interaction between the backward-traveling laser field and the forward-traveling Stokes field is ignored. Thus, we use the following equations to describe the interaction among the forward-traveling laser field $E_l(z, t)$, the backward-traveling Stokes field $E_s^b(z, t)$, and the material density $\rho(z, t)$[11]:

$$\frac{\partial E_l}{\partial z} + \frac{n}{c}\frac{\partial E_l}{\partial t} = -\alpha E_l + i\kappa\rho E_s^b, \tag{1a}$$

$$\frac{\partial E_s^b}{\partial z} - \frac{n}{c}\frac{\partial E_s^b}{\partial t} = \alpha E_s^b - i\kappa\rho^* E_l ,$$

$$\frac{\partial \rho}{\partial t} + \frac{\Gamma}{2}\rho = i\Lambda E_l E_s^{b*} + f , \qquad \text{(1c)}$$

where α is the linear absorption coefficient, γ is the nonlinear index parameter, Γ is the phonon decay rate, κ and Λ are Brillouin coupling constants, and the Langevin noise source $f(z, t)$ describes the thermal fluctuations in the density of the medium that lead to spontaneous Brillouin scattering. We have neglected the nonlinear index change experienced by the Stokes field and the laser field since in an optical fiber the Kerr effect is typically less important than the Brillouin effect for continuous-wave fields[12] (i.e. $n_2 k/g_0 \approx 0.015$ where n_2 is the nonlinear index coefficient, k is the wave vector amplitude of the laser field, and g_0 is the SBS gain factor).

The spectrum of the output Stokes field (i.e. at $z=0$) is derived using the equations for the Stokes field [Eqs. (1b) and (1d)] and the acoustic density [Eq. (1c)] in the limit of an undepleted laser field ($E_l(z)=E_l(0)$). The details of the derivation are given in the Appendix, and the power spectrum $\langle F_s^{b*}(0, \delta)\, F_s^b(0, \delta)\rangle$ of the backward-traveling Stokes field is found to be

$$\langle F_s^{b*}(0, \delta)\, F_s^b(0, \delta)\rangle = \frac{4\pi k_B T}{n v A}\,\frac{\left[e^{|\mathcal{L}|^2 G_{\text{eff}}} - 1\right]\left[\dfrac{2\alpha L_{\text{eff}}}{\mathcal{L} G_{\text{eff}}} + e^{-2\alpha L}\right]}{\left|1 - r_1 r_2 \exp\left[\dfrac{1}{2}\mathcal{L} G_{\text{eff}} - 2\alpha L + i(\phi - 2\,\delta T_t)\right]\right|^2} \qquad (2)$$

where $G_{\text{eff}}=g_0 I_l L_{\text{eff}}$ is the effective single-pass Brillouin gain due to the forward-traveling laser field inside the cavity, $g_0=32\pi\kappa\Lambda/nc\Gamma$ is the SBS gain factor, $I_l=(nc/8\pi)|E_l(0)|^2$ is the input laser intensity, $L_{\text{eff}} = [1-\exp(-2\alpha L)]$ is the effective interaction length, $T_t=nL/c$ is the transit time of light through the medium, ϕ is the relative detuning of the nearest longitudinal mode of the cavity from the center of the Brillouin spectral line (e.g. $\phi = \pm\pi$ corresponds to the situation in which the peak of the spontaneous Brillouin spectrum lies exactly between two cavity modes), $\mathcal{L}=(1-i2\delta/\Gamma)^{-1}$ and $g=\mathcal{L}g_0$. In the limit where α, r_1, $r_2 \to 0$, the spectrum reduces to the gain-narrowed spectrum associated with usual SBS. The threshold for Brillouin-oscillation, which is an absolute instability, occurs when the denominator of the right-hand side of Eq. (2) vanishes. By setting both the real and the imaginary parts of the term within the absolute-value brackets equal to zero, we find that the threshold gain and the oscillation frequency are given by the expressions

$$G_{\text{eff}}^{\text{th}} = [4\alpha L - 2\ln(r_1 r_2)]\left\{1 + \left[\frac{\phi}{\Gamma T_t - 2\alpha L + \ln(r_1 r_2)}\right]^2\right\},$$

and

$$\frac{\delta_{\rm th}}{\Gamma} = -\frac{\phi}{2[\Gamma T_t - 2\alpha L + \ln(r_1 r_2)]} \tag{3b}$$

