### Optical phase conjugation of nonclassical fields

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We investigate theoretically the quantum-noise properties of optical phase conjugation via four-wave mixing of nonclassical signal fields. We show that while the field generated at a phase-conjugate mirror (PCM) can be in a nonclassical state, the generated field is not necessarily useful within the context of phase conjugation. For the case in which a quadrature-squeezed signal field is incident at a PCM, we determine that the signal-to-noise ratio of one of the quadrature components of the generated field can approach the signal-to-noise ratio of the corresponding quadrature component of the signal field as the amount of squeezing of the vacuum field injected at the rear of the PCM is increased. For the case in which the signal field is amplitude squeezed, we determine that the signal-to-noise ratio of the conjugate field is always much smaller than that of the signal field. These results demonstrate that the phase-conjugation process can preserve the desirable quantum-noise properties of quadrature-squeezed fields but not those of amplitude-squeezed fields.

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#### I. INTRODUCTION

The nonlinear optical technique of optical phase conjugation is useful for correcting distortions imparted on an optical wave front [1]. Recent quantum-mechanical treatments [2-5] of phase conjugation have shown that noise is inherent in the phase-conjugation process and that the conjugate-reflected field is typically noisier than the incident signal field. For the case of optical phase conjugation by four-wave mixing, the noise can be attributed to a vacuum field that is incident at the rear port of a phase-conjugate mirror (PCM) and that is amplified as a result of the four-wave mixing process [6], as shown in Fig. 1. Previous work [5] has shown that for these conditions the conjugate field has a positive, semidefinite phase-space density, which implies that the conjugate field cannot exhibit any nonclassical features. These results have implications for performing phase conjugation of quadrature-squeezed fields, amplitude-squeezed fields, and fields having only a few photons per mode [7,8]. More recently, Bajer and Perina [9] have shown that, for the case in which a quadrature-squeezed field or a Fock state field is incident at the rear of the PCM, the generated field can exhibit either quadrature squeezing or amplitude squeezing, respectively. Demonstrating that the generated field is nonclassical does not guarantee, however, that it is in fact the desired phase conjugate of the signal field. For example, in the limit in which the phaseconjugate reflectivity vanishes, the noise and wave-front properties of the generated field are in fact those of the

In this paper, we analyze the ability of a PCM to produce an ideal phase conjugate of a quadrature-squeezed field and an amplitude-squeezed field. We show that the field generated by the PCM can also be in a nonclassical state, as obtained previously [9], but that this field is not necessarily the ideal phase conjugate of the nonclassical signal field. We determine the quantum-noise properties of the generated field for conditions under which the field injected at the rear of the PCM is in a vacuum state, a quadrature-squeezed vacuum state, or a Fock state. For the case in which the signal field is in a quadraturesqueezed state, the generated field preserves the signalto-noise ratio of one of the quadrature components of the signal field only when the corresponding quadrature component of the rear field is perfectly quadrature squeezed. For the case in which the signal field is in an amplitudesqueezed state, the signal-to-noise ratio of the generated field is typically much less than that of the signal field. We conclude that the process of optical phase conjugation can preserve the signal-to-noise ratio of only those signal fields that are quadrature squeezed.

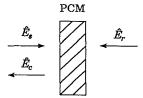


FIG. 1. Schematic illustration of the signal field  $\hat{E}_r$ , the rear field  $\hat{E}_r$ , and the conjugate field  $\hat{E}_c$  interacting via four-wave mixing at a phase-conjugate mirror (PCM).

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rear field rather than those of the conjugate of the signal field.

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# II. PHASE CONJUGATION OF QUADRATURE-SQUEEZED FIELDS

We consider a single-mode signal field  $\hat{E}_s$  incident at the front of a four-wave-mixing PCM. As a result of the four-wave-mixing process, the signal field is coupled to the rear field  $\hat{E}_r$ , incident on the rear port of the PCM to produce the conjugate-reflected field  $\hat{E}_c$ , as shown in Fig. 1. For the case of degenerate four-wave mixing, the two modes incident at the PCM are represented in the scalar approximation by the expressions

$$\hat{E}_s(\vec{r},t) = C\hat{a}_s \exp[i(\vec{k}\cdot\vec{r} - \omega t)] + \text{H.a.} , \qquad (1a)$$

$$\hat{E}_r(\vec{r},t) = C^* \hat{a}_r \exp[i(-\vec{k} \cdot \vec{r} - \omega t)] + \text{H.a.} , \qquad (1b)$$

