Quantum-noise limit on optical amplification by two-beam coupling in an atomic vapor

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We investigate theoretically the noise properties of the amplification of a weak probe beam in an atomic vapor pumped by an intense nearly resonant pump field. For the case of gain via the three-photon effect, we find that only in the limit in which collisional broadening is absent can the system operate as an ideal quantum-noise limited optical amplifier for the probe wave. For the case of gain via stimulated Rayleight scattering, we find that the minimum noise figure is four times that of an ideal optical amplifier, and that it occurs when the atomic system is predominantly collisionally broadened.

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It is well known that even an ideal linear optical amplifier must introduce noise into the amplified field. Louisell, Yariv, and Siegman [1] showed that for an optical parametric amplifier, the amplified field has an uncertainty product that is twice that of the coherent-state input field. Several workers [2] have shown in general that any linear optical amplifier must introduce a minimum amount of noise such that the signal-to-noise ratio of the amplified field is twice that of the input field. Their results were based on the fundamental consideration that the amplified field must satisfy the appropriate commutation relations. Hong, Friberg, and Mandel [3] showed that all nonclassical features (e.g., squeezing) of a field are lost if the field is amplified by more than a factor of 2. Collet and Walls [4] have also studied the effects of nonlinearity in a parametric amplifier as a result of pump depletion.

Mollow [5] first showed theoretically that the spectrum of a strongly driven two-level atomic system contained two spectral features that could lead to gain for a weak probe field. One of these features is at the Rabi sideband, and the other feature is at frequency close to the strong laser field and is known as stimulated Rayleigh scattering. This spectrum was observed by Wu *et al.* [6] using an atomic beam. In an atomic vapor cell where high number densities can be achieved, high amplification can occur in a relatively short interaction length. Gruneisen, MacDonald, and Boyd [7] realized as much as a 38-fold increase in the intensity of the probe wave. Several workers have used these gain features to construct dressed-state lasers [8,9]. The effects of Doppler broadening [10] as well as that of a strong probe wave have also been considered.

The theoretical treatments discussed in the preceding paragraph treat the atomic-field interaction through use of a semiclassical description. The interaction of a strong classical pump field and two weak sidebands through a forward four-wave-mixing process in an atomic vapor has been treated quantum mechanically by several workers [11,12]. A linear combination of the two sidebands was shown to result in a field that exhibited strong squeezing characteristics. The amplification of a weak probe wave as a result of the three-photon effect has also been treated quantum mechanically [13,14] in the context of a dressed-state laser.

In this paper, we formulate a quantum theory of two-beam coupling between a weak signal wave and a strong classical pump wave in a homogeneously broadened two-level system with the allowance of collisional broadening. Under conditions in which the signal wave experiences gain, we determine the amount of noise that is introduced by the amplification process. We find that gain as a result of the three-photon effect can have a noise level equal to that of an ideal optical amplifier only in the absence of collisions. The amplification process resulting from stimulated Rayleigh scattering is generally an inherently noisier process. We predict that the minimum noise level that can be achieved is four times that of the ideal amplifier, and unlike the three-photon case, it is achieved when the atomic system is collisionally broadened.

The theory of two-beam coupling in an atomic system can be developed through use of the general quantum-mechanical theory of multiwave mixing [12]. The geometry we treat is shown in Fig. 1, in which the pump field

$$\mathbf{E}_p(\mathbf{r},t)=\mathbf{A}_p e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}+\text{ c.c.} \tag{1}$$

is taken to be a classical quantity. The signal field is represented by the quantum-mechanical field operator

$$\hat{\mathbf{E}}_s(\mathbf{r},t)=\beta_s\boldsymbol{\epsilon}_s\hat{a}\, e^{i(\mathbf{k}_s\cdot\mathbf{r}-\omega_s t)}+\text{ H.c. }, \tag{2}$$

where $\hat{a}$ is the annihilation operator for the signal field, $\boldsymbol{\epsilon}_s$ is the signal-field unit vector, $\beta_s=-i(2\pi\hbar\omega_s/V)^{1/2}$, and V is the quantization volume. The interaction energy between the fields and the atomic system can be described by the interaction Hamiltonian

$$\hat{H}_1=-\int\hat{\mathbf{P}}(\mathbf{r})\cdot[\mathbf{E}_p(\mathbf{r},t)+\hat{\mathbf{E}}_s(\mathbf{r},t)]d^3r \;, \tag{3}$$