For the case in which the cavity is sufficiently long that many modes of the cavity lie beneath the Brillouin gain curve ($\Gamma T_t \gg 1$), the oscillation frequency occurs roughly at the peak of the Brillouin line ($\delta_{\rm th} \sim 0$). The threshold laser intensity is then given by

$$G_{\rm eff}^{\rm th} = 4\alpha L - 2\ln(r_1 r_2) \tag{4}$$

Thus, even for small values of the reflectivity product (e.g. $|r_1 r_2|^2 = 10^{-4}$) and small absorption, the threshold for Brillouin oscillation ($G_{\rm eff}^{\rm th} \approx 9$) can be considerably smaller than the threshold value of $G_{\rm eff}^{\rm th} = 20$ for usual single-beam SBS.[11]

When the term within the absolute-value brackets in the expression for the Stokes spectrum [Eq. (2)] becomes small, the output spectrum can become much narrower than either the gain-narrowed spectrum or the empty-cavity lineshape. The origin of this extreme spectral narrowing is that the new modes of the cavity in the presence of Stokes gain, resemble a cavity whose reflectivity product $r_1 r_2$ is enhanced by the factor $\sim \exp[G_{\rm eff}/2]$ over its empty cavity value of $r_1 r_2$. Thus, even in the limit of very small feedback ($r_1 r_2 \ll 1$), even a modest amount of gain is sufficient to lead to a considerable narrowing of the Stokes spectrum. Figure 2 shows plots of the Stokes spectrum for various values of $G_{\rm eff}$ for the cases $\Gamma T_t = 5$ and $\Gamma T_t = 100$ and no absorption ($\alpha L = 0$). In both cases, the values of the reflectivities are taken to be $|r_1|^2 = |r_2|^2 = 0.04$ and the peak of the spontaneous spectrum is assumed to lie at a cavity resonance such that $\phi = 0$. The spontaneous Brillouin spectrum is also shown for comparison. For a value of $G_{\rm eff} = 3$, the spectrum for both values of ΓT_t possess nearly the same linewidth, but for $\Gamma T_t = 100$, the effect of the cavity on the lineshape is more pronounced as a result of peaks at the cavity-mode frequencies. For a value of $G_{\rm eff}$ close to the threshold for Brillouin oscillation ($G_{\rm eff}^{\rm th} = 6.43$), the spectrum for $\Gamma T_t = 5$ has narrowed considerably, but the narrowing is even more dramatic for the case $\Gamma T_t = 100$.

2. Temporal Evolution of the Stokes Output Intensity

In order to determine the temporal behavior of the Stokes field for input intensities above the threshold for Brillouin oscillation, the full set of stochastic differential equations [Eq. (1)] for the laser field, Stokes field and the density are numerically integrated with the appropriate boundary conditions. Figures 3 and 4 show the temporal evolution of the Stokes intensity for various values of $G_{\rm eff}$ for the cases $\Gamma T_t = 5$ and 100, respectively. In all cases, the output is shown for times such that the transients associated with the turn-on of the laser field are no longer

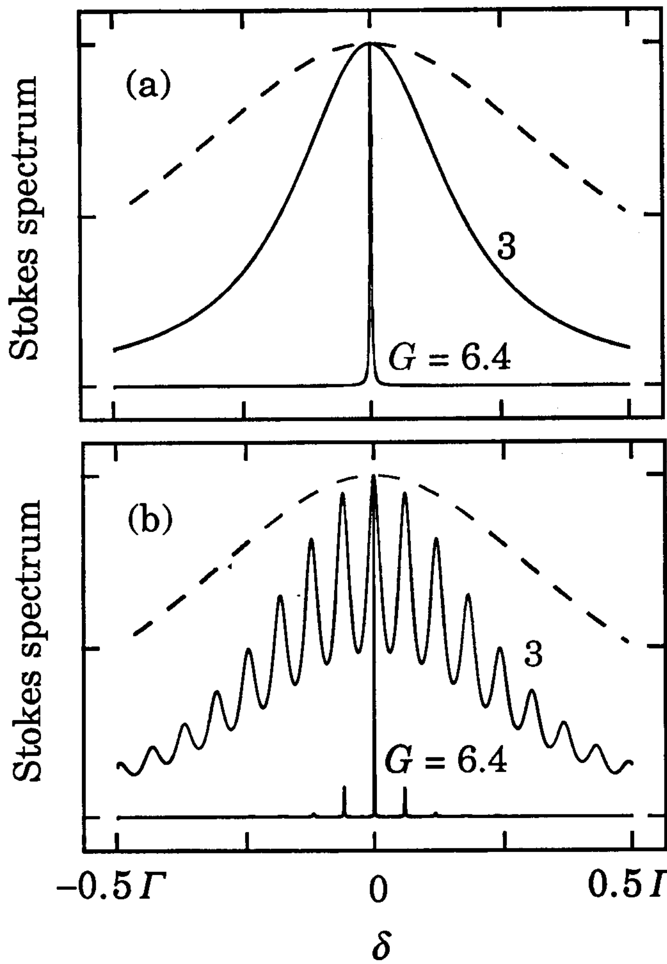

Fig. 2. Plots of the Stokes spectrum in the presence of feedback ($|r_1|^2=|r_1|^2=0.04$) for two different values of G for the case of (a) a relatively short medium with $\Gamma T_t=5$, and (b) a long medium with $\Gamma T_t=100$. The dashed curve in each case represents the spontaneous Brillouin spectrum.