where the constant  $C = -i(2\pi\hbar\omega/V)^{1/2}$  and V is the quantization volume. For the case of nearly degenerate four-wave mixing, the two fields no longer have the same frequencies; nevertheless, the following analysis can still be applied. A quadrature-squeezed signal field typically used in ultraprecision measurements is a superposition of a signal contribution and a squeezed vacuum background. For example, the signal contribution can be produced by a weakly modulated laser field [10]. For these conditions, the annihilation operator  $\hat{a}_s$  of the signal field is described by the expression

$$\hat{a}_s = \gamma \hat{a}_\alpha + \sqrt{1 - \gamma^2} \hat{a}_b , \qquad (2)$$

where  $\hat{a}_{\alpha}$  is the photon annihilation operator associated with a strong coherent laser field, and  $\hat{a}_b$  is the photon annihilation operator of the background vacuum field that may be quadrature squeezed. The parameter  $\gamma$  is much less than unity and represents the fraction of the strong laser field that is coupled into the signal field mode to create the signal contribution. The expectation values  $\langle \hat{a}_{\alpha} \rangle$  and  $\langle \hat{a}_{s} \rangle$  of the photon annihilation operators of the laser field and signal field are then given by the relations

$$\langle \hat{a}_{\alpha} \rangle = \alpha = |\alpha| \exp[i\theta_{\alpha}],$$
 (3a)

$$\langle \hat{a}_s \rangle = \alpha_s = |\alpha_s| \exp[i\theta_s] = \gamma \alpha$$
, (3b)

where the parameters  $\theta_{\alpha}$  and  $\theta_{s}$  are the phases of the laser field and signal field, respectively.

The field  $\hat{E}_c$ , generated at the PCM, is represented in the scalar approximation by the expression

$$\hat{E}_c(\vec{r},t) = C^* \hat{a}_c \exp[i(-\vec{k} \cdot \vec{r} - \omega t)] + \text{H.a.} , \qquad (4)$$

where  $\hat{a}_c$  denotes the photon annihilation operator of the conjugate field. The photon operators  $\hat{a}_s$ ,  $\hat{a}_{\alpha}$ ,  $\hat{a}_r$ , and  $\hat{a}_c$  satisfy the canonical boson commutation relations

$$[\hat{a}_i, \hat{a}_i^{\dagger}] = 1 , \qquad (5)$$

where  $j = s, \alpha, r, c$ . The signal field operators and rear field operators are assumed to commute. It has been shown [6] that for a field generated at a four-wave-mixing PCM, the annihilation operator of the field is related to the creation and annihilation operators of the signal field and rear field as

$$\hat{a}_c = v_{\rm pc} \hat{a}_s^{\dagger} + \mu_{\rm pc} \hat{a}_r , \qquad (6)$$

where  $v_{\rm pc}$  is the phase-conjugate amplitude reflectivity, and where the relation  $|\mu_{\rm pc}|^2 - |v_{\rm pc}|^2 = 1$  ensures that  $\hat{a}_c$  satisfies Eq. (5). The expectation value of the photon annihilation operator of the generated field is then  $\langle \hat{a}_c \rangle = v_{\rm pc} \langle \hat{a}_s^{\dagger} \rangle + \mu_{\rm pc} \langle \hat{a}_r \rangle$ . The phase-conjugate reflectivity  $R_{\rm pc}$  is  $R_{\rm pc} = |v_{\rm pc}|^2$ .

The quadrature components  $\hat{a}_{j1}$  and  $\hat{a}_{j2}$  of any of the field operators are given by the relations

$$\hat{a}_{j1} = \frac{(\hat{a}_j + \hat{a}_j^{\dagger})}{2} , \qquad (7a)$$

$$\hat{a}_{j2} = \frac{(\hat{a}_j - \hat{a}_j^{\dagger})}{2i} , \tag{7b}$$

where j = s, b, r, c. We allow either the background mode of the signal field or rear field to be in a quadrature-squeezed vacuum state by representing the annihilation and creation operators of these fields with the relations [4]

$$\hat{a}_k = \mu_k \hat{b}_k + \nu_k \hat{b}_k^{\dagger} \,, \tag{8}$$

where k = b, r,  $|\mu_k|^2 - |\nu_k|^2 = 1$ , and  $\hat{b}_k$  is the annihilation operator of the appropriate vacuum field mode. In the following analysis, we assume that the parameters  $\mu_k$  and  $\nu_k$  are real quantities, to ensure that the noise in each quadrature of the vacuum modes contributes only to the noise in the corresponding quadrature of the conjugate mode.