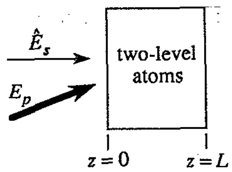

FIG. 1. Geometry for the two-beam coupling interaction.

where $\hat{\mathbf{P}}(\mathbf{r})$ is the polarization operator which is related to dipole moment operator $\hat{\mathbf{d}}^{(i)}$ of an atom at position $\mathbf{R}^{(i)}$ through the sum

$$\hat{\mathbf{P}}(\mathbf{r})=\sum_i \delta(\mathbf{r}-\mathbf{R}^{(i)})\hat{\mathbf{d}}^{(i)} . \tag{4}$$

The equation of motion for the density operator $\hat{\rho}$ for the coupled atom-field system is given by

$$\frac{\partial\hat{\rho}}{\partial t}=-\frac{i}{\hbar}[\hat{H}_{0A}+\hat{H}_{0F}+\hat{H}_1,\hat{\rho}]+\hat{L}_A\hat{\rho} , \tag{5}$$

where $\hat{H}_{0A}$ and $\hat{H}_{0F}$ are the unperturbed Hamiltonians for the atoms and the field, respectively, and where $[\hat{A},\hat{B}]=\hat{A}\hat{B}-\hat{B}\hat{A}$ denotes the commutator of any operators $\hat{A}$ and $\hat{B}$. The relaxation Liouville operator $\hat{L}_A$ includes the contributions from both spontaneous emission and atomic collisions. In the limit in which the frequencies of the pump and signal fields are close to the atomic resonance frequency, we can make the rotating-wave approximation, in which case the expression (3) for the interaction Hamiltonian becomes

$$\hat{H}_1=-\int \hat{\mathbf{P}}^-(\mathbf{r})\cdot[\mathbf{E}_p^+(\mathbf{r},t)+\hat{\mathbf{E}}_s^+(\mathbf{r},t)]d^3r+\text{ H.c. } , \tag{6}$$

where the superscript $+$ $(-)$ denotes the positive- (negative-) frequency parts of the field. We next calculate the equation of motion for the density operator for the field $\hat{\rho}_s$ by tracing the complete density operator $\hat{\rho}$ over the atomic variables and by using projection-operator techniques [15]. The signal field is assumed to be weak in comparison with the pump field, so that only terms up to second order in the signal field operator $\hat{a}$ are retained. The details of the derivation are discussed in Ref. [12], and we give only the result for the master equation for $\hat{\rho}_s$:

$$\frac{\partial\hat{\rho}_s}{\partial t}=-\frac{|\beta_s|^2N}{2\hbar^2}(\tilde{C}^{+-}(i\nu_s)[\hat{a}^\dagger,\{\hat{a},\hat{\rho}_s\}] +\tilde{Q}^{+-}(i\nu_s)[\hat{a}^\dagger,[\hat{a},\hat{\rho}_s]])+\text{ H.c. } , \tag{7}$$

where N is the number of atoms in the interaction region, $\nu_s=\omega-\omega_s$ is the detuning of the signal field from the pump field, and $\{\hat{A},\hat{B}\}=\hat{A}\hat{B}+\hat{B}\hat{A}$ denotes the anticommutator of operators $\hat{A}$ and $\hat{B}$. The quantities $\tilde{C}^{+-}$ and $\tilde{Q}^{+-}$ are given by

$$\tilde{C}^{+-}(i\nu_s)=\int_0^\infty d\tau\, e^{-i\nu_s\tau}C^{+-}(\tau) \tag{8a}$$