present. For an input laser intensity ($G_{\text{eff}}=4$) below the threshold value for Brillouin oscillation, the Stokes intensity fluctuates in a stochastic fashion and remains at a small value for both cases of $\Gamma T_t=5$ and $\Gamma T_t=100$. For a value of $G_{\text{eff}}=8$, which is just above the threshold for Brillouin oscillation, the Stokes output undergoes a transition to an extremely stable state for the case $\Gamma T_t=5$ with an SBS reflectivity greater than 15%. However, for $\Gamma T_t=100$, the output intensity self-pulses with a period equal to the round-trip time ($2T_t$) of the cavity. This temporally unstable deterministic behavior corresponds to the behavior predicted by Bar-Joseph et al.[2] through a stability analysis of the Stokes intensity. At higher input intensities ($G_{\text{eff}}=25$), the output intensity for the case $\Gamma T_t=100$ becomes stable.

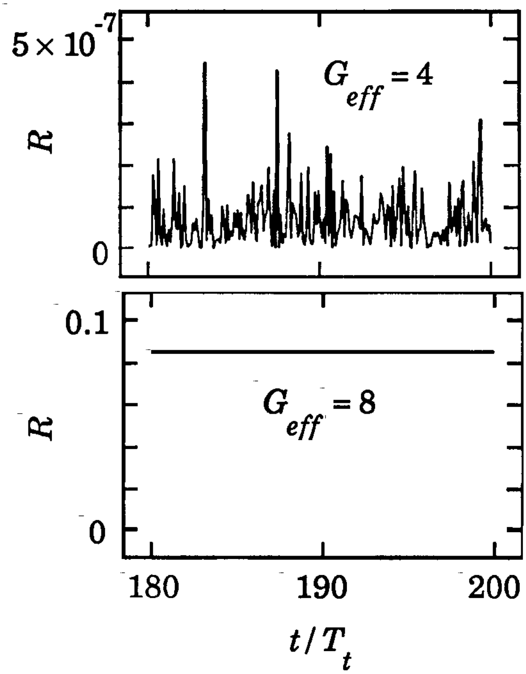

Fig. 3. Theoretically predicted SBS reflectivity R as a function of time below and above the threshold for Brillouin oscillation ($G_{\text{eff}}^{\text{th}} = 6.4$) for the case $\Gamma T_t = 5$, and $|r_1|^2 = |r_2|^2 = 0.04$. Above Brillouin oscillation threshold the Stokes output is stable.

3. Experimental Measurements of the Threshold for Brillouin Oscillation and the Stokes Output Spectrum

As predicted in Sec. 1, the threshold for SBS in the presence of feedback can be lower than the threshold for usual single-beam SBS. Our experimental setup for measuring the backscattered Stokes power as a function of input laser power from a single-mode argon-ion laser ($\lambda_l = 0.5145\,\mu$m) is shown in Fig. 5. The fiber length is 100 m and the absorption in the fiber is $\alpha L = 0.22$. Figure 6 shows plots of the SBS reflectivity as a function of G_{eff} both in the presence and in the absence of feedback. Reflections from the glass-air interface at the fiber ends provided the weak feedback ($\sim 4\%$) necessary to observe Brillouin oscillation. The conditions for normal SBS (i.e. without feedback) were created by cleaving the fiber ends at an oblique angle and placing the back end of the fiber into index-matching fluid. The experimentally measured value of the single-pass gain G_{eff} is determined by utilizing the following expression:

$$G_{\text{eff}} = 0.7 g_o P_l L_{\text{eff}} / A \,, \tag{5}$$

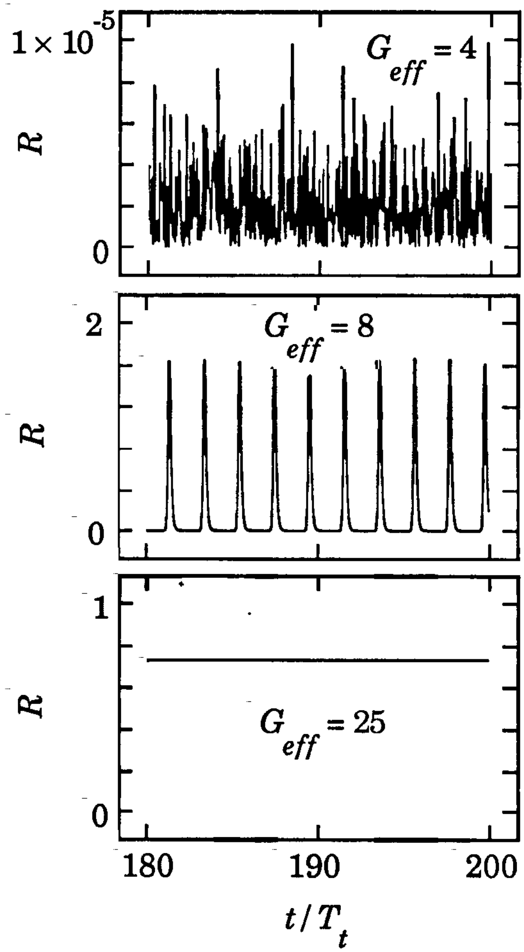

Fig. 4. Theoretically predicted SBS reflectivity R as a function of time for various values of $G_{\rm eff}$ for the case $\Gamma T_t = 100$, and $|r_1|^2 = |r_2|^2 = 0.04$. Oscillatory behavior is predicted in the regime just above Brillouin oscillation threshold, and at higher values of $G_{\rm eff}$ the output becomes stable.

where P_l is the input laser power inside the front end of the fiber, $g_0 = 2.5 \times 10^{11}$ m/W is the experimentally measured value of the gain factor for the fiber,[13] $A = 1 \times 10^{-11}$ m^2 is the area of the fiber core, and $L_{\rm eff} = 81$ m is the effective interaction length. Since the output Stokes light contained approximately 70% of its power in the polarization direction parallel to that of the input laser light, a factor of 0.7 is included to account for the depolarization inside the fiber.[12] The threshold value of $G_{\rm eff}$ (i.e. the value of $G_{\rm eff}$ at which the SBS reflectivity reaches 1%) for the case of no feedback is $G_{\rm eff}^{\rm th} = 12.4$ and in the presence of feedback from ends of the fiber is $G_{\rm eff}^{\rm th} = 7.7$. The lower value for $G_{\rm eff}^{\rm th}$ in the presence of feedback suggests that feedback from the ends of the fiber could be playing a role in the dynamics of the

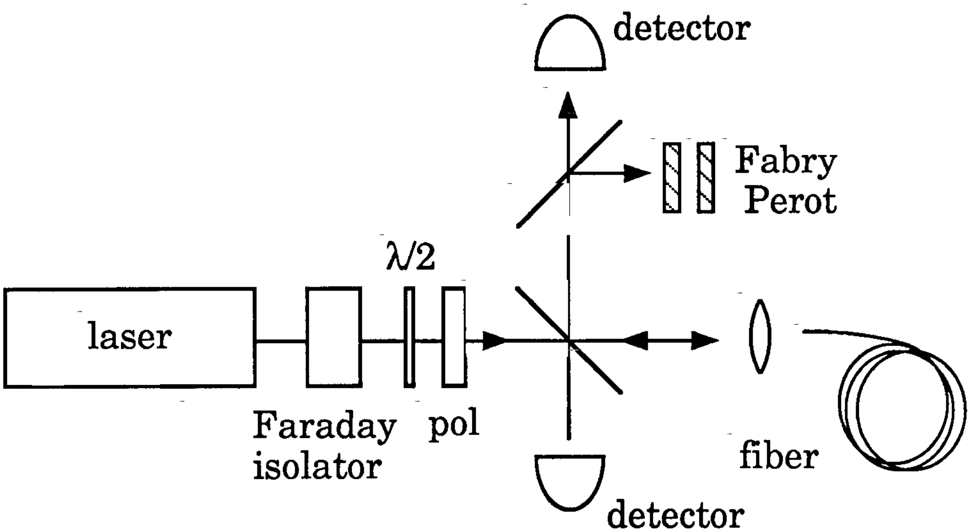

Fig. 5. Experimental setup used to study SBS in an optical fiber.

Stokes light. The prediction for the threshold value of G_{eff} for $|r_1|^2 = |r_2|^2 = 0.04$ [Eq. (4)] is equal to 7.3 which agrees reasonably well with the experimentally observed value.