We identify the noise  $N_{k1}$  and  $N_{k2}$  in each quadrature component of the two vacuum modes with the variance of the corresponding quadrature component, such that

$$N_{k1} = \langle (\Delta \hat{a}_{k1})^2 \rangle = \frac{1}{4m_k} , \qquad (9a)$$

$$N_{k2} = \langle (\Delta \hat{a}_{k2})^2 \rangle = \frac{\eta_k}{4}$$
, (9b)

where the noise parameter  $\eta_k$  is given by the relation

$$\eta_k = \frac{\mu_k - \nu_k}{\mu_k + \nu_k} \quad (10)$$

The noise parameter  $\eta_k$  can take on values between zero and unity, which correspond to the cases for which the field is in a perfectly quadrature-squeezed state or coherent state, respectively. As the value of the noise parameter is decreased from unity, Eqs. (9a) and (9b) indicate that the noise in the first quadrature is increased above the shot-noise limit of  $\frac{1}{4}$ , while the noise in the second quadrature is decreased below the shot-noise limit

We now consider the noise properties of the signal field and generated field and determine the conditions under which the optical phase-conjugation process preserves the noise properties of the signal field. We assume that the phase-conjugate amplitude reflectivity  $v_{\rm pc} = \sqrt{R_{\rm pc}}$  and the four-wave-mixing amplitude gain  $\mu_{\rm pc} = \sqrt{1 + R_{\rm pc}}$  are also real quantities, to ensure that the noise in each

quadrature of either input mode contributes only to the noise in the corresponding quadrature of the generated

The signal-to-noise ratios  $\mathcal{R}_{jl}$  for the quadrature components of both fields are defined to be

$$\mathcal{R}_{jl} = \frac{\langle \hat{a}_{jl} \rangle^2}{\langle (\Delta \hat{a}_{jl})^2 \rangle} , \qquad (11)$$

where j=s,c and l=1,2. Using Eqs. (2), (3), and (9)-(11), we find that the signal-to-noise ratios of each of the quadrature components of the signal field are

$$\mathcal{R}_{s1} = 4\eta_b |\alpha_s|^2 \cos^2 \theta_s , \qquad (12a)$$

$$\mathcal{R}_{s2} = \frac{4}{\eta_b} |\alpha_s|^2 \sin^2 \theta_s , \qquad (12b)$$

where we have used the assumption that the parameter  $\gamma$ of Eq. (2) is much less than unity. Equations (12a) and (12b) show that the signal-to-noise ratio in the second quadrature is enhanced when the quadrature squeezing of the background vacuum field is increased. For the case in which the signal field is perfectly quadrature squeezed, the signal-to-noise ratio of the second (first) quadrature diverges (vanishes).

Using Eq. (6), the signal-to-noise ratios for the quadrature components of the generated field are given by the expressions

$$\mathcal{R}_{c1} = \frac{4\eta_r \eta_b M_{\rm pc} |\alpha_s|^2 \cos^2 \theta_s}{\eta_b + M_{\rm pc} \eta_r} , \qquad (13a)$$

$$\mathcal{R}_{c1} = \frac{4\eta_r \eta_b M_{\rm pc} |\alpha_s|^2 \cos^2 \theta_s}{\eta_b + M_{\rm pc} \eta_r} , \qquad (13a)$$

$$\mathcal{R}_{c2} = \frac{4M_{\rm pc} |\alpha_s|^2 \sin^2 \theta_s}{\eta_r + M_{\rm pc} \eta_b} , \qquad (13b)$$

where the parameter  $M_{pc}$  is given by  $M_{pc} = R_{pc}/(1+R_{pc})$ .

We now calculate the information transfer coefficients  $T_l$  (l=1,2) [11] for both quadrature components of the generated field to characterize the extent to which they retain the quantum-noise properties of the signal field. From Eqs. (12) and (13), we obtain the expressions

$$T_1 = \frac{\mathcal{R}_{c1}}{\mathcal{R}_{s1}} = \frac{M_{\rm pc}\eta_r}{\eta_b + M_{\rm pc}\eta_r}, \qquad (14a)$$

$$T_2 = \frac{\mathcal{R}_{c2}}{\mathcal{R}_{s2}} = \frac{M_{\rm pc}\eta_b}{\eta_r + M_{\rm pc}\eta_b}$$
 (14b)