and

$$\tilde{Q}^{+-}(i\nu_s)=\int_0^\infty d\tau\, e^{-i\nu_s\tau}Q^{+-}(\tau) , \tag{8b}$$

where C^{+-} and Q^{+-} can be expressed in terms of the correlation functions of the polarization field as

$$C^{+-}(\tau)=\lim_{t\to\infty}\{[\langle\hat{P}_s^+(t+\tau)\hat{P}_s^-(t)\rangle -\langle\hat{P}_s^+(t+\tau)\rangle\langle\hat{P}_s^-(t)\rangle] -[\langle\hat{P}_s^-(t-\tau)\hat{P}_s^+(t)\rangle -\langle\hat{P}_s^-(t-\tau)\rangle\langle\hat{P}_s^+(t)\rangle]\} \tag{9a}$$

and

$$Q^{+-}(\tau)=\lim_{t\to\infty}\{[\langle\hat{P}_s^+(t+\tau)\hat{P}_s^-(t)\rangle -\langle\hat{P}_s^+(t+\tau)\rangle\langle\hat{P}_s^-(t)\rangle] +[\langle\hat{P}_s^-(t-\tau)\hat{P}_s^+(t)\rangle -\langle\hat{P}_s^-(t-\tau)\rangle\langle\hat{P}_s^+(t)\rangle]\} , \tag{9b}$$

where $\hat{P}_s^+=\hat{\mathbf{P}}_s^+\cdot\boldsymbol{\epsilon}_s^*$ and $\hat{P}_s^-=\hat{\mathbf{P}}_s^-\cdot\boldsymbol{\epsilon}_s$ are the operators associated with polarization that drives the signal field, and where it is understood that the expectation value of operators in (9) are all evaluated at the same point $\mathbf{r}$ inside the medium. The polarization correlation function C^{+-} corresponds to the susceptibility that is calculated from a semiclassical theory; the function Q^{+-} cannot be described semiclassically and corresponds to quantum fluctuations of the atomic system. The polarization correlation functions are calculated by solving the optical Bloch equations in steady state for an atom with a resonance frequency ω_0 located at $\mathbf{R}^{(i)}$. The resulting expressions for $\tilde{C}^{+-}$ and $\tilde{Q}^{+-}$ [Eqs. (8)] are

$$\tilde{C}^{+-}(i\delta_s)=-\mu^2T_2(2U_2\Phi_3-U_3\Phi_1) , \tag{10a}$$

$$\tilde{Q}^{+-}(i\delta_s)=-\mu^2[2U_1\Phi_1^2-U_2(1-2\Phi_1\Phi_2)+2U_3\Phi_1\Phi_3] , \tag{10b}$$

where $\mu=|\boldsymbol{\mu}|$ is the transition dipole matrix element and T_2 is the dipole dephasing time and where

$$\Phi_1=\frac{\Omega^*T_2(\delta-i)}{2P(0)} , \tag{11a}$$

$$\Phi_2=\Phi_1^* , \tag{11b}$$

$$\Phi_3=-\frac{(1+\delta^2)}{2P(0)} , \tag{11c}$$

$$U_1=\frac{\Omega^2T_1T_2}{2P(\delta_s)} , \tag{11d}$$

$$U_2=-\frac{2(\gamma_{12}\delta_s-i)(\delta_s-\delta-i)-|\Omega|^2T_1T_2}{2P(\delta_s)} , \tag{11e}$$

$$U_3=\frac{\Omega T_1(\delta_s-\delta-i)}{P(\delta_s)} , \tag{11f}$$

$$P(\delta_s)=(1+i\gamma_{12}\delta_s)[(1+i\delta_s)^2+\delta^2]+(1+i\delta_s)|\Omega|^2T_1T_2 , \tag{11g}$$

where $\delta=(\omega_0-\omega)T_2$ is the relative detuning of the pump field from the atomic resonance, $\delta_s=\nu_sT_2$ is the pump-probe detuning, $\Omega=2\boldsymbol{\mu}\cdot\mathbf{A}_p/\hbar$ is the Rabi frequency associated with the pump field, T_1 is the population relaxation time, and $\gamma_{12}=T_1/T_2$.