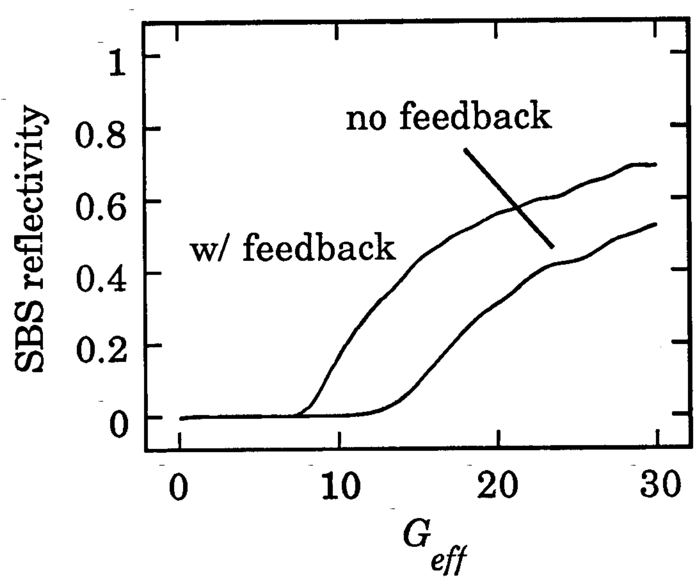

Fig. 6. Experimental results for the SBS reflectivity both in the presence and in the absence of feedback plotted as a function of the input laser power for the 100-m-long fiber.

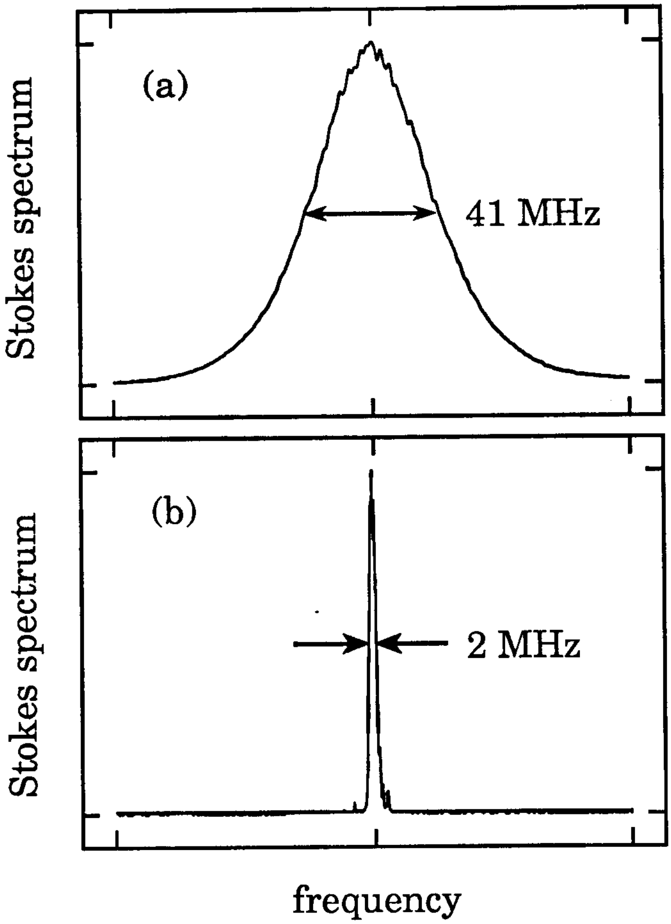

Fig. 7. Spectrum of the Stokes light for the 100-m-long fiber (a) just below the threshold for Brillouin oscillation ($G_{eff}=6$), and (b) just above the threshold for Brillouin oscillation ($G_{eff}=7.8$).

The spectrum of the Stokes light was measured with a Coherent 699-21 frequency-stabilized (<2 MHz linewidth) dye laser operating at a wavelength of 0.57 μm. A "super-cavity" scanning Fabry–Pérot interferometer with a spectral resolution of 500 kHz was used to monitor the Stokes spectrum. Figure 7a is a plot of the Stokes spectrum for an input laser power that corresponds to a value of $G_{eff}=7.1$. The measured spectral width of 41 MHz is approximately equal to the gain-narrowed linewidth that is measured in the absence of feedback from the fiber endfaces. However, at slightly higher input laser powers the spectrum suddenly collapses with a width approximately equal to that of the laser spectrum. Figure 7b is a plot of the Stokes spectrum for a laser power equal to 48 mW ($G_{eff}=9$) which is just above the threshold for Brillouin oscillation.