Equations (14a) and (14b) show that the transfer coefficients in the two quadrature components are always equal to or less than unity. As the amount of squeezing in the rear vacuum field is increased, the transfer coefficient  $T_1$  for the first quadrature is decreased, while the transfer coefficient  $T_2$  for the second quadrature is increased. For the case in which the rear vacuum field is perfectly quadrature squeezed, the transfer coefficient for the first quadrature vanishes, and the transfer coefficient for the second quadrature achieves the maximum value of unity, since no noise is added to this quadrature. These results indicate that when the rear vacuum field is perfectly quadrature squeezed, optical phase conjugation can

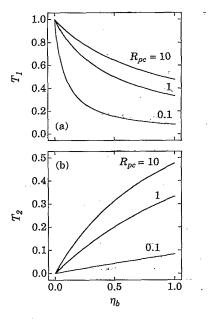


FIG. 2. Information transfer coefficients  $T_1$  (a) and  $T_2$  (b) of the two quadrature components of the conjugate field as functions of the background field noise parameter  $\eta_b$  for different values of the PCM reflectivity  $R_{\rm pc}$ , for the case in which the rear field is in a vacuum state.

preserve the noise properties of the second quadrature component of the signal field.

For the case in which the field  $\hat{E}_r$  injected into the rear port of the PCM is in an ordinary vacuum state, the information transfer coefficients for both quadrature components become

$$T_1 = \frac{R_{\rm pc}}{R_{\rm pc} + (1 + R_{\rm pc})\eta_b}, \qquad (15a)$$

$$T_2 = \frac{R_{\rm pc} \eta_b}{(1 + R_{\rm pc}) + R_{\rm pc} \eta_b} \tag{15b}$$

Figures 2(a) and 2(b) are plots of these transfer coefficients as functions of the noise parameter  $\eta_b$  for different values of the phase-conjugate reflectivity  $R_{pc}$ . The signal-to-noise ratio of the initially squeezed quadrature is always degraded, and this degradation is most pronounced for large squeezing  $(\eta_b \ll 1)$  of the signal field. These results are consistent with those of previous work [5], which showed that the conjugate field cannot exhibit nonclassical features such as squeezing when the rear field is in an ordinary vacuum state.

### III. PHASE CONJUGATION OF AMPLITUDE-SQUEEZED FIELDS

We now consider the effect of optical phase conjugation on a general single-mode signal field  $\hat{E}_s$  that is amplitude squeezed. The expressions for the signal field, rear field, and conjugate-reflected field interacting at a PCM

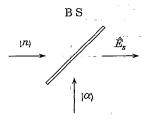


FIG. 3. Production of an amplitude-squeezed signal field  $\hat{E}_s$  by combining a Fock state  $|n\rangle$  and a coherent state  $|\alpha\rangle$  at a beam splitter (BS) having a reflection coefficient R.

are still given by Eqs. (1a), (1b), and (4), respectively. We create an amplitude-squeezed signal field by combining a Fock state field with a coherent-state field at a beam-splitter with reflection coefficient R, as shown in Fig. 3. The degree of amplitude squeezing of the field can be varied continuously by changing the value of the beam-splitter reflection coefficient. The annihilation operator  $\hat{a}_s$  of the amplitude-squeezed signal field is given by the expression

$$\hat{a}_s = \sqrt{1 - R} \, \hat{a}_n + i \sqrt{R} \, \hat{a}_\alpha \,\,, \tag{16}$$

where  $\hat{a}_n$  is the photon annihilation operator of the Fock state field  $|n\rangle$ , and  $\hat{a}_{\alpha}$  is the photon annihilation operator of the coherent-state field  $|\alpha\rangle$ . The expectation value of the photon-number operator of the signal field  $\langle \hat{n}_s \rangle$  is

$$\langle \hat{n}_s \rangle = (1 - R) \langle \hat{n}_n \rangle + R \langle \hat{n}_\alpha \rangle , \qquad (17)$$

where  $\hat{n}_n$  and  $\hat{n}_\alpha$  are the photon-number operators for the Fock state field and the coherent-state field, respectively. By setting the expectation values of both photon number operators  $\hat{n}_n$  and  $\hat{n}_\alpha$  equal to  $n_s = \langle \hat{n}_s \rangle$ , the photon number of the amplitude-squeezed state is fixed at  $n_s$ , regardless of the value of the beam splitter reflection coefficient R.