In order to make the comparison of our theory of amplification in a two-level atomic system with the phenomenological theories of quantum amplifiers [2], we convert the dynamical equation (7) for the density operator $\hat{\rho}_s$ for the signal field into the following Langevin equation for the annihilation operator $\hat{a}$,

$$\frac{d\hat{a}}{dz}=-\alpha\hat{a}+\hat{f}(z) , \tag{12a}$$

where

$$\alpha=\frac{\alpha_0\tilde{C}^{+-}}{2|\mu|^2T_2} \tag{12b}$$

is the absorption for the probe that is calculated from the semiclassical theory [7]. Here $\alpha_0=4\pi N\mu^2\omega nT_2/Vc$ is the unsaturated line-center absorption coefficient, and we set $t=nz/c$, where c/n is the phase velocity of the signal wave. The first- and second-order correlation functions of the Langevin noise operator that appears in Eq. (12a) are given by

$$\langle\hat{f}(z)\rangle=\langle\hat{f}^\dagger(z)\rangle=0 , \tag{13a}$$

$$\begin{aligned}\langle\hat{f}^\dagger(z)\hat{f}(z')\rangle&=\frac{\alpha_0}{2|\mu|^2T_2}\mathrm{Re}[\tilde{Q}^{+-}(i\delta_s)\\&\quad-\tilde{C}^{+-}(i\delta_s)]\delta(z-z')\\&=2D_{+-}\delta(z-z') ,\end{aligned} \tag{13b}$$

$$\begin{aligned}\langle\hat{f}(z)\hat{f}^\dagger(z')\rangle&=\frac{\alpha_0}{2|\mu|^2T_2}\mathrm{Re}[\tilde{Q}^{+-}(i\delta_s)\\&\quad+\tilde{C}^{+-}(i\delta_s)]\delta(z-z')\\&=2D_{-+}\delta(z-z') .\end{aligned} \tag{13c}$$

where $2D_{+-}$ and $2D_{-+}$ are elements of the diffusive matrix [12]. Since $\hat{f}$ is Gaussian in nature, all higher-order correlation functions can be determined from the first- and second-order correlation functions.

The Langevin equation (12a) should be compared with the ideal standard amplifier in which the probe field is propagating through a system of perfectly inverted atoms with no external coherent pump field acting on the system. One obtains Eqs. (12) and (13) but with the coefficients $\tilde{C}^{+-}$ and $\tilde{Q}^{+-}$ given by

$$Q^{+-}(i\nu_s)=-C^{+-}(i\nu_s)=T_2|\mu|^2/(1+i\delta_s) . \tag{14}$$

Thus for the ideal standard quantum amplifier, the diffusion coefficients D_{+-} and D_{-+} become

$$2D_{+-}=\frac{\alpha}{|\mu|^2T_2(1+\delta_s^2)} , \quad D_{-+}=0 . \tag{15}$$

For the case in which the atoms are not perfectly inverted, the diffusion coefficient $D_{-+}\neq0$, which leads to the standard amplifier imparting additional noise onto the amplified field. We will show that in certain cases the diffusion coefficient D_{-+} does vanish for a two-beam coupling amplifier which corresponds to the ideal amplifier limit.

Equation (12a) is integrated form $z=0$ to $z=L$, and the solution can be written in the form

$$\hat{a}(L)=g\hat{a}(0)+\hat{F} , \tag{16a}$$

where $g=\exp(-\alpha L)$ is the gain (or loss) experienced by the signal field and

$$\hat{F}=\int_0^L dz'\hat{f}(z')e^{-\alpha(L-z')} , \tag{16b}$$

is the noise operator for amplification in a strongly driven two-level atomic system. Note that $\hat{F}$ satisfies the commutation relation $[\hat{F},\hat{F}^\dagger]=1-|g|^2$, thus preserving the commutator for $\hat{a}$.