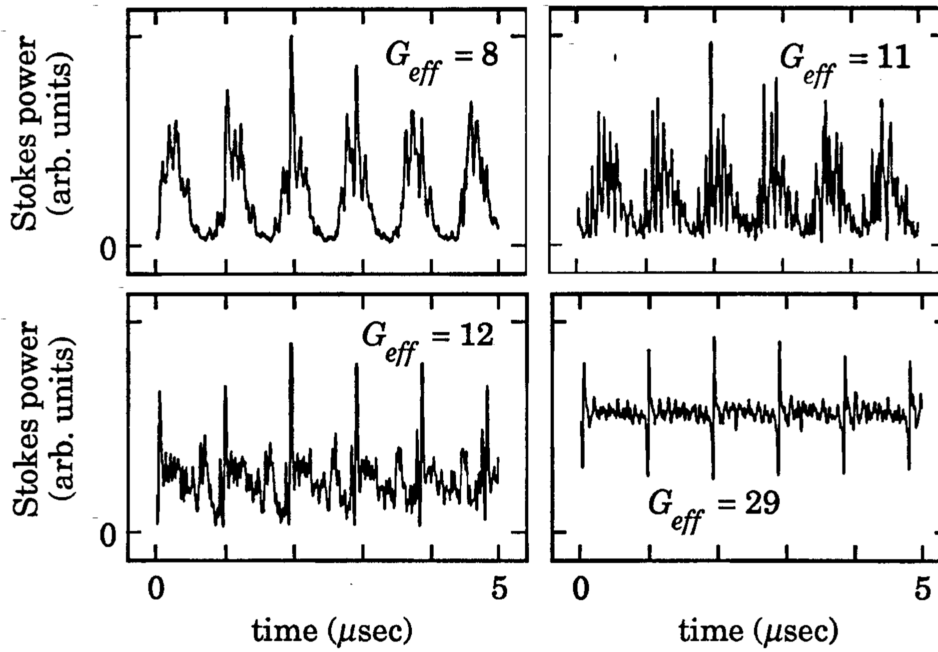

Fig. 8. Experimentally measured temporal evolution of the Stokes output power for several values of the input laser power. In all cases, the basic period of the self-pulsing is equal to the round-trip time (1 μsec) of the fiber.

4. Experimental Results for the Temporal Evolution of Stokes Output in the Presence of Feedback

To monitor the temporal evolution of the Stokes output power in the presence of feedback an argon-ion laser ($\lambda_l = 0.5145$ μm) was used. As predicted in the Sec. 2, the temporal evolution of the Stokes output above the threshold for Brillouin oscillation takes on a deterministic nature. However, for a fixed input laser power, the output exhibited transitions between different forms of periodic pulsations and intervals of stable output consisting of almost no fluctuations. The duration of a particular type of behavior generally lasted no more than 1 msec, and the transitions occurred in a time equal to roughly 5 μsec.

The period of oscillation that was most commonly observed was equal to the round-trip time ($2T_t = 1$ μsec) through the fiber. Figure 8 shows several examples of the sorts of oscillations that were observed having a 1 μsec period for various input laser powers. Generally, oscillations with this period were easier to achieve at laser powers close to Brillouin oscillation threshold power (~50 mW) than at the higher powers. Oscillations at higher harmonics of the fundamental oscillation frequency ($1/2T_t$) were also observed. Figures 9a–9e show the temporal evolution of the Stokes output power for the cases in which it exhibits oscillations at the second, third, fourth, fifth and sixth harmonic of the fundamental oscillation frequency,