The noise  $N_s$  in the photon number of the signal field is identified with the variance of the signal-field photon-number operator such that

$$N_{\rm s} = \langle (\Delta \hat{n}_{\rm s})^2 \rangle = 2n_{\rm s}^2 R (1-R) + n_{\rm s} R (2-R)$$
 (18)

We further quantify the photon statistics of the signal field by calculating the Mandel Q parameter [12]  $Q_s$  for the amplitude-squeezed signal field, which is given by the expression

$$Q_{s} = \frac{N_{s}}{\langle \hat{n}_{s} \rangle} - 1 = R (1 - R)(1 + 2n_{s}) + R - 1 .$$
 (19)

The parameter  $Q_s$  can take on any value between zero and -1. For the case in which the parameter  $Q_s$  is less than zero, the field exhibits sub-Poissonian statistics and

is amplitude squeezed. The signal-to-noise ratio  $\mathcal{R}_s$  for the photon number in the signal field can be expressed in terms of the parameter  $Q_s$  as

$$\mathcal{R}_s = \frac{n_s^2}{\langle (\Delta \hat{n}_s)^2 \rangle} = \frac{n_s}{Q_s + 1} . \tag{20}$$

We now characterize the noise properties of the photon number of the generated field by calculating the corresponding Q parameter  $Q_c$ , the signal-to-noise ratio  $\mathcal{R}_c$ , and the information transfer coefficient  $T_c$ . The expectation value of the number of photons  $n_c$  in the generated field is nonzero, even when the signal field and rear field are both in ordinary vacuum states [5]. It is therefore physically meaningful to define the signal-to-noise ratio  $\mathcal{R}_c$  and the transfer coefficient  $T_c$  to be

$$\mathcal{R}_c = \frac{(n_c^{\text{on}} - n_c^{\text{off}})^2}{N_c}$$
 (21)

and

$$T_c = \frac{\mathcal{R}_c}{\mathcal{R}_c} , \qquad (22)$$

where  $n_c^{\text{on}}$  is the expected value of the photon number in the generated field when the signal field is "on," and  $n_c^{\text{off}}$  is the expected value of the photon number when the signal field is "off" (i.e., is in the ordinary vacuum state). Using Eqs. (6) and (16), we calculate the Q parameter, the signal-to-noise ratio  $\mathcal{R}_c$ , and the information transfer coefficient  $T_c$  to be

$$Q_c = \frac{N_c}{R_{pc}(n_s + 1) + (R_{pc} + 1)n_r} - 1, \qquad (23a)$$

$$\mathcal{R}_c = \frac{R_{\text{pc}}^2 n_s^2}{N_c} \,, \tag{23b}$$

and

$$T_c = \frac{n_s R_{\rm pc}^2(Q_s + 1)}{N_c} \,, \tag{23c}$$

where  $n_r = \langle \hat{n}_r \rangle$  is the expectation value of the number of photons in the rear field and  $N_c = \langle (\Delta \hat{n}_c)^2 \rangle$  is the noise in the photon number of the generated field.

For the case in which the rear field  $\hat{E}_r$  is in a Fock state  $|n_r\rangle$ , the noise in the generated field is given by the expression

$$N_c = R_{pc}(1 + R_{pc})[1 + n_s + n_r + 2n_s n_r] + R_{pc}^2 n_s (Q_s + 1) .$$
(24)

In this case, the expressions for the Q parameter, the signal-to-noise ratio, and the transfer coefficient become

$$Q_{c} = \frac{R_{\text{pc}}^{2} + n_{r}(R_{\text{pc}}^{2} - 1) + 2R_{\text{pc}}n_{s}n_{r}(1 + R_{\text{pc}}) + n_{s}R_{\text{pc}}^{2}(Q_{s} + 2)}{n_{r}(1 + R_{\text{pc}}) + R_{\text{pc}}(n_{s} + 1)},$$
(25)

$$\mathcal{R}_c = \frac{R_{pc} n_s^2}{(1 + R_{pc})(1 + n_r + n_s + 2n_s n_r) + R_{pc} n_s (Q_s + 1)},$$
(26)

and

$$T_c = \frac{R_{\rm pc} n_s(Q_s + 1)}{(1 + R_{\rm pc})(1 + n_r + n_s + 2n_s n_r) + R_{\rm pc} n_s(Q_s + 1)}$$
 (27)