We define the following noise factor N_f to characterize the amplification process:

$$N_f=\frac{\langle\hat{F}^\dagger\hat{F}\rangle}{\langle[\hat{F},\hat{F}^\dagger]\rangle}=\frac{1}{2}\left[1-\frac{\mathrm{Re}(\tilde{Q}^{+-})}{\mathrm{Re}(\tilde{C}^{+-})}\right]\geq1 \quad \text{for } |g|>1 . \tag{17}$$

The noise factor in (17) is equal to unity only in the ideal-amplifier limit as described above in Eq. (14), and it depends only on the ratio of the polarization correlation function $\tilde{Q}^{+-}$ to the semiclassical susceptibility function $\tilde{C}^{+-}$. This ratio can be interpreted as the amount of quantum noise introduced by atomic fluctuations relative to the efficiency the amplification process.

The expectation value for the number of photons in the transmitted signal field is given by

$$\langle\hat{n}(L)\rangle=|g|^2\langle\hat{n}(0)\rangle+N_f(|g|^2-1) . \tag{18}$$

As seen by the above expression, the number of "noise" photons that are added to the output signal field is equal to the product of the noise factor N_f and $|g|^2-1$. Through the use of Eqs. (16) and (18), the signal-to-noise ratio of the photon number of the output field in the high-gain limit ($|g|\gg1$) is calculated to be

$$\begin{aligned}\left[\frac{S}{N}\right]^2_{\mathrm{out}}&=\frac{\langle\hat{n}(z)\rangle^2}{\langle\Delta\hat{n}^2(z)\rangle}\\&=\frac{\langle\hat{n}(0)\rangle^2+2N_f\langle\hat{n}(0)\rangle+N_f^2}{\langle\Delta\hat{n}^2(0)\rangle+(2N_f-1)\langle\hat{n}(0)\rangle+N_f^2} .\end{aligned} \tag{19}$$

For the case in which the input field is in a coherent state [i.e., $\langle\Delta\hat{n}^2(0)\rangle=\langle\hat{n}(0)\rangle$], the output signal-to-noise ratio becomes

$$\left[\frac{S}{N}\right]^2_{\mathrm{out}}=\frac{[\langle\hat{n}(0)\rangle+N_f]^2}{N_f[2\langle\hat{n}(0)\rangle+N_f]} . \tag{20}$$

In the limit in which $N_f=1$ and $\langle\hat{n}(0)\rangle\gg1$, we reach the result that the square of the signal-to-noise ratio of the output field is half that of the input field.

In Fig. 2 we plot of the gain $|g|^2$ experienced by the signal field as a function of pump-signal detuning for the case $\Omega T_2=5$, $\delta=-5$, $\alpha_0L=300$, and $T_2/2T_1=0.3$. The two gain mechanisms are known as the three-photon (TP) effect and stimulated Rayleigh (SR) scattering. We will study the quantum-noise properties of each one separately.

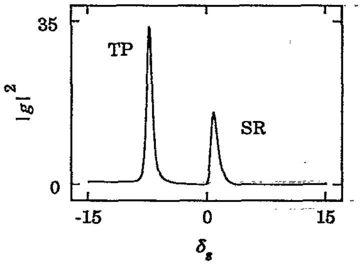

FIG. 2. The gain for the probe field as a function of the pump-probe detuning δ_s for the case $\Omega T_2=5$, $\delta=-5$, $a_0L=300$, and $T_2/2T_1=0.3$. The resulting gain features are known as the three-photon (TP) effect and stimulated Rayleigh (SR) scattering.