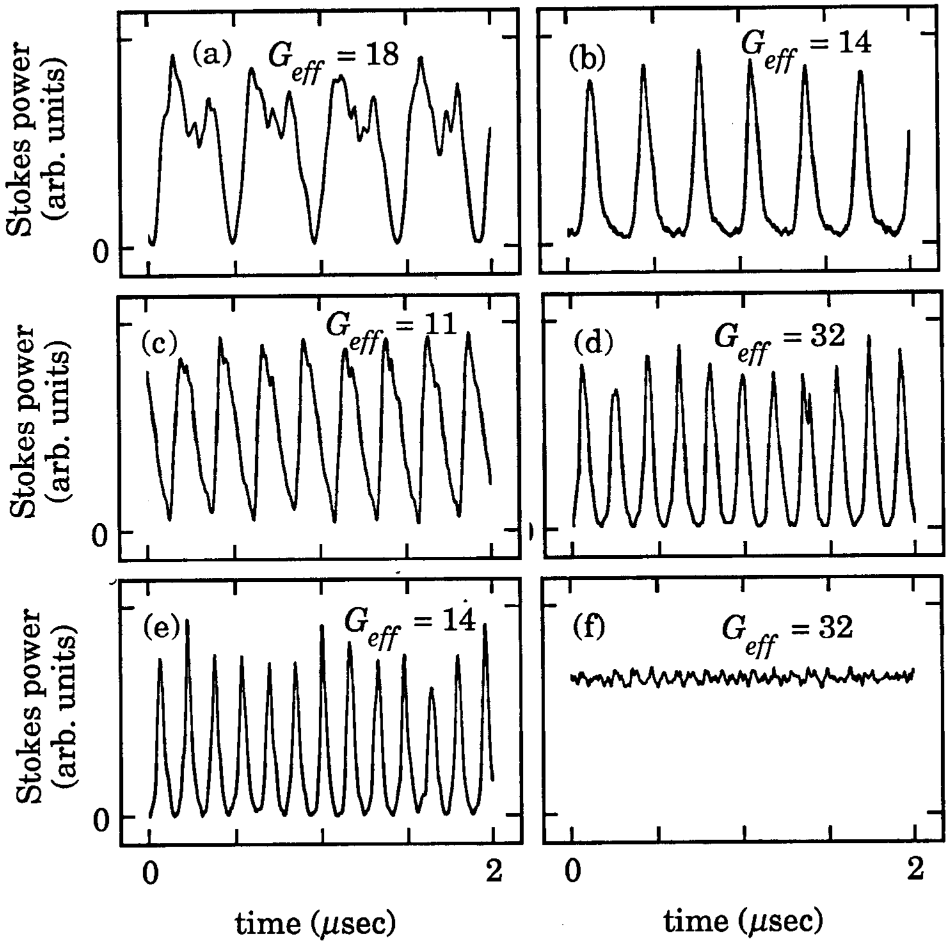

Fig. 9. Experimentally measured temporal evolution of the Stokes output power for several values of the input laser powers. Plots (a)–(e) show evidence of second, third, fourth, fifth and sixth harmonics, respectively, of the fundamental oscillation frequency $(1/2T_t)$. Plot (f) demonstrates stable Stokes output.

respectively. In Fig. 9f, the Stokes output is seen to be relatively stable, as would be expected from theoretical simulations shown in Sec. 2.

We were also able to observe (Fig. 10) the transition from the stochastic behavior associated with normal SBS to the deterministic evolution of Brillouin oscillation which is predicted in our theoretical simulations (Fig. 4). By index-matching the back end of the fiber, the threshold for normal SBS was lower than the threshold for Brillouin oscillation. Thus, the stochastic behavior associated with normal SBS is observed (Fig. 10a) for input laser intensity ($G_{\text{eff}}=14$) in the intermediate regime, and self-pulsing at the fundamental oscillation frequency $(1/2T_t)$ is observed (Fig. 10b) for an input intensity ($G_{\text{eff}}=18$) slightly above the Brillouin oscillation threshold ($G_{\text{eff}}=16$).

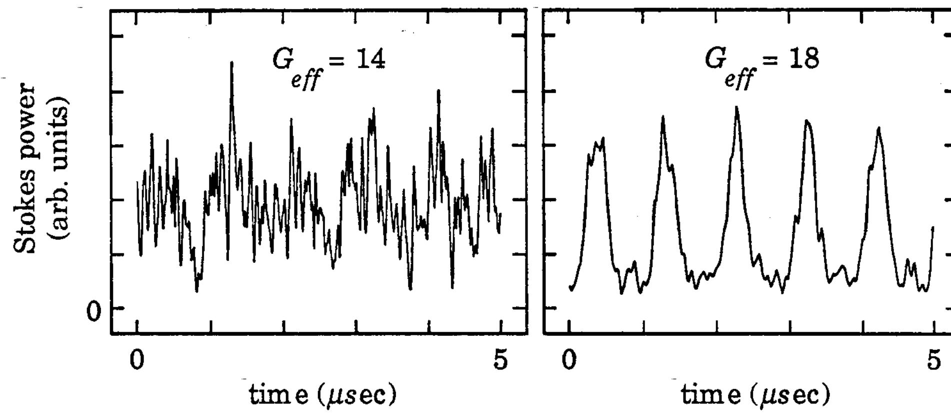

Fig. 10. Experimentally measured temporal evolution of the Stokes output power for two different values of the input laser power, which show the transition from the stochastic behavior of normal SBS to the periodic pulsation of Brillouin oscillation.

The theoretical model presented here does not predict the occurrence of transitions among different types of periodic behavior, nor does it predict the appearance of oscillatory behavior at harmonics of the fundamental oscillation frequency ($1/2T_t$). A mechanism that may contribute to the observed behavior is fluctuations in the optical pathlength of the fiber due to changes in the ambient temperature. This effect could lead to a relatively slow modification of the mode structure of the Brillouin oscillator within a time interval of a few milliseconds.

In summary, we have investigated the behavior of SBS in the presence of weak external feedback. The threshold for Brillouin oscillation can be significantly lower than the threshold for normal SBS, and above this threshold the Stokes spectrum is found to undergo extreme narrowing. Under conditions of Brillouin oscillation, the Stokes output intensity exhibits a wide range of dynamical behavior.

5. Acknowledgments

We acknowledge useful discussions with P. Narum and K. Rzazewski on the theoretical aspects of this work. We are also grateful to A. Stentz for his assistance with the experimental measurements and to D. Hall for information concerning the optical properties of the Corning optical fibers. This work was supported by the sponsors of the New York State Center for Advanced Optical Technology and by the Army Research Office University Research Initiative.