For the case in which the field injected at the rear of the PCM is in a vacuum state (i.e.,  $n_r = 0$ ), Eq. (25) indicates that the photon statistics of the generated field are always super-Poissonian for an amplitude-squeezed signal field, since the parameter  $Q_c$  is always greater than zero. This result is consistent with previous work [5], which demonstrated that under these conditions the generated field cannot exhibit any nonclassical features. Accordingly, the information transfer coefficient  $T_c$  is always less than unity and reaches a maximum value of one-half in the limit in which  $Q_s = 0$  and  $n_r = 0$  and the number of photons  $R_{pc}n_s$  expected in the classical limit in the conjugate field is large. Figure 4 is a plot of the signal-to-noise ratio  $\mathcal{R}_c$  as a function of the phase-conjugate reflectivity  $R_{\rm pc}$  for a signal field having various values of  $Q_s$  and  $n_s = 5$ . The signal-to-noise ratio is maximized for the case in which the signal field is in a Fock state ( $Q_s = -1$ ).

Figures 5(a) and 5(b) are plots of the parameter  $Q_c$  and information transfer coefficient  $T_c$  as functions of the phase-conjugate reflectivity  $R_{\rm pc}$  for different values of the photon number  $n_r$  in the rear field, where we have set  $n_s = 5$  and  $Q_s = -0.5$ . We have allowed for a nonzero number of photons in the rear field to compare our results with those of Bajer and Perina [9]. Figure 5(a) shows that the photon statistics of the generated field are sub-Poissonian when the phase-conjugate reflectivity  $R_{pc}$ is less than unity and when the number of photons in the rear field is greater than zero. This result of sub-Poissonian statistics, which was also obtained previously [9], requires careful interpretation, since it is precisely in this limit that the photon statistics of the generated field are dominated by the statistics of the rear Fock state (for which  $Q_r = -1$ ) rather than by the statistics of the amplitude-squeezed signal field. Furthermore, the wavefront properties of the field produced by the PCM are also associated in this limit with those of the rear field

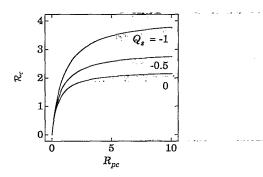


FIG. 4. Signal-to-noise ratio  $\mathcal{R}_c$  of the generated field as a function of the phase-conjugate reflectivity  $R_{\rm pc}$ , for different photon statistics in the amplitude-squeezed signal field. The number of photons in the signal field  $n_s$  and in the rear field  $n_r$  are equal to 5 and 0, respectively.

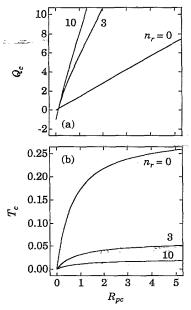


FIG. 5. Parameter  $Q_c$  (a) and information transfer coefficient  $T_c$  (b) of the conjugate-reflected field as a function of the phase-conjugate reflectivity  $R_{\rm pc}$  for different values of the photon number  $n_r$  in the rear field. The number of photons  $n_s$  in the signal field is 5, and the parameter  $Q_s$  of the signal field is -0.5.

rather than with those of the desired conjugate of the signal field. Alternatively, an appropriate measure of the extent to which the generated field can in fact retain the desirable quantum-noise properties of the signal field is the information transfer coefficient  $T_c$  of the photon number of the generated field. Figure 5(b) shows that the transfer coefficient is always much less than unity for conditions in which the signal field is in an amplitudesqueezed state. In fact, the transfer coefficient is optimized when no photons are injected into the rear port of the PCM. This result signifies that under conditions in which the signal field is in an amplitude-squeezed state, the signal-to-noise ratio of the field generated through phase conjugation can never equal that of the signal field. Therefore, although the generated field can exhibit nonclassical statistics, this field is not necessarily useful within the context of optical phase conjugation.

### IV. SUMMARY AND CONCLUSIONS

We have calculated the quantum-noise properties of fields generated by optical phase conjugation via four-wave mixing from nonclassical signal fields under conditions in which the field injected at the rear of the PCM can be in a vacuum state, a quadrature-squeezed vacuum state, or a Fock state. We have used the information transfer coefficient as a measure of the extent to which the field generated at the PCM is the desired phase conjugate of the nonclassical signal field. Specifically, we find that, while the generated field may be nonclassical, this field is not necessarily useful within the context of optical phase conjugation in the sense of being the desired ideal conjugate of the signal field. For the case in which the

signal field is in a quadrature-squeezed state, we have determined that the signal-to-noise ratio of one of the quadrature components of the generated field can preserve the signal-to-noise ratio of the signal field. For the case in which the signal field is in an amplitude-squeezed state, we have determined that the signal-to-noise ratio of the generated field is always less than that of the signal field.

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