We first consider amplification as result of the three-photon effect. In all the ensuing analysis for the TP feature, we choose the pump-probe detuning to be at the peak of the gain curve which occurs at the generalized Rabi frequency such that $\delta_s=\delta[1+(\Omega T_2/\delta)^2]^{1/2}$. We show the effect that collisions have on the TP gain feature by plotting the gain $|g|^2$ [Fig. 3(a)] and the noise factor N_f [Fig. 3(b)] as functions of the ratio $T_2/2T_1$ for various values of the pump detuning for the case $\Omega T_2=100$ and $\alpha_0L=200$. For a particular value of δ, the highest gain always occurs in the radiatively broadened limit at $T_2/2T_1=1$. In general, we find that the highest gain for all ratios of $T_2/2T_1$ occurs approximately at a pump detuning of $\delta=\Omega T_2/3$. In addition, the noise factor is always minimized in the radiatively broadened limit [see Fig. 3(b)], and in this limit the amplification process is close to that of the ideal amplifier for larger pump detunings.

The increase in the noise factor for the TP feature as the atomic system becomes collisionally broadened is primarily a result of a decrease in the efficiency of the gain process as $T_2/2T_1$ decreases rather than a result of the collisions introducing noise to the field through atomic fluctuations. This point is illustrated in Fig. 4, where we plot as a function of $T_2/2T_1$ the quantities $\tilde{C}^{+-}$, which is a measure of how efficient the gain process is, and $\tilde{Q}^{+-}$, which represents the contribution from the atomic fluctuations to the noise in the signal field for the curves in Fig. 3 that corresponds to $\delta=30$. We see that the quantity $\tilde{Q}^{+-}$ is relatively constant, whereas the value of $\tilde{C}^{+-}$ drops by nearly two orders of magnitude as the effect of collisions are increased. This decrease in $\tilde{C}^{+-}$ accounts for the corresponding increase in the noise factor in Fig. 3(b). Thus, increasing both Rabi frequency and the pump detuning increases the efficiency of the amplification process, and correspondingly lowers the noise factor.

We next consider amplification through use of stimulated Rayleigh scattering. The peak value of the gain occurs at a value of the pump-probe detuning ν_s between $1/T_1$ and $1/T_2$; the exact value depends on the Rabi frequency, on the pump detuning, and on the ratio $T_2/2T_1$. In all our ensuing analysis of gain at the SR feature, we adjust the pump-probe frequency such that the gain is always maximized. The gain $|g|^2$ and the noise factor N_f are plotted for various values of the pump detuning as a function of $T_2/2T_1$ in Figs. 5(a) and 5(b), respectively, for the case $\Omega^2T_1T_2=50$ and $\alpha_0L=300$. We choose to keep the intensity $I_p=\Omega^2T_1T_2$ constant (as opposed to ΩT_2 for the TP effect) as $T_2/2T_1$ is varied, since the gain from SR scattering is a result of population oscillations that occur due to incoherent population transfer. We find that as the amount of collisional broadening increases, the gain [Fig. 5(a)] for the signal wave increases [7]. In addition, Fig. 5(b) shows that collisional broadening actually reduces the noise factor N_f. The maximum gain is achieved for a $\delta=3$, whereas the lowest value of N_f occurs for a $\delta=10$. We find in general that for a particular value of the pump intensity I_p, the most efficient amplification occurs at a pump detuning of $\delta^2\approx I_p/4$, and the least amount of noise with a relatively high amplification efficiency occurs at a pump detuning of $\delta^2\approx I_p$. We have not found, for any values of the parame-

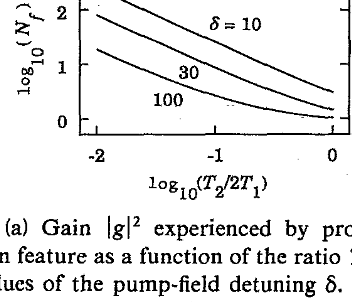

FIG. 3. (a) Gain $|g|^2$ experienced by probe field for the three-photon feature as a function of the ratio $T_2/2T_1$ for three different values of the pump-field detuning δ. The presence of collisions (i.e., $T_2/2T_1<1$) always leads to a reduced value of the gain. (b) The corresponding noise factor N_f for each of the cases in (a). Only in the absence of collisions (i.e., $T_2/2T_1=1$) does the amount of noise in the amplified field approach the value for the ideal quantum amplifier.