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Appendix. Spectrum of the Stokes Light in the Presence of Feedback

We derive the output Stokes spectrum in the limit in which the laser field is undepleted by the Brillouin interaction and Eqs. (1) are linearized in the Stokes field. In this limit, Eqs. (1) reduce to the following equations:

$$E_l(z) = E_l(0)\, e^{-\alpha z}$$

$$\frac{\partial E_s}{\partial z} - \frac{n}{c}\frac{\partial E_s}{\partial t} = \alpha E_s - i\kappa\rho^* E_l\,,$$

and

$$\frac{\partial \rho}{\partial t} + \frac{\Gamma}{2}\rho = i\Lambda E_l E_s^* + f \qquad \text{(A1c)}$$

Equations (1b) and (1c) are Fourier transformed in time, and the spatial evolution for the temporal Fourier transform of the Stokes field $F_s(z, \delta)$ is given by

$$\frac{dF_s}{dz} = \left(\alpha + \frac{in\delta}{c} + \frac{g}{2} I_l e^{-2\alpha z}\right) F_s + i\sigma E_l(z)\tilde{f}^*\,,$$

where $I_l = (nc/8\pi)|E_l(0)|^2$ is the input intensity of the laser field, $g = \mathcal{L} g_0$, $g_0 = 32\pi\kappa\Lambda/nc\Gamma$, $\mathcal{L} = (1\ \ i2\delta/\Gamma)^{-1}$, $\sigma = 2\mathcal{L}\kappa/\Gamma$, and $\tilde{f}^*$ is the temporal Fourier transform of f^*. Since no Stokes field is injected into the cavity, the homogeneous solution to Eqs. (A2) vanishes. The particular solution to Eq. (A2) is

$$F_s(z,\delta) = C(z,\delta)\exp\left[\left(\alpha + \frac{in\delta}{c}\right)z + \frac{g}{2}I_l z_{\text{eff}}\right]$$

where $z_{\text{eff}} = [1 - \exp(-2\alpha z)]/2\alpha$ and $C(z, \delta)$ is given by

$$C(z,\delta) = C(L,\delta) - i\mathcal{L}\kappa\int_0^L dz'\,\tilde{f}(z')E_l(z')\exp\left[\left(\alpha + \frac{in\delta}{c}\right)z' + \frac{gI_l}{4\alpha}(1 - e^{-2\alpha z'})\right] \tag{A4}$$

The boundary condition on the total Stokes field leads to the following relation:

$$F_s(L,\delta) = r_1 r_2 F_s(0,\delta)e^{-\alpha L + i2k_s L}$$

where k_s is the Stokes wave vector, and where we have taken into account the absorption experienced by the reflected Stokes field in traveling from $z = 0$ to $z = L$.

The power spectrum of the Stokes field at $z = 0$ is then given by

$$\langle F_s^*(0,\delta)F_s(0,\delta)\rangle = \langle C^*(0,\delta)C(0,\delta)\rangle$$

The expression for $C(0, \delta)$ is found by applying the boundary conditions [Eqs. (A5)] to Eqs. (A3) and (A4). We assume that $f(z, t)$ is a Gaussian random process with zero mean and is delta correlated in space and time in the sense that $\langle f(z,t) f^*(z',t')\rangle = Q\delta(z - z')\delta(t - t')$, where $Q = 2k_B T\rho_0\Gamma/v^2 A$, k_B is Boltzman's constant, T is the temperature, ρ_0 is the mean density of the material, v is the velocity of sound in the material, and A is the cross-sectional area of the interaction region. The resulting expression for $C(0, \delta)$ is then substituted into Eq. (A6) to yield the power spectrum

$$\langle F_s^*(0,\delta)F_s(0,\delta)\rangle = \frac{4\pi k_B T}{nvA}\,\frac{\left[e^{|\mathcal{L}|^2 G_{\text{eff}}} - 1\right]\left[\dfrac{2\alpha L_{\text{eff}}}{\mathcal{L}G_{\text{eff}}} + e^{-2\alpha L}\right]}{\left|1 - r_1 r_2\exp\left[\dfrac{1}{2}\mathcal{L}G_{\text{eff}} - 2\alpha L + i(\phi - 2\delta T_t)\right]\right|^2}$$

where $G_{\text{eff}} = g_0 I_l L_{\text{eff}}$ is the effective single-pass Stokes gain, $L_{\text{eff}} = (1 - e^{-2\alpha L})/2\alpha$ is the effective interaction length, and $\phi = (\omega_s - 2\pi\nu_c)T_t$ represents the relative detuning of the Stokes frequency from the nearest cavity-mode frequency ν_c (i.e. $\pi < \phi \leq \pi$).