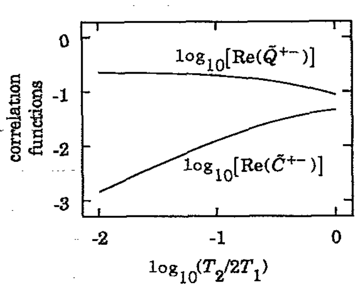

FIG. 4. The real part of the polarization correlation functions $\tilde{Q}^{+-}$ and $\tilde{C}^{+-}$ for the three-photon feature are plotted as a function of the ratio $T_2/2T_1$ for the curves in Fig. 3 that correspond to $\delta=30$.

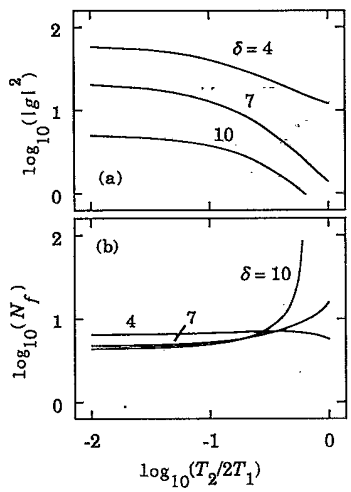

FIG. 5. (a) Gain $|g|^2$ experienced by probe field for the stimulated Rayleigh feature as a function of ratio $T_2/2T_1$ for three different values of the pump-field detuning δ. The presence of collisions (i.e., $T_2/2T_1<1$) leads to an increase in the value of the gain. (b) The corresponding noise factor N_f for each of the cases in (a). The amount of noise in the amplified field is minimized only in the limit in which the atoms are collisionally broadened (i.e., $T_2/2T_1 \ll 1$).

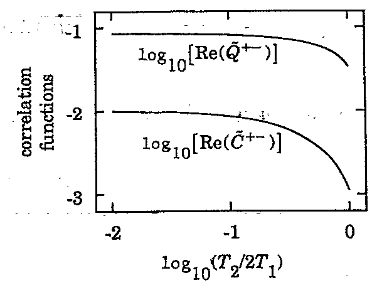

FIG. 6. The real part of the polarization correlation functions $\tilde{Q}^{+-}$ and $\tilde{C}^{+-}$ for the stimulated Rayleigh feature are plotted as a function of the ratio $T_2/2T_1$ for the curves in Fig. 5 that correspond to $\delta=7$.

ters for the pump field and the atomic system, a regime in which amplification as a result of SR scattering can operate at the ideal amplifier limit. The minimum value of the noise factor that can be reached is $N_f \approx 4$.

The decrease in the noise factor as the amount of collisional broadening is increased is a result of the increase in the efficiency of the SR gain. This effect is even more pronounced for SR gain than for TP gain, since the collisions increase the amount of noise into the signal field due to atomic fluctuations. We illustrate this point by plotting (Fig. 6) the quantities $\tilde{C}^{+-}$ and $\tilde{Q}^{+-}$ as a function of $T_2/2T_1$ for the curves in Fig. 5 that correspond to $\delta=7$. Although $\tilde{Q}^{+-}$ increases with the level of collisional broadening, $\tilde{C}^{+-}$ increases by a greater amount, which leads to a corresponding decrease in the noise factor N_f.

We have also studied the dependence of the noise factor N_f as a function of pump-probe detuning for both the TP and the SR features. We find that the noise factor is minimized at pump-probe detunings that correspond to the maximum gain in all cases. As in the cases discussed above, this behavior can be easily understood from the fact that the efficiency of the gain process relative to the amount of quantum noise present is maximum at these detunings.

In conclusion, we have determined the quantum-noise limits on amplification by two-beam coupling in an atomic vapor. Amplification via the three-photon effect can lead to a noise level that is equal to that of an ideal optical amplifier only in the limit in which the atoms are radiatively broadened. We find that stimulated Rayleigh scattering is an inherently noisier gain process, and the lowest noise level that can be achieved is predicted to be four times the ideal amplifier limit. Surprisingly, this noise level is minimized only under conditions of strong collisional broadening.